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THE

PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND
THEORETICAL PHYSICS

CONDUCTED BY

EDWARD L. NICHOLS, ERNEST MERRITT,
AND FREDERICK BEDELL

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HENRY A. ROWLAND

Volume XIII. *July, 1901.* *Number 1.*

THE

PHYSICAL REVIEW.

THE EFFECT OF GRAVITY AND PRESSURE ON ELECTROLYTIC ACTION.

By Rolla R. Ramsey.

WHEN a current of electricity is passed through an electrolytic cell the anode is dissolved while the cathode is plated. If the cell is made in the form of a tube, with the electrodes in the ends, so mounted as to rotate from a horizontal to a vertical position about a horizontal axis, the law of conservation of energy would lead us to expect a greater consumption of energy in the case in which the current passes up through the tube than when the tube is rotated 90° making the flow along a horizontal plane. The difference in the two cases of the amount of energy consumed in a unit of time will be

$$m\,g\,h,$$

where m is the mass of metal transferred in unit of time, g the force of gravity, h the distance between the electrodes.

If the current strength is kept a constant value of 1 C.G.S. unit, the difference of the potential differences of the two ends in the two cases is

$$E = mgh,$$

where m is 10 times the electro-chemical equivalent of the cation. The above equation can also be put in the form

$$E = \text{Kat}\,gqh,$$

where Kat is the equivalent weight of the cation and q is 10 times the electro-chemical equivalent of hydrogen. Since, as was first shown by Hittorf,[1] when the cations move with the current the anions move in the opposite direction with a certain definite velocity, the velocity ratios being expressed by the migration constant, n: we have,

$$E = [\text{Kat}\,(1-n) - n\,\text{An}]gqh,$$

or

$$E = [\text{Kat} - N(\text{Kat} - \text{An})]gqh,$$

where An is the equivalent weight of the anion.

Since E can be regarded as a counter E.M.F. a small current should be detected by short-circuiting the voltameter through a delicate galvanometer.

HISTORICAL.

The Effect of Gravity.—To Maxwell belongs the honor of having first shown that such an electro-motive force should exist.[2] In 1878 Maxwell communicated to Nature,[3] a letter written to himself by F. J. Pirani, in which Pirani stated that he had detected such an electro-motive force. In an added note Maxwell spoke of his own attempt to repeat the experiment but found that the greater part of the deflection, if not all, was due to an air bubble floating up and striking the upper electrode. Pirani found as did Maxwell that the electro-motive force persists after the tube had been placed in the horizontal position. In the same volume of Nature,[4] R. Colley, of Moscow, pointed out that he had published results,[5] which established the fact that such an E.M.F. exists and that he had also calculated its magnitude and verified his results experimentally.

Colley, in his successful method, used a glass tube with the electrodes fused through the walls near the ends. One end of the tube was fused shut while the other had a rubber tube placed over it so as to be closed by means of a pinchcock. Two such tubes were used in the experiment. The electrodes in one case were of silver and the electrolyte was silver nitrate; in the other they were of cadmium and the electrolyte was cadmium iodide. Wires connected to the electrodes led the current through a resistance box and a sensitive galvanometer, the sensibility of the galvanometer being 8×10^{-9} with a period of 80 sec. He pointed out that it is necessary to remove all air from the electrolyte by means of an air pump. He found by calculation and also by experiment that in the case of silver nitrate the E.M.F. is down, while with cadmium iodide it is up through the tube. The following table contains his results.

---

[1] Pogg. Ann., Vol. 98, p. 5, 1856.

[2] Maxwell, Vol. I., p. 317, 1st edition.

[3] Nature, Vol. 17, p. 180.

[4] Nature, Vol. 17, p. 282.

[5] Jour. Phy. Chem. of St. Petersb., 1876; Pogg. Ann., 157, pp. 370, and 628, 1876; Phil. Mag., s. 5, Vol. I., p. 419.

TABLE I.

Tube.	Length.	Resistance.	Deflection.	
			Calculated.	Observed.
300 to 381 grams $AgNO_3$ per liter.				
No. I	160 cm.	774 Siemens	8.32	4.3
No. II	360 "	1500 (?) "	16.22	6.9
Saturated Solution CdI_2				
No. II	360 cm.	7976	21.55	6.8

He explained the discrepancy by a statement of Helmholtz, that Ohm's law seems to be inapplicable to weak currents, or that they behave as if the cell had a much larger resistance than calculated. He determined the resistance of the cell by Ohm's method and finds it to be 18240. Using this resistance the calculated deflection was 6.61 compared with 6.83 by experiment.

Gore has done considerable work which has more or less bearing upon this subject. In an article entitled Relation of Volta Electro-Motive Force to Pressure,[1] he described a long series of experiments with glass tubes 3 meters long and 1 cm. bore, fitted with paraffined corks through which wires passed for electrodes. The tubes were filled with the electrolyte through a T at the middle leaving an air bubble of .5 cm. In some cases as many as five of these tubes were mounted side by side and connected in series, the lower end of one being connected to the top of the next. These were connected through a galvanometer and a resistance of 50,000 ohms. The method of procedure was to place the tubes in a horizontal position until the galvanometer came to a steady position and then to turn the tubes to the vertical position and note the maximum deflection which was obtained in from 4 to 10 minutes. He says, "No perceptible amount of interference was caused by the small difference of temperature between the top and the bottom of the room usually equal to about 1°.5 C." The electrodes were made from the same piece of metal. The electrolytes were solutions of various salts of varying proportions. The results are given in terms of galvanometer deflection, the sensibility not being given except in one case. (Expt. 93, 8° corresponds to an E.M.F. of .00572 volt.) Experiment 93 was with zinc electrodes in an electrolyte of KCl. Two tubes were in series making $h = 600$ cm., which will give approximately 1×10^{-5} volts per cm. height (compared with 2.4×10^{-8} volts for zinc in $ZnSO_4$). Of 91 experiments, 41 showed a perceptible current. Of the 41, 39 were up. With zinc in zinc sulphate he detects no current. Dr. Gore also tried the effect of pressure by making a tube with a porous biscuit-ware diaphragm inserted between the electrodes. By means of this and a tube fused into the first he was able to keep both electrodes at atmospheric pressure while their difference in height was 3 meters. Upon connecting to the galvanometer he was unable to detect a current. He next placed both electrodes in the same horizontal plane with the diaphragm inserted between them and by means of a rubber tube filled with mercury he was able to vary the pressure on one while the other was at atmospheric pressure. A pressure of 30 inches of mercury gave a deflection of 6° and a pressure of 52 inches gave 12°. The electrode which was under pressure was positive to the other.

---

[1] Phil. Mag., Vol. 35, p. 97, 1893.

Dr. Gore's second article is interesting on account of its errors rather than results. The article is entitled Influence of Proximity of Substance upon Voltaic Action.[1] In his experiments he made use of 72 pigs of lead, weighing 8,271 lbs., as an influencing mass. From his calculation he shows the looked-for effect to be $1/24 \times 10^{-8}$ of the effect due to the force of gravity. (Which would be 1×10^{-17} in the case of zinc in zinc sulphate solution.

---

[1] Phil. Mag., Vol. 43, p. 440, 1897.

With potassium chloride the effect will be even less since the equivalent weights of the ions are nearly equal and the migration constant, $n = .516$.) The apparatus used was a number of cells in series mounted so as to rotate in a horizontal plane about a vertical axis, thus presenting either end to the mass of lead. Dr. Gore says there were several disturbances but they were sufficiently small to permit a very minute deflection of the galvanometer due to the presence of the lead provided a proper method of averaging the magnitudes was employed. The second apparatus consisted of a number of tubes in series mounted so as to slide endwise into and out of lead tubes, one end entering as the other receded. The third apparatus was built on the same general plan with the tubes of larger cross section, thus reducing the resistance. He concludes from his results that the proximity of lead increases the positive condition of the electrodes. A galvanometer was used in this experiment; probably the one used in the first experiment. Judging from the calculated values and the sensibility of the galvanometer, we are pretty safe in saying that the effect detected by Dr. Gore was not due to the gravitational attraction of the lead on the ions.

In an article entitled Unpolarizable Electric Cells under the Influence of Centrifugal Force, Th. Des Coudres[1] gives results obtained with two cadmium voltameters mounted as to be rotated about a vertical axis. The tubes were each 20 cm. long so placed that the electrodes were approximately 10 and 20 cm. from the axis of rotation. The electrolyte was a solution of cadmium iodide. This salt is especially suited to show the gravitational effect on the ions, since the equivalent weight of cadmium is 56 and that of iodine is 126, while the migration constant is greater than 1. In the solution used by Des Coudres it was 1.12. The results are given in terms of revolutions per sec.

TABLE II.

1 Part CdI_2 to 3.6 H_2O.			
E.M.F.	155.	75.3	37.4 micro volts.
Revs. obs.	5.8	3.87	2.9 per sec.
" Calc.	5.2	3.7	2.6 " "

---

[1] Wied. Ann., Vol. 49, 254, 1893.

It is to be noted that the value given for the E.M.F., as is pointed out by Des Coudres in a later article, should be divided by two since the values used for the equivalent weights are twice too large.

Des Coudres has also measured the direct effect of gravitation on the ions.[1] In this work he uses a glass tube with cadmium amalgam electrodes; the tube being mounted so as to revolve about a horizontal axis. The method of measurement was by means of a potentiometer which gave the result in terms of a Daniell cell. Connection from the voltameter to the galvanometer was made by means of glass and rubber tubing filled with mercury. The reversing key was a four-way glass stopcock. A detailed discussion is given of the various corrections which enter into the result. First, the correction for the buoyant force of the electrolyte; second, a small E.M.F. due to concentration caused by compression; third, the compression changes the density of the salt in solution making a correction to the first correction; fourth, a concentration change due to diffusion; fifth, a thermal E.M.F. due to heat liberated by compression. He shows that all these corrections except the first are so small as to be neglected without impairing the observed results. The results are given in terms of the migration constant. He finds that for a 23.8% solution of cadmium iodide $n = 1.236$; Hittorf's[2] result being 1.184, that of Lenz[3] 1.165 to 1.161.

Des Coudres then discarded cadmium iodide and used more simply constituted electrolytes. A voltameter was built of two tubes of such form as to be kept in water-baths. These baths were suspended from pulleys over head in such a manner as to raise one while the other is lowered through a distance of 4 meters. The two glass tubes were connected by a rubber tube filled with the electrolyte. The electrodes were made of mercury.

His results are given in the following table. They are calculated from the mean of from four to eleven determinations of a particular resistance, w_3 on the potentiometer.

---

[1] Wied. Ann., Vol. 57, p. 232, 1896.

[2] Pogg. Ann., Vol. 106, p. 543, 1859.

[3] Mem. Acc. Petersb. (9), Vol. 30, p. 36, 1882.

TABLE III.

Salt.	NaCl	BaCl	LiCl	KCl	HCl	$CdCl_2$	
						Cal. El	Amalg. E
Sol. %	19.9	17.	17.3	16.8	3.56	30.1	30.1
E	−0.315	+11.7	−1.09	+10.51	− .218	−1.83	−2.21
n Calc.	.66	.64	.77	.50	.150	1.10	1.02
n Obs.	.65	.65	.75	.52	.175	.82	.82

THE EFFECT OF PRESSURE.

H. Wild[1] has done some work upon the change of E.M.F. between metal and fluids. He concluded that the E.M.F. between amalgamated zinc and zinc sulphate by increasing the pressure two-thirds of an atmosphere does not change by an amount of 1/40000 of a Daniell. Bichat and Blondlot[2] have detected a slight increase of E.M.F. when platinum copper and platinum silver couples immersed in electrolytes were subjected to pressure.

The variation of E.M.F. of various forms of cells with pressure has likewise been studied by Henri Gilbault.[3] The method used was to make a small cell in a test-tube which was placed in a piesometer filled with oil. Pressure was applied by means of a Ducretet pump. Variations of the E.M.F. were measured by the potentiometer method, method of opposition, and a reverse potentiometer method in which the auxiliary cell had a lower E.M.F. than the cell investigated. A Lippmann electrometer was used to indicate the points of perfect balance. Gilbault has shown that the variation of the E.M.F. with pressure in non-polarizing cells may be expressed by the equation:

$$E_p - E_0 = Pdv.$$

Where P is the pressure expressed in dynes and dv is the change in volume of the contents of the cell when 1 C.G.S. unit of quantity has passed through the cell. His results are as follows:

---

[1] Pogg. Ann., Vol. 125, p. 119, 1865.

[2] J. de Phys. (2), Vol. 2, p. 503, 1883.

[3] Lum. Él., Vol. 42, pp. 7, 63, 175, 220, 1891; C. R., Vol. 113, p. 465, 1891, and Electrician, Vol. 27, p. 711, 1891.

TABLE IV.

Pressure — 100 Atmospheres.

Cell.	Per cent. Solution.	Obs.	Calc.
Daniell.	13 % $ZnSO_4$	6.6	7.18×10^{-4} volts.
"	20 % "	5.0	5.17
"	27.56 "	2.2	2.00
Warren de la Rue.	1 % $ZnCl_2$	13.	13.24
"	40 % "	–5.	–5.04
Lalande.		–10.	–13.5
Planté Accumulator.	8.8 H_2SO_4	–12.	–12.7
"	9.5 "	–5.	–4.95
Volta.		–600.	–586.
Bunsen.		–405.	–383.
Gas battery.		845.	865.

This survey of the work done upon the effects of gravity and pressure shows that the looked-for effect in the case of gravity has been obtained with considerable degree of accuracy by using the chlorides of the metallic earths and with a much less degree of accuracy by using cadmium iodide; while the effect of pressure on the more common forms of commercial cells gives results which accord with theory in a very remarkable manner.

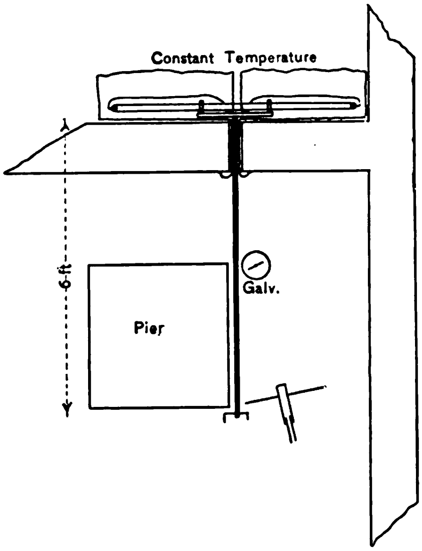

Fig. 1.

EFFECT OF GRAVITY.

Apparatus and Methods.—The work about to be described has been done with zinc and cadmium sulphates, the two most important electrolytes perhaps used by the physicist. The voltameter was mounted so as to rotate end over end about a horizontal axis in the constant temperature room of the Physical Laboratory. This room is a small inside room surrounded with brick walls and closed with double doors. The voltameter was turned by means of a brass rod, which served as the axis of rotation, and which passed through a small hole in the wall into the observation room where it extended past and was supported by a brick pier (Fig. 1). The length of this rod was about six feet. The pier, which extended to the second floor, also served as a support for a bracket to which the galvanometer was suspended. The galvanometer used had a resistance of 733 ohms and was capable of giving a sensibility of 2×10^{-10} with a period of 18 sec. The connecting wires were insulated copper, all connections being made with binding posts, mercury or solder. To guard against the possible contact of the wires by the rotation of the voltameter one of the wires leading from the cell passed through a rubber tube. Mercury reverse and break keys were inserted so as to reverse the current and to cut the voltameter out of the galvanometer circuit while finding the sensibility.

Voltameter No. I. (Fig. 2) was constructed of two small bell-jars

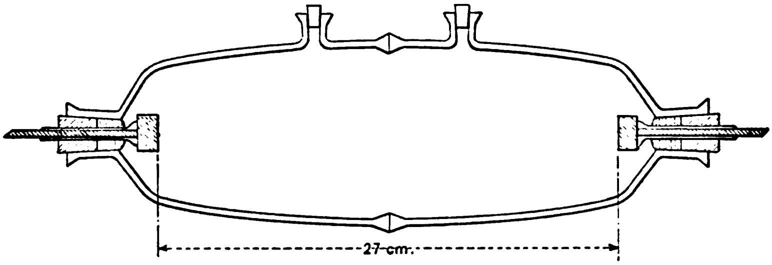

Fig. 2.

with openings in the top and sides. These were cemented together with bicycle cement after having electrodes fixed in the ends by means of corks and cement. The electrodes were cast from C. P. zinc. When constructed the distance between electrodes was 27 cm., the diameter 7.5 cm. The electrolyte was a 10-per-cent. solution of zinc sulphate (10 grams of salt dissolved in 90 cc. water). The method first used was to place the cell in one arm of a wheatstone bridge and to balance it against an equal resistance and observe the deflection of the galvanometer when the voltameter was reversed. From the figure of merit of the galvanometer, the average difference of deflection in the two cases, the resistance of the various arms, and the E.M.F. of the battery the counter E.M.F. in the voltameter was calculated using Kirchhoff's laws. By this method the counter E.M.F., *i. e.*, the E.M.F. due to gravity, was found to be down through the voltameter. In 1898 the mean of 1,350 readings taken by this method showed the E.M.F. down to be 4.48×10^{-4} volts. This was with a voltameter diameter of which was 4 cm. and length of which between electrodes was 9 cm. A possible explanation of this deviation from the results found later is the polarization of the electrodes due to the large current and the friction of eddy currents in the electrolyte caused by the rotation of the voltameter. As this method promised very little the voltameter was short-circuited through the galvanometer and a resistance box. Upon rotating the voltameter a throw of the galvanometer needle was observed. In every case the throw indicated a momentary E.M.F. opposing the constant E.M.F. This momentary E.M.F. was large enough to give a throw of 20 mm. when the E.M.F. per scale diversion of the galvanometer was 1.3×10^{-4} volts. The constant E.M.F. referred to above is always present and of considerable magnitude in voltameters with metal electrodes. With amalgam electrodes it is considerably smaller and at times almost nil. It is caused, no doubt, by the small differences in the chemical and physical properties of the two electrodes and of the electrolyte surrounding the two electrodes.

To prevent the free circulation of the electrolyte in the voltameter, which was thought to be the cause of the disturbance in No. I., voltameter No. II. was built. (Fig. 3.) This was made from glass tubing 1–½ cm. diameter. The length between the electrodes was 31 cm. A short distance from the electrodes the tube was

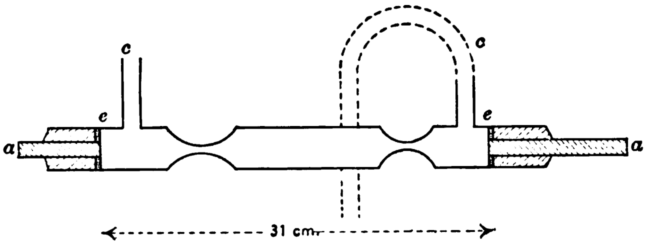

Fig. 3.

drawn down to small diameter. T tubes were fused in the ends near the electrodes for convenience in filling. The electrodes, *a*, were made of a bar of C. P. zinc cut in two in the middle; the two contiguous parts being exposed to the electrolyte. These were fastened in the tube by means of short perforated corks, *e*, and a filling back of the corks of melted cement. This tube was provided with the proper mountings and placed on the end of the brass shaft after being filled with the electrolyte. To exclude air bubbles the tubes, *c*, were closed carefully with ordinary corks. This tube also gave a throw when rotated from horizontal position to vertical, or from the vertical position through 180° to vertical. When rotated from horizontal through 180° to horizontal no deflection was detected. Short-circuiting the voltameter by a wire connecting the two electrodes together removed the disturbance, showing that it was in the tube and not due to magnetic influence. To try the effect of pressure on the electrodes without changing their position, a U tube was fused to one of the tubes, *c*, while the other was fused shut. Pressure was applied by placing over the U a rubber tube filled with mercury. This tube passed through the wall along a shelf and was connected to a reservoir which could be raised and lowered by the observer at the telescope. The galvanometer gave a throw in the same direction both when putting on and relieving the pressure. The variation of pressure was approxi-

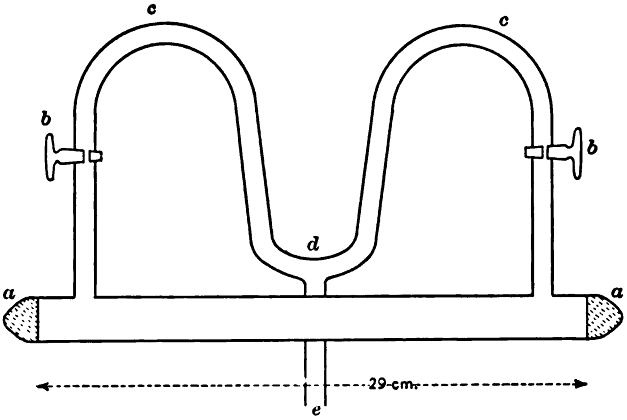

Fig. 4.

mately from zero to two atmospheres. The momentary E.M.F. opposed the E.M.F. of the voltameter.

To more fully investigate the cause of the pressure disturbance tube No. III. was built (Fig. 4). This tube was made by taking two pieces of glass tubing and fusing a platinum wire in one end of each. T's were also fused into these a short distance from the ends. To these T's long narrow tubing with glass stopcocks were fastened. In the end containing the platinum wire zinc amalgam was placed. After this operation the two tubes were fused together making an enclosure which would be absolutely rigid except for the elasticity of the glass. The smaller tubes were bent in the form of a U above the stopcocks and joined to a T by means of rubber tubing. After the voltameter had been filled with the electrolyte up to the bend in the side tubes and the rubber tube, filled with mercury, had been attached to the T tubes, *e*, the pressure could be varied at will by raising or lowering the reservoir of mercury in the observation room. By means of the stopcocks the pressure could be applied through one or both ends. By this means the slight flow of the electrolyte due to the expansion of the glass due to pressure could be made to wash either electrode. The effect of variation of pressure on this voltameter was not very marked. By a careful comparison of a large number of observations a slight increase of current is noted when the pressure is increased. This effect can be explained to some extent at least by the decrease of resistance due to pressure. Upon relieving the pressure the effect is not so marked, if present at all. From these results it would seem that the effect noted in voltameter No. I. and No. II. was due to the kind of electrodes and the manner of inserting them in the tube. Since the cement used was of about the consistency of Burgundy pitch the added pressure would move the electrodes to a certain extent. To obviate the pressure due to the electrolyte, a voltameter (Fig. 5) was constructed in which zinc sulphate paste was used as an electrolyte. This tube was made in the form of an H, the zinc amalgam electrodes being placed in the diagonally opposite ends of the arms. The zinc sulphate paste was made by grinding zinc sulphate and water together in a mortar until it was a semi-fluid. This was placed in the tube and the ends of the arms opposite to the electrodes were closed with corks over which melted bicycle cement was placed. On account of the great difference in the electrolyte immediately next to the electrodes a comparatively high E.M.F. was always present. This required a high

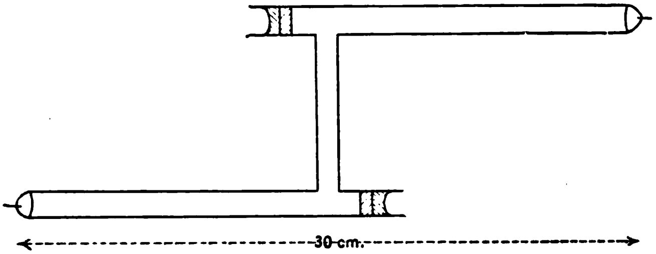

Fig. 5.

resistance to keep the deflection of the galvanometer on the scale. This resistance cut down the sensibility to such an extent that readings were impossible.

During these experiments the disturbing elements due to the Dynamo Laboratory, and to varying currents in the neighboring power and car lines rendered the reading very uncertain and at times impossible. To better the conditions observations were made between 11 P. M. and 6 A. M. It was found that after midnight the disturbances were approximately one-tenth of those in the daytime. Never in any case was it possible to read the deflection to within 1 mm. with any degree of accuracy when the galvanometer was in its most sensitive position. Finally after wasting several nights the method of multiplication was tried. In this method the E.M.F. of the circuit is reversed in unison with the swing of the galvanometer needle until a final maximum deflection is reached, the maximum depending upon the E.M.F. and the damping factor of the galvanometer.

Let A_0 be the deflection of galvanometer.
" A_1 " " " " first swing.
" A_2 " " " " second swing.
" A_n " " " " nth swing.

In case of no damping

$$A_1 = 2A_0.$$

With damping,

$$A_1 = A_0 + A_0 e^{-\lambda} = A_0(1 + q) \qquad \text{where} \qquad q = e^{-\lambda}$$

$$A_2 = A_0 + A_1 q + A_0 q = A_0(1 + q)(1 + q)$$

$$A_3 = A_0 + A_2 q + A_0 q = A_0(1 + q)(1 + q + q^2)$$

$$A_4 = A_0(1 + q)(1 + q + q^2 + q^3)$$

$$A_n = A_0(1 + q)(1 + q + q^2 + q^3 + \dots + q^{n-1})$$

$$A_n = A_0\left(\frac{1 + q}{1 - q}\right)$$

$$A_0 = A_n\left(\frac{1 - q}{1 + q}\right) = A_n\left(\frac{1 - e^{-\lambda}}{1 + e^{-\lambda}}\right).$$

The damping factor was computed from the observed value of the logarithmic decrement λ but it was found that better determinations could be had by placing a reverse key in the circuit and reversing a known E.M.F. in unison with the swing of the needle. This damping factor varied with the sensibility of the galvanometer. A number of results at different sensibilities were found, and with the results a curve (Fig. 6) was plotted using galvanometer sensibilities as abscissas and damping factor as ordinates. The damping factors used in the later calculations are read from the curve.

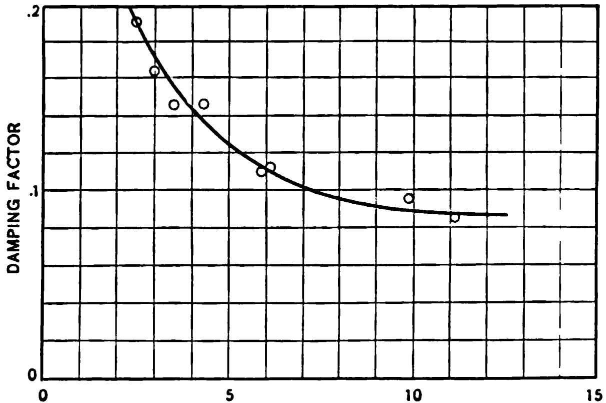

Fig. 6.

To use this method in the case of the determination of the E.M.F. due to gravity, we start with the voltameter horizontal and the galvanometer at rest; suddenly turn the voltameter to a vertical position and at the instant the needle reaches its greatest elongation turn the voltameter through 180°, repeating the operation when the needle reaches its greatest elongation on the other side. This process is continued until the deflection reaches a maximum. If the magnetic field changes a larger or smaller deflection will be noted depending upon the position of the needle at the instant, but by rotating in unison with the needle the old maximum will be obtained. The advantages of this method of observation are: large deflections, the elimination of the effect of disturbances in the magnetic field, and the elimination by repeated reversals of the tube of the thermal E.M.F. due to the difference of the temperature of the two ends. The disadvantages are: errors due to not rotating the tube at the proper instant, and errors due to duration of rotation. With proper precautions the errors of this method are very small. The cycle of operations by means of which the results given in the following tables were obtained was as follows:

(*a*) Starting with the tube in normal position (horizontal and commutator *straight*) the tube was turned to the vertical position and reversed repeatedly in unison with the galvanometer needle until a maximum deflection was obtained.

(*b*) The above operations were repeated with the commutator reversed. These operations were followed until six determinations of the maximum deflection was obtained.

(*c*) Starting with the tube horizontal but turned 180° from the normal position, the tube was turned to the vertical and rotated until a maximum deflection was obtained. The commutator was straight and reversed alternately as before until six more determinations were obtained. The mean of the twelve determinations of the maximum deflection was taken.

Table V. contains a set of readings which will illustrate the method.

Voltameter No. V. (Fig. 7) which may be described as a sample of all made afterwards was made of glass tubing the inside diameter of which was about 1 cm. The amalgam electrodes were made by first solidifying the amalgam in the ends of short pieces of

TABLE V.

Dec. 27-5-15-*P.M.* *Tube No. VIII.* *Length* 144 *cm.* *R. of tube* 9,300 *ohms.*
Galv. $R = 733$ *ohms.* *Fig.* $M. = 1.5 \times 10^{-10}$.

Voltameter Normal.					
Commutator Straight.			**Commutator Reversed.**		
Right End.			**Right End.**		
Down.	**Up.**	**Difference.**	**Up.**	**Down.**	**Difference.**
146	137	9	231	220	11
160	152	8	214	205	9
178	166	12	193	180	13
Voltameter Reversed.					
186	175	11	184	174	10
208	202	6	174	165	9
208	200	8	168	158	10

Mean difference 9.5 mm.
Damping factor .23.
E.M.F. per cm. length of voltameter 2.34×10^{-8} volts.

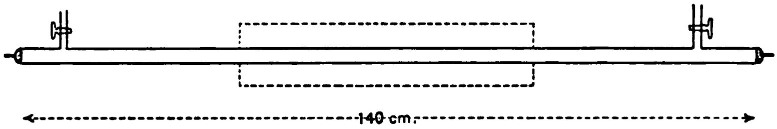

Fig. 7.

tubing and subsequently fusing these to the long tube which formed the body of the voltameter. Side tubes with glass stopcocks were fused on at either end. Some later tubes had only one stopcock the electrolyte being run in through a small capillary tube. The amalgam was made after the formula given by Kahle.[1] This amalgam, however, contained too much mercury to be satisfactory. In order to remove the air bubbles from the electrolyte it was necessary to attach the tube to an air pump. This caused mercury to come to the surface of the electrode and at times to be detached entirely. After letting the pressure of the atmosphere on the electrode, the mercury would disappear from the surface but the metallic lustre between the surface of the glass and the amalgam had disappeared also. This metallic lustre seemed to be an important factor. In every case with zinc amalgam after the electrodes became granular, disturbances inside the voltameter became so great as to render the tube worthless. The method pursued was to build the tube and place it under the pump, then shake off the extra mercury and allow it to pass out through the stopcock. After this the tube was broken, the amalgam melted and poured out, the short ends with the platinum wire were fused again or new ones made. The amalgam was again placed in the short tubes which were again fused to the longer. To obtain a permanent metallic surface in contact with the inner surface of the glass it was found that it was necessary to place the amalgam in the tube immediately after the latter had been fused. Any moisture in the tubes after being fused gave a poor contact between the amalgam and the glass. Good contact or surface lasted a few days only. In the case of cadmium amalgam, no trouble of this kind was experienced. With amalgam made of 1 part cadmium to 6 parts mercury there was no tendency for mercury to be drawn out when under the pump.

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[1] Wied. Ann., Vol. 51, p. 203, 1894.

The following table gives the results for 10 per cent. solution of zinc sulphate:

Table VI.

Date.	Tube.	Length.	Resistance		Fig. of M.	Damping factor.
			of tube.	in box.		
Dec. 8.	VI.	143.7 cm.	4353	1000	2.1×10^{-10}	.21
" 11.	VI$_b$.	144.01	4360	2000	2.08	.21
" 27.	VIII.	144.	9500	0	1.5	.23
" 27.	"	144.	"	0	1.5	.23
" 29.	"	144.	"	0	2.1	.21
Jan. 2.	"	144.	"	1000	3.57	.158

E. per Scale Division.	Max. deflect. An.	A_0	E. obs.	E. per cm. Height.	E. cal.
1.26×10^{-6}	34.5 mm.	7.42	9.36×10^{-6}	-6.5×10^{-8}	-2.4×10^{-8}
1.45	18.5	3.88	5.62	−3.9	
1.54	9.5	2.19	3.37	−2.34	
1.52	12.41	2.86	4.34	−3.02	
2.14	11.16	2.33	5.00	−3.45	
4.05	7.5	1.11	4.5	−3.13	

It will be noted that the values for tube No. VIII., are lower than the values obtained with previous tubes and also that the values obtained with No. VIII. increase as the tube becomes older. Tube

No. VIII. was made with special care to get the electrodes set firmly in the end of the tube. The metallic lustre was perfect when first built, but this passed away with age.

The following is the theoretical calculation of the E.M.F. per cm. height. The value of the migration constant used in this calculation is taken by interpolation from the three values given for zinc sulphate in a table of electro-chemical properties of aqueous solution by Rev. T. C. Fitzpatrick.[1] The value used is $n = .715$ for 10 per cent. solution.

In the formula

$$E = [\text{Kat} - n(\text{Kat} + \text{An})]gqh.$$

we assign for zinc sulphate the following values:

Kat = ½(65) = 32.5 (equivalent weight of Zn).
An = ½(96) = 48 (equivalent weight of SO_4).
$g = 980.$
$q = .0001035$ (ten times the electro-chem. equiv. of H).
$h = 1$ cm.
$n = .715.$

The particular solution of E is therefore:

$$E_1 = [32.5 - .715 \times 80.5]\ 980 \times .0001035$$
$$= [32.5 - 57.7]\ .0001013.$$
$$E_1 = -\ 2.53 \text{ C.G.S.}$$
$$E_1 = -\ 2.53 \times 10^{-8} \text{ volts.}$$

From this is to be subtracted the correction for the buoyant force of the liquid, viz.:

$$e = D[\text{Kat}\ V - nV'(\text{Kat} + \text{An})]gqh:$$

where

D = density of the electrolyte,
V = specific volume of the cathode.
V' = specific volume of the salt in solution.

putting

$$D = 1.1.$$
$$V = 1/7.$$
$$V' = 1/10.$$

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[1] See Whetham's Solution and Electrolysis, pp. 215–283. B. A. Report 1893.

We have —

$$e = 1.1[32.5/7 - n\ 80.5/10]gqh.$$

$$e = 0.124 \times 10^{-8} \text{ volts.}$$

$$E = E_1 - e.$$

$$E = -2.4 \times 10^{-8} \text{ volts.}$$

The sign — indicates that the current flows up through the tube opposite to gravity. This, perhaps, would seem at first thought to be inconsistent with the law of conservation of energy. It will be noted that when 32.5 grams of zinc is carried with the current that 57.7 grams of SO_4 moves in the opposite direction. The E.M.F. is due to the excess of the mass of SO_4 over the mass of zinc. This E.M.F. will continue until the E.M.F. due to difference in concentration is large enough to balance it. In case the rate of diffusion is great enough to prevent an appreciable difference in concentration the source of energy must be the heat absorbed by diffusion.

With cadmium sulphate the results are a great deal more consistent than with zinc sulphate. The voltameters were made after the same plan as in the case of zinc. The cadmium amalgam was made after the formulæ given by Jaeger & Wachmuth.[1] Although the amalgam became crystalline at the surface of the glass in a short time there seemed to be no tendency for the electrolyte to creep between the two as in the case of zinc amalgam. A difference was noticed between zinc and cadmium amalgam in that the zinc seemed to adhere better to the platinum wire. After melting and pouring the amalgam out of the tube, a coating of the zinc amalgam adhered to the wire while the cadmium amalgam left the wire perfectly clean. While removing the air from the cadmium sulphate solution small bubbles would form constantly. The vacuum was perfect enough to give a metallic click when the electrolyte was shaken. This action seemed to indicate that there might be a slow chemical reaction taking place in the electrolyte.

The results for 10% cadmium sulphate are given in the following table.

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[1] Wied. Ann., Vol. 59, p. 575, 1896.

TABLE VII.

Date.	Tube.	Length.	Resistance of tube.	Resistance in box.	Fig. of m.	Damp. Factor.
Jan. 3	Cd No. I	128 cm.	11240	0	4.8×10^{-10}	.127
" 7	"	"	"	0	3.3	.163
" 16	" II	130 cm.	11300	0	3.06	.17
" 19	"	"	"	0	2.27	.202

E. per Scale Division.	Max. deflect. A_n	A_0	E_h	E per cm. Height.
5.82×10^{-6}	2.5	.317	1.85×10^{-6}	-1.44×10^{-8}
3.96	3.51	.556	2.20	−1.72
3.68	4.	.680	2.51	−1.92
2.76	4.5	.910	2.51	−1.92
			Mean	−1.75

The migration constant for cadmium sulphate seens never to have been experimentally determined. The constant corresponding to any E.M.F. may however be determined from the equation,

$$E = \{[\mathrm{Kat} - n(\mathrm{Kat} + \mathrm{An})] - D\,[\mathrm{Kat}\,V - nV'(\mathrm{Kat} + \mathrm{An})]\}\,gqh,$$

where the following values are substituted,

$\mathrm{Kat} = 55.7$

$\mathrm{An} = 48.$

$D = 1.08$

$V = 1/9.39$

$V' = 1/8.4.$

We have:

$$E = 5.02 - 9.17\,n \times 10^{-8} \text{ volts.}$$

Solving from n we have:

$$n = (E + 5.02)/9.17.$$

For

$$E = -1.75 + n = .738.$$

With a nearly saturated solution of cadmium sulphate the E.M.F. is positive or the current flows down through the tube. The following table gives the results for a 46% solution (46 grams of $CdSO_4$ crystals dissolved in 56 grams of water).

TABLE VIII.

Date.	Tube.	Length.	Resistance		Fig. m.	Damp. Factor.
			of tube.	in box.		
Jan. 29.	Cd No. 11.	130 cm.	7560	1000	6.04×10^{-10}	.11
“ 29.	“	“	“	0	4.3	.138
“ 29.	“	“	“	10000	4.28	.137
Feb. 1.	“	“	“	0	4.74	.127

E. per Scale Division.	Max. deflect. A_n	A_0	E_h	E per cm. Height.
5.48×10^{-6}	9.4 mm.	1.36	7.5×10^{-6}	5.77×10^{-8} volt.
3.47	11.4	1.59	5.5	4.25
8.1	5.6	.77	6.25	4.8
3.85	11.	1.40	5.36	4.12
			Mean	4.78

Calculation gives the following equation for E:

$$E_1 = 4.95 - 8.67\,n \times 10^{-8} \text{ volts.}$$

For

$$E = 4.78,\ n = .02.$$

The meaning of the small value of n is that the anions are moving with a very small velocity in the direction opposite to that of the current. The tendency in a concentrated solution would be for both ions to settle to the lower end of the tube and produce a difference in concentration.

In the above work the resistance of the voltameter was determined by taking the galvanometer deflections with various resistances in the box when the voltameter was in the horizontal position, the constant E.M.F. of the voltameter being used as the E.M.F. of the cell. Plotting the reciprocal of deflections as ordinates and resistances as abscissas, the intercept on the x axis will give the resistances of the cell plus the galvanometer resistance. The results obtained in this manner agree with the accepted values. It will be remembered that R. Colley, in the paper cited in an earlier paragraph of this article, found the resistance to be much larger by this method than the usual method of determination.

EFFECT OF PRESSURE.

Apparatus and Methods.—The effect of pressure on various cells was determined by connecting in opposition two cells whose E.M.F. were nearly equal and measuring the variations of the difference of voltage when one was subjected to pressure. In the first method the cell was made, in the H form, of heavy walled glass tubing with small inside diameter. This cell, which was a cadmium one, was made according to the directions given by Jaeger and Wachsmuth.[1] The electrodes were placed with mercurous sulphate paste over the mercury, and cadmium sulphate crystals over the amalgam. One arm of the tube was fused shut, while the other, which was longer, served as an opening through which the cadmium sulphate solution was passed by means of a capillary tube. By patient work and care the H was entirely filled with the liquid. A long capillary tube was then fused to the cell which connected it to the piesometer. (Fig. 8). Pressure was applied by means of a compression pump.

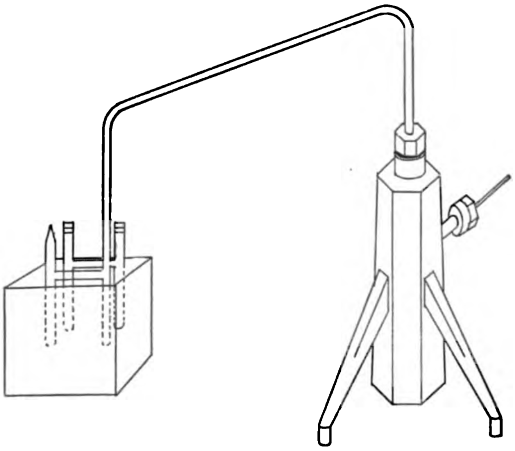

Fig. 8.

The two cells were placed side by side in a block of wood and were connected through a resistance box to the galvanometer. The variation of the E.M.F. was determined by the deflection and sensibility of the galvanometer and the total resistance in the circuit.

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[1] Wied. Ann., Vol. 59, p. 575, 1896.

The accompanying curve (Fig. 9) gives the results up to 75 atmospheres at which pressure the connecting tube burst. As the strength of the glass tubing limited the pressure applied, the cells were constructed so as to be placed in a piesometer the inside diameter of which was 2.5 cm. The cells were made in a modified H form (Fig. 10) in which the cross tube was omitted and the vertical ones fused together. This was done by blowing out a small bulb on the side of each tube and then blowing a hole in the bulbs and fusing together at these openings. The electrodes were placed in the ends and the paste and crystals placed over them. Next the cell was heated above the connection and the two tubes drawn down to capillaries. The cell was now filled and the capillaries broken off at the smallest point. It was then placed in the piesometer which was filled with kerosene. By this means the pressure on the inside and outside of the cell will be the same and any possible

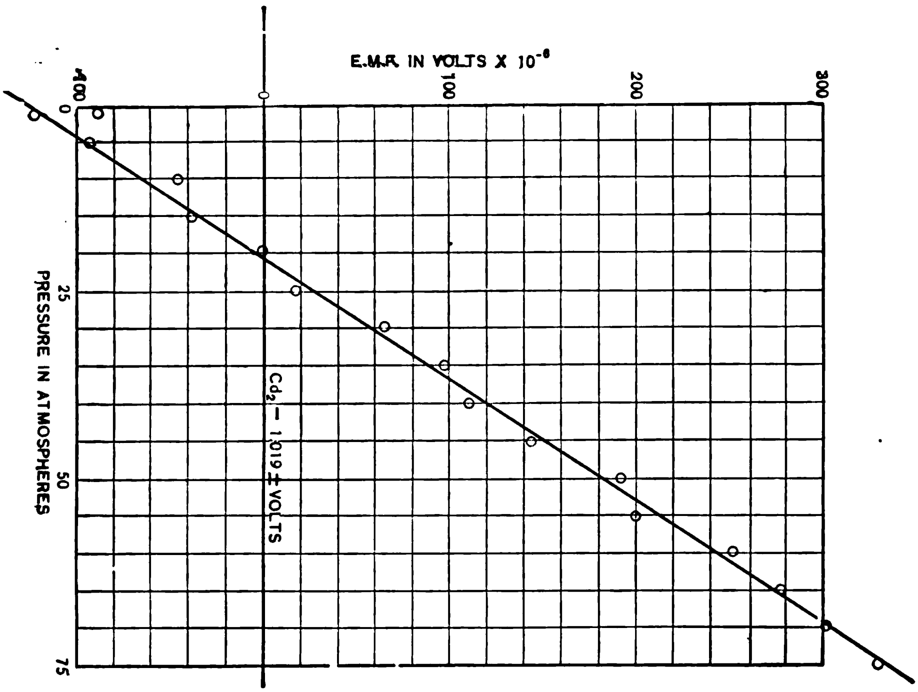

Fig. 9.

solution of the kerosene in the electrolyte will be avoided by the small surface of contact in the capillary tubes. Electric connection was made through two insulated conductors passing through the cap of the piesometer. To avoid errors due to change of temperature the piesometer was immersed in a large water-bath and the compensating cell in a kerosene-bath which was in turn immersed in the water-bath. This water-bath was surrounded with an air-jacket and this with a jacket of sawdust. The covering was made of cotton waste. By this means the temperature of the bath, which was approximately the same as that of the room, could be kept constant during an entire set of observations. Never in any case could a change be detected by means of a thermometer which was graduated to tenths of a degree.

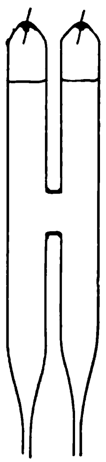
Fig. 10.

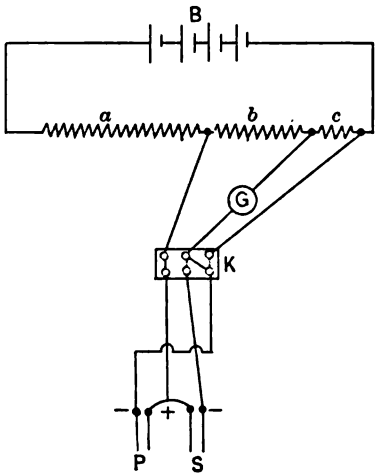

Fig. 11.

Instead of determining the change of E.M.F. by means of galvanometer deflections, a potentiometer method was substituted. In this method *S* (Fig. 11) is the cell under pressure, *P* is the compensating standard cell, *B* is the auxiliary battery consisting of four gravity cells, *a* is a high resistance of from 50,000 to 70,000 ohms; *b* is a resistance adjusted in proportion to the difference of the E.M.F. of *P* and *S*; *G* is galvanometer; and *K* is triple mercury key by means of which the cells *P* and *S* can be balanced on the potentiometer. The connections as marked by the heavy lines, permit the cell *P* to be balanced by the resistance *b*, while the connections as shown by the dotted lines permit the difference of the two cells in case the E.M.F. is less than *P*, being balanced by *c*. When the cell under pressure *S* has a higher E.M.F. than *P*, the balance can be made on *c* by diagonally connecting the four right hand mercury cups. The smaller the difference of the E.M.F. of the two cells the less will be the error introduced by any variation of the E.M.F. of the battery *B*.

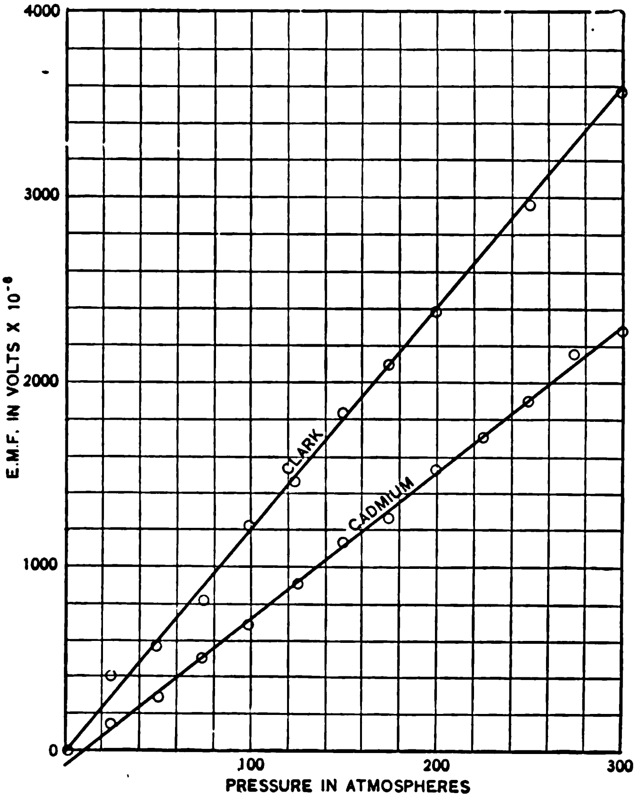

Fig. 12.

By means of this method the change of a cadmium cell with pressure up to 300 atmospheres, was determined. It was found to be linear as is shown by the curve (Fig. 12). In a Clark cell, made according to the directions given by Kahle,[1] the change is linear as shown by curve (Fig. 12), but it is necessary to allow the cell to stand a few minutes at each pressure in order to allow the E.M.F. to become constant. The time lag is shown by the curve (Fig. 13) in which time is the abscissa and E.M.F. is the ordinate. This may be explained by supposing that the change of E.M.F. at any time is due to the superposition of two changes ; a positive one due to pressure and a negative one due to increase of temperature caused by pressure.

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[1] Wied. Ann., 51, p. 203, 1894.

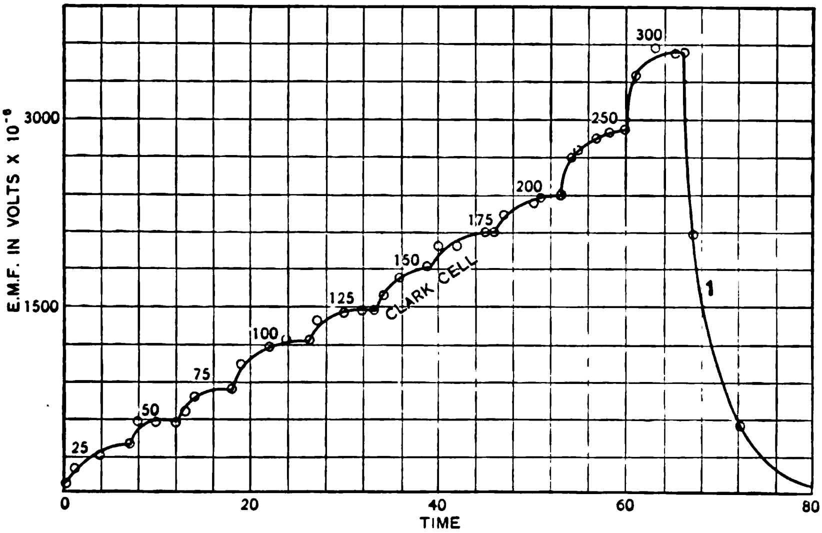

Fig. 13.

After a few minutes the cell regains its original temperature and the increase is that due to pressure alone. In a Carhart Clark cell, made in the H form with zinc amalgam electrodes but otherwise according to the directions given by Carhart,[1] the time lag is more pronounced than in the Clark. The curve (Fig. 14) gives both the time lag and the linear variation of E.M.F. with pressure, since the time intervals are equal. In the one-volt calomel cell, made in the H form but otherwise according to Carhart,[2] the curve (Fig. 15) rises rapidly for the first 25 atmospheres and then bends over, reaching a small linear increment at 100 atmospheres. The satu-

[1] Carhart, Primary Batteries, p. 90.

[2] Am. J. Soc., Vol. 46, p. 60, 1893.

rated calomel, made the same as the zinc calomel, except that the zinc chloride solution is saturated, gives the same kind of curve

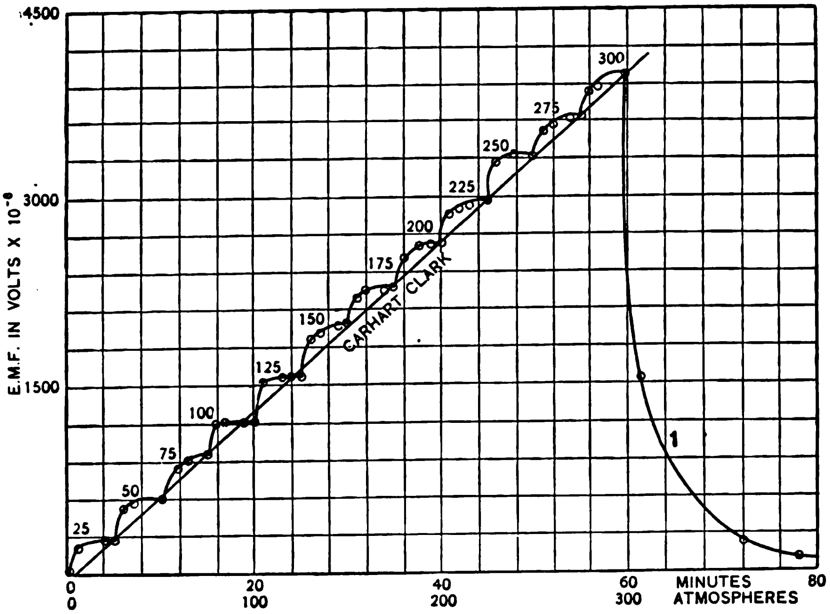

Fig. 14.

(Fig. 15) as the one-volt calomel. The rise at first is not so great and the linear part of the curve is slightly steeper than that of the one-volt calomel. The E.M.F. of this cell was .856 volt. In the

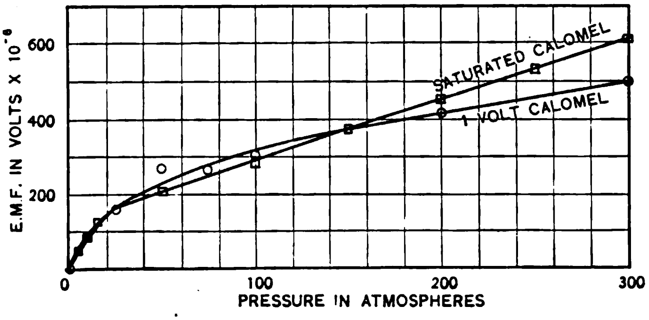

Fig. 15.

copper sulphate cell the variation was quite small and in the negative direction. This cell was made in the H form with zinc amalgam and copper, placed on a platinum wire, as electrodes. The zinc amalgam was covered with zinc sulphate paste, and the copper electrode was surrounded with a saturated solution of copper sulphate containing an excess of the crystals. After a diaphragm of blotting paper had been inserted over the copper sulphate solution the cell was filled with a saturated solution of zinc sulphate. The E.M.F. of this cell was 1.078. The time lag was very great, requiring fifteen minutes to settle to a constant value. This rendered the observations slow and uncertain. The silver chloride cell made

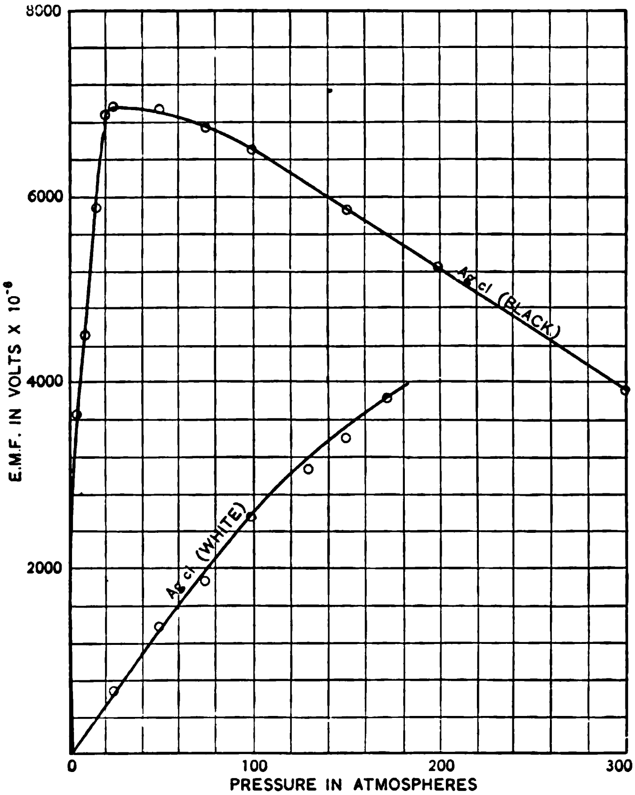

Fig. 16.

according to the directions given by Carhart,[1] except that zinc amalgam was used instead of zinc, gave a curve (Fig. 16) which is nearly linear but curves slightly to the right. The silver chloride used in this cell was freshly precipitated from a silver solution, and cast about the silver wire. The same cell with an old silver chloride electrode which had been in the laboratory twenty or more years, and had turned black, gave a very rapid increase of E.M.F. for the first 25 atmospheres and then diminished as shown by the curve marked AgCl (black) (Fig. 16).

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[1] Carhart, Primary Batteries, p. 62.

A comparison of the pressure effects on the various cells investigated are given by the curves (Fig. 17) and also a tabulation of results in Table XI. In Fig. 17 the zero line for the curve AgCl (black) has been dropped to — 4000.

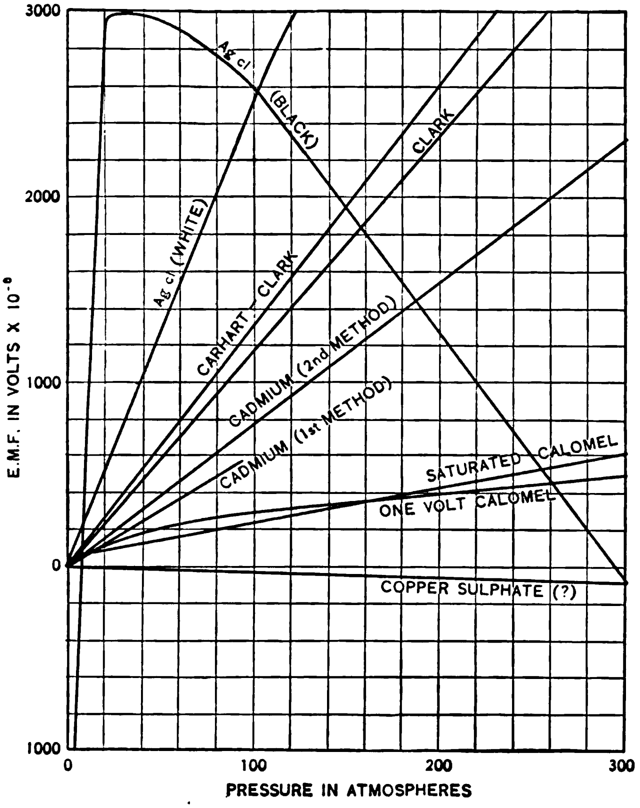

Fig. 17.

The E.M.F. of the cells were determined by comparison with cadmium and Clark cells, assuming the E.M.F.'s and temperature coefficients as given by the Reichsanstalt.

Table XI.

Variation of Pressure given in Volts × 10^{-6} per Atmos.

	E.M.F. of Cell.	Temp.	Increase of E.M.F. to .100.	Mean Increase from 100 to 300 Atmos.
Cadmium (1st method).	1.019	20°.4	6.02	
Cadmium (2d method.)	1.019	19°.	7.6	7.6
Clark.	1.440	16°.6	11.6	11.6
Carhart-Clark.	1.443	16°.3	13.1	13.1
One Volt Calomel.	.996	15°.	3.	.98
Saturated "	.805	17°.	2.8	1.63
Copper Sulphate.	1.078	17°.	−1.(?)	−1.(?)
AgCl (white).	1.076	18°.	25.7	15.
AgCl (black).	1.164	15°.6		−13.

This investigation was made under the direction of Professor E. L. Nichols to whom I express my thanks for suggestions made and for the apparatus placed at my disposal.

Cornell University, May, 1901.

ON A NEWLY DISCOVERED PHENOMENON PRODUCED BY STATIONARY SOUND-WAVES.

By Bergen Davis.

IN the course of an investigation upon the action of stationary sound-waves, under certain conditions I have observed, that if a small cylinder which is closed at one end, is placed in a stationary sound-wave, it will not only arrange itself perpendicular to the wave, but will also move across it in a direction perpendicular to the stream-lines. The force producing this motion is of considerable magnitude, and it acts normally to the closed end of the cylinder, causing it to move in the direction of the closed end, *i. e.*, the closed end is driven towards the wall of the organ-pipe.

A short description of this phenomenon has already been published,[1] since which a more complete investigation has been in progress.

Description of Apparatus.

The stationary wave used was that obtained in a stopped organ-pipe when blown so as to produce its first overtone. Two nodes were thus obtained, one at the stopped end and the other near the mouth. A thin rubber diaphragm was inserted in the pipe at this node as shown at *d* in Fig. 1. I have found, as has been observed by Lord Rayleigh,[2] that a thin diaphragm at a node does not alter the intensity of the sound appreciably. This diaphragm kept the interior of the pipe free from any disturbances that might arise from blowing.

Two pipes were used in these experiments, a long one and a short one. A section of the longer pipe is shown in Fig. 1. The length of the pipe was 216 cm., from the mouth to the end. It was provided with a movable stop which could be placed at various

[1] Am. Journal of Science, Sept., 1900.

[2] Phil. Mag., Vol. XIV., p. 186, 1882.

positions as *A*, *C*, *I* and *H*. One side of the pipe from the diaphragm to the end was removed and a glass panel substituted for it. This arrangement made the whole of the interior of the pipe visible from the diaphragm to any position of the movable stop. A slit about 8 mm. wide, extending from near the diaphragm to near the end, was cut into the upper side of the pipe. This slit was covered by a long wooden bar *B* of about 2.5 by 3 cm. in cross-section. The side of the pipe around this slit was lined with kid skin. The bar when pressed down upon this closed the slit and rendered the pipe air-tight. A glass tube *g*, carrying at the top a circular, graduated scale *s*, was inserted in the bar near its center. The torsion wire passed down this tube into the pipe. By this arrangement the torsion balance *T* could be moved along the pipe to any desired position.

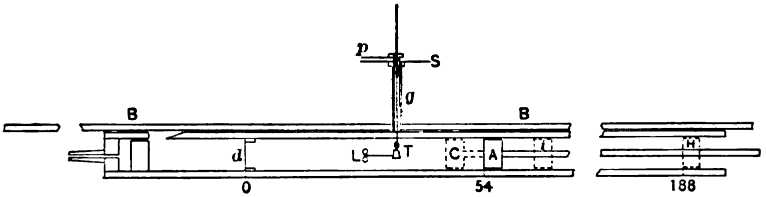

Fig. 1.

The diaphragm was placed at 16 cm. from the lip, as this was found to be the position of the node. A centimeter scale was placed along the pipe, the zero being at the diaphragm, *i. e.*, the first node.

The readings along this scale will also represent distances along the stationary wave. The pipe was 6.7 cm. by 5.5 cm. in cross-section, the height of the mouth opening being 3 cm.

The shorter pipe may also be represented by Fig. 1, by considering the end of the pipe to be at stop *A*, and this stop immovable. This pipe was 67.8 cm. long from lip to stop and 5.3 cm. by 6.4 cm. in cross-section. The height of the mouth opening was 1.8 cm. The arrangements for torsional measurements were the same as those of the longer pipe. The diaphragm was placed at 15.8 cm. from the lip as this was found to be the position of the node.

A mercury manometer was used to regulate and compare the forces of blowing in the various experiments. It consisted of a U tube of small bore containing mercury, which was inclined at an angle of 30° to the horizontal, so that the total difference in height of the mercury surfaces in the two branches was equal to the reading on a scale placed along one of the branches.

The torsion wire was No. .0035 platinum-silver. Its constant was determined by two methods: By observing the time of vibration of a suspended circular disc of known dimensions and mass; and by connecting the end of the torsion arm to the pointer of a chemical balance, and on turning the torsion head, observing the number of milligram necessary to bring the pointer back to zero.

The two methods gave the same result. The constant was such that a force of one dyne acting at the end of an arm one centimeter in length, attached to a wire one centimeter long produced a deflection of .3° of arc.

The cylinders, the force of whose tendency to move was to be measured, were of four sizes. Three of them were made from gelatine capsules such as are used for medical purposes. The other, the largest one, was made of stiff paper. The dimensions of the cylinders were as follows:

TABLE I.

	Dia. cm.	Area sq. cm.	Length cm.
No. 5.	.421	.14	1.3
No. 2.	.575	.26	3.1
No. 00.	.794	.495	3.1
Paper.	1.15	1.04	3.1

The hemispherical end of the capsule cylinder was removed, and a paper disc inserted in its stead, thus giving a flat end to the cylinder. The caps were removed and two or more joined end to end to form a cylinder of desired length.

EXPERIMENTS WITH ROTATING MILL.

A small mill was constructed by fastening four No. 00 capsule cylinders each 1.7 cm. long, on cardboard arms as shown in Fig. 2. The arms of the mill were each two centimeters long. The mill was mounted, by means of a glass pivot, upon a fine steel point which was fastened to the end of a long rod. By means of this rod the mill could be carried to any desired part of the stationary wave. The shorter pipe which was used in this experiment was placed in a vertical position. The rod bearing the mill was introduced through the stop at the lower end. The mill was thus enclosed in the pipe with its plane of rotation perpendicular to the stream-lines, *i. e.*, the vibrations were always perpendicular to the axes of the cylinders.

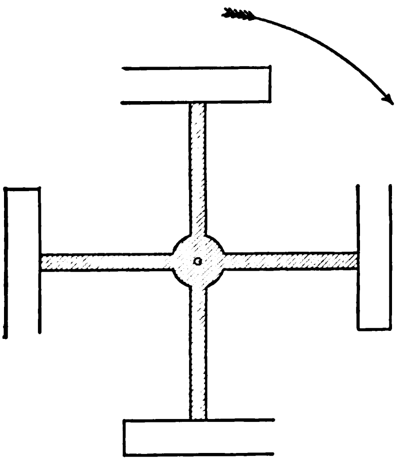

Fig. 2.

Matters being thus arranged when the pipe was blown, the mill rotated with a high velocity. The rate of rotation was greatest at the middle of the loop and it came to rest at the nodes.

A series of experiments were made to determine the variation of the rate of rotation with the position along the wave. A constant pressure of blowing was maintained throughout the experiments. The zero of the scale on the pipe is at the node at the diaphragm. A stroboscopic disc was used to count the rotations. The results of a number of experiments are given in the table below:

Table II.

Position along Pipe.	Relations per Second.	Position along Pipe.	Relations per Second.
0	0	27	9.00
4	.92	30	8.74
9	4.69	34	7.70
14	7.00	39	6.43
19	8.12	44	3.70
22	8.44	49	.77
25	8.80	52	.0

This table is plotted in the curve shown in Fig. 3. The ordinates are rates of rotation per second and the abscissæ are distances along pipe. The full line is the observed curve and the broken line is a sine curve. The agreement of the observed curve and the sine curve is very close.

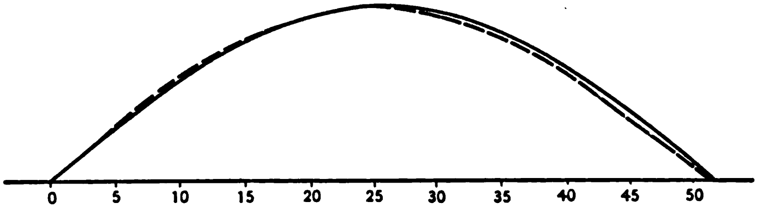

Fig. 3.

The force driving the cylinder across the pipe was measured by means of a torsion balance mounted in the longer pipe as shown in Fig. 1. Two cylinders *L* are mounted at the end of a cardboard arm, their plane being perpendicular to the pipe. They are placed a small distance apart to allow the vibration of the air between them. When the pipe was blown the cylinder moved towards the wall of the pipe and the torsion head was turned until the cylinders came back to their zero position. The amount of torsion was indicated by the pointer *p* on the scale *s*.

THE EFFECT OF AREA OF CROSS-SECTION OF THE CYLINDERS.

In making this investigation the mounted cyliders of various sizes were brought to the middle of the loop, the force being a maximum at that point. A table is given below of the degrees of torsion for the various cylinders. The sizes corresponding to the number of cylinders are given in Table I. The readings have all been reduced to the same area (1 sq. cm.) and the same length of arm. The manometer pressures are in centimeters of mercury.

It will be observed that for low pressure the small cylinders are affected the most. The No. 00 cylinders experience the largest force for moderate pressures as those near the pressure of maximum efficiency, while for the highest pressures the paper cylinder is most strongly affected. The No. 5 and No. 2 cylinders give almost identical results.

The shape of the body in the sound-wave also affects its behavior in a marked manner. Instead of the cylinders a hollow rectangular parallelopiped was used, which was 30 mm. long, 20

TABLE III.

Degrees of Torsion.

Pressures.	No. 5.	No. 2.	No. 00.	Paper.
1½	20	22	10	8.6
2	66	64	89	46
3	198	215	320	176
4	362	306	410	354
5	460	409	550	620
6	530	500	690	630
7	644	610	750	745
8	715	714	830	820
9	811	780	920	969
10	862	834	1000	1090
11	898	891	1030	1220
12	925	907	1055	1318
13½	928	923	1080	1360
15	927	923	1060	1260
16	Overtone.			

mm. wide and 2 mm. thick. The area of the closed end was 40 sq. cm. This was placed in the sound-wave successively in two positions. (1) The 20 mm. edge was placed *perpendicular* to the directions of the stream-lines (position A). (2) The 20 mm. edge was placed *parallel* to the direction of the stream-lines (position B).

The results of the experiment are given in Table IV. A and B are the readings obtained at the given pressures in the two positions mentioned above. A' and B' are the same readings reduced to the

TABLE IV.

Degrees of torsion.

Pressures.	A	B	A'	B'	C
1	2.5	0	3.8	0	5
2	45	0	68	0	89
3	116	5	174	7.6	320
4	150	30	228	46	410
5	179	92	270	140	550
7	190	120	288	185	750
10	193	150	290	228	1000
12	198	160	300	245	1055
15	228	215	345	325	1060
17	255	256	436	436	

area of 1 sq. cm. and same length of arm used with No. 00 cylinders in Table III. Column *C* contains the results for No. 00 cylinders brought over from Table III. for comparsion.

The force depends quite decidedly on the position of the parallelopiped with reference to the stream-lines except in the case of large amplitudes, when the values approach and become equal just as the pipe passed over into the second overtone. By comparison with *C* it will be noticed that the rectangular shape gives much less effect than the cylindrical.

VARIATION OF THE FORCE ALONG THE STATIONARY WAVE.

The No. 2 capsule cylinders were used in these experiments. Two of them were fastened to the end of the arm of the torsion balance. The arm was 5.33 cm. long. The torsion balance was moved along the pipe and readings taken at intervals from near the diaphragm to the stopped end. The pipe was blown at the constant pressure of 3 cm. of mercury throughout. The note obtained at this pressure was quite free from overtones. The results of the experiments are given in Table V.

TABLE V.

Position along Pipe.	Torsions.	Sq. root of Torsion.	Position along Pipe.	Torsions.	Sq. root of Torsion.
0	0°	0	29	190°	13.8
6	36	6	32	173	13.1
11	79	8.9	35	145	12
16	148	12.1	39	102	10.1
21	185	13.6	42	73	8.6
25	194	13.9	48	25	5.
27	198	14.1	54	0	0

This table is plotted in Fig. 4. The abscissæ are positions along pipe and the ordinates are *square roots* of torsions. The full line is

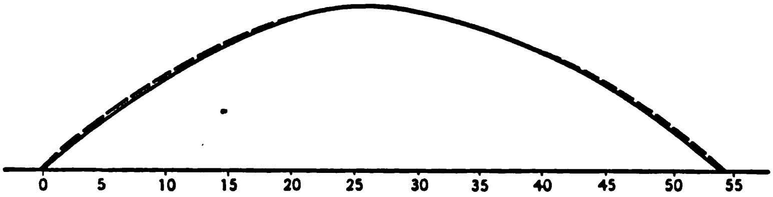

Fig. 4.

the observed curve and the broken line is a sine curve. The agreement of the two curves is quite close and indicates that the cylinders experience a force that is proportional to the square of the amplitude of vibration at any position.

EXPERIMENTS PERFORMED IN VARIOUS GASES.

The portion of the organ-pipe between the diaphragm and the stopped end forms a closed chamber which is kept free from all disturbances except the vibratory motion of air or gas. Various gases were introduced into this chamber and the force determined as before. The stationary wave that was formed was transmitted through two media: the gas back of the diaphragm and the air in front of it. Both pipes were used in these experiments, those with the shorter pipe are described first.

The stop was fixed and the length of the pipe could not be altered. When a dense gas as carbon dioxide was used, the pitch was lowered and the node was moved a little toward the stopped end. The pitch was raised when illuminating gas and hydrogen were used, and the node, at least in the case of the illuminating gas, was moved toward the mouth. The rise in pitch was quite noticeable with hydrogen. Measurements were made in the middle of the loop as before, care being taken to keep the pressure of blowing constant throughout the series of experiments.

The mean results of a number of experiments are given in the table below, the torsion readings in air as well as its density being taken as unity.

	Torsion Deflections.	Density of Gas.
Air	1.	1.
CO_2	1.47	1.52
Illum. gas	.77	.75
Hydrogen	.064	.069

The force obtained is closely proportioned to the density of gas used. The above results were obtained when the pipe was blown at moderate pressures. The relation does not hold for all pressures, as will be shown later in this paper.

The stop of the shorter pipe was fixed, and the node, when the chamber contained air, was at the diaphragm. The densities of carbon dioxide and illuminating gas do not differ from air sufficiently to displace the node very much and the change of pitch obtained was approximately as should be expected. When hydrogen was used the pitch of the pipe was raised, but not at all in proportion to the inverse of the square root of its density. The pitch was such as to throw the node, had a simple wave been formed, just outside the mouth of the pipe. This would seem to indicate that a simple wave was not formed in this case, but that the mode of vibration was influenced in an indeterminate manner by the air space between the diaphragm and the mouth. The compartment of air between the diaphragm and the mouth possibly acted partially as a pipe having a stop at the position of the diaphragm. On account of the small density of the hydrogen its energy was inconsiderable compared to that of the air column, so that the tone obtained was a compromise between that natural to the air space outside the diaphragm and that natural to the hydrogen column within.

The pitch natural to the pipe with a stop at the position of the diaphragm is about 330 vibrations per second, while that natural to the hydrogen column behind the diaphragm would have been above 1,200 vibrations per second. This would seem to affirm that the characteristic pitch was largely determined by the air column in front of and not by the hydrogen column back of the diaphragm.

The longer pipe was constructed at the suggestion of Dr. F. L. Tufts for the purpose of overcoming these difficulties. This pipe had a movable stop, which was placed so as to give the same note as the shorter pipe, and then the diaphragm was adjusted to the node. The shorter pipe was used as a standard of pitch. On introducing carbon dioxide the stop was moved up until the pipe spoke in unison with the shorter one. In the case of illuminating gas or hydrogen the stop was moved back until unison was obtained. The positions of the stop were: for CO_2, at 45 cm.; for air, at 54 cm.; for illuminating gas, at 62 cm., and for hydrogen, at 188 cm. from the diaphragm. These distances are nearly equal to one-half of the length of the stationary wave formed in each case. This arrangement made the conditions at the mouth and diaphragm constant for all the gases. The form of the exterior wave was not altered.

The efficiency curves of the organ-pipe for the gases were obtained by this arrangement, the torsion balance being moved so as to be in the middle of the loop of the stationary wave in all cases. The pipe was first blown at low pressure, and then the pressure was increased until it passed over into the next overtone. Table VI. contains the results of the experiments with various gases. The range of pressure of blowing in the case of hydrogen was so small that it was necessary to use a water manometer and afterward reduce the pressure to the corresponding mercury manometer readings. The torsion balance carried two No. 00 cylinders each 3.1 cm. long.

Table VI.

	Degrees of Torsion.			*Hydrogen.*	
Pressure.	**Air.**	**Illum. gas.**	**CO_2.**	**Pres.**	**Torsion.**
1	9	5	12	1.22	1.3
1½	69	45	90	1.47	3.8
2	195	120	290	1.62	10.9
2½	400	275	530	1.76	15.
3	510	365	690	1.91	26.
4	760	530	880	2.06	34.
5	920	670	1100	2.20	53.
6	1120	875	1170	2.41	64.
7	1230	1140	1200	2.50	overtone.
8	1250	1160	1220		
9	1260	1240	1210		
10	1270	1260	1230		
11	1290	1260	1230		
12	1320	1210	1250		
13	1470	1130	1280		
14	1500	overtone.	1260		
15	1570		1250		
16	overtone.		1350		
17			1520		
18			1560		
19			1560		
21			1500		
22			overtone.		

This table is plotted in Fig. 5, with pressures of blowing as ordinates and the square roots of torsion as abscissæ. It will be shown later that the amplitude of vibration is proportional to the square root of the force acting on the cylinders, hence the curves obtained represent the relation between the pressures of blowing and the amplitude of vibration of the gas particles in the middle of the loop.

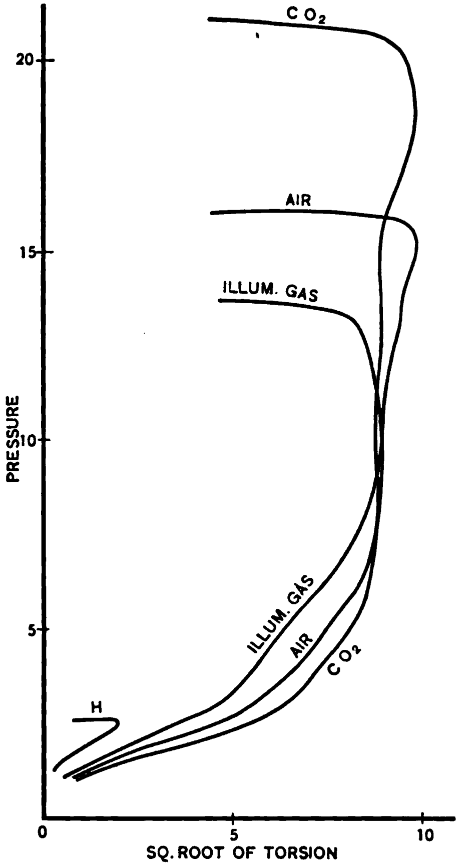

Fig. 5.

As a result of some experiment conducted by Dr. F. L. Tufts, it has been found that the cube of the velocity of efflux of air from the slit of an organ-pipe is approximately proportional to the square of the pressure. The energy of an escaping jet of air is also proportional to the cube of the velocity, hence the energy delivered to the pipe is proportional to the square of the pressure of blowing. The energy contained in the vibrating air is proportional to the square of the amplitude, consequently the torsions represent the energy converted into sound.

The curves of relation between torsions and squares of pressures plotted in Fig. 6, are the efficiency curves of the organ-pipe. The pressure of maximum efficiency is indicated by the ordinates of the points of tangency of the tangents to the curves through the origin. This pressure was found to be nearly the same for all gases, being somewhat less for hydrogen. In the case of carbon dioxide, air and illuminating gas it was about 2.3 cm. of murcury. The torsions obtained at this pressure are given in the table below, the values for air as well as its density being taken as unity.

This is nearly the same result obtained with the shorter pipe.

Table VII.

	Torsions.	Density of Gas.
Air	1.	1.
CO_2	1.53	1.52
Illum. gas	.71	.75
Hydrogen	.153	.069

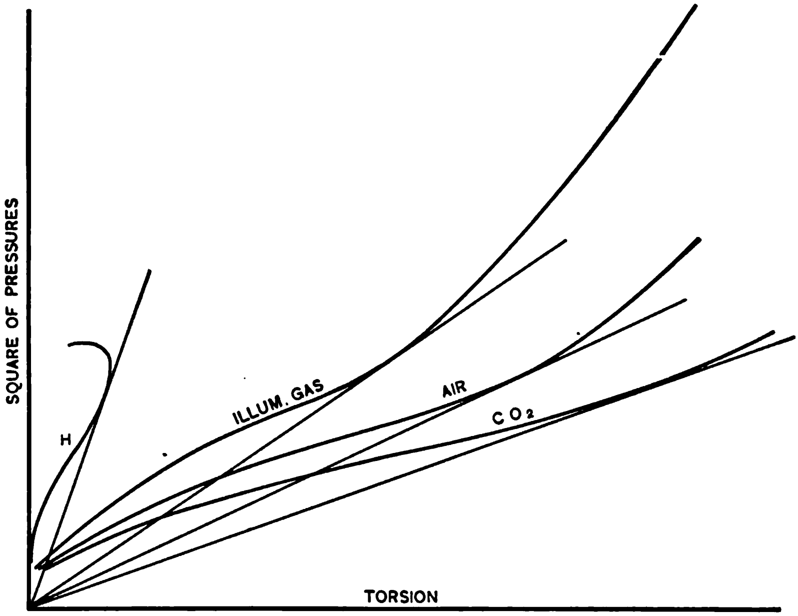

Fig. 6.

In the experiments the results of which are given in Table VI., the pressure was increased from zero until the note passed over into the next overtone. The pressure at which the second overtone was obtained was not sharply defined. At a high pressure the tone was in an unstable condition, so that when the finger was brought near the mouth of the pipe, the equilibrium was destroyed and the second overtone obtained. The pressure at which the second overtone appeared varied with the gas. The length of the pipe varied inversely as the density of the gas, so this shows the well-known fact that the longer a vibrating column, the more easily it breaks into shorter waves. The hydrogen wave being very long bu a small pressure was necessary to force the next overtone.

The rates of rotation of the mill previously mentioned were also taken in the various gases. The shorter pipe was again used for this purpose. It was placed vertically and the mill introduced from below as before. The pressure of blowing was kept constant throughout the experiments. The mean results obtained are given below, the rate of rotation in air as well as its density being taken as unity.

Air.	Illum. Gas.	CO_2.	Hydrogen.
1	1	1.04	.59

The rate is apparently constant for all gases, although the force acting on the mill to produce this rotation is proportional to the density of the gas. This can be explained by the circumstance that the resistance to motion is also proportional to the density. The deviation from this result in the case of hydrogen is probably due to the fact that the friction of the supporting pivot was not an inconsiderable quantity compared to the total resistance experienced by the cylinders, and this factor is proportionately larger in the case of hydrogen.

DISCUSSION.

This phenomenon is another instance of the relation between pressure, density and velocity of a fluid first expressed analytically by Daniel Bernouilli.[1] A fluid in motion is less dense than the same fluid at rest. The air within the small cylinder remains nearly at rest, while that without is in rapid motion. The air on the outside is less dense than that within the cylinder. The force exerted is due to the difference in density and pressure on the two sides of the closed end of the cylinder. It is evident from the formula deduced below, that if the force is measured in absolute units then a value for the velocity and amplitude may be found.

The relation between the velocity, pressure, and density of a fluid may be expressed by Bernouilli's equation since for this case the average state of the fluid only is under consideration.

$$\int \frac{dp}{\rho} = R - \tfrac{1}{2}u^2.$$

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[1] Hydrodynamica, Argentorati, 1728.

R is the potential due to external forces and is negligible in this case; hence

$$\int \frac{dp}{\rho} = -\tfrac{1}{2}u^2.$$

The adiabatic gas relation gives $p = C\rho^\gamma$, where C is a constant and γ is the ratio of the specific heat at constant pressure to that at constant volume.

$$C^{\frac{1}{\gamma}} \int p^{-\frac{1}{\gamma}}\, dp = \tfrac{1}{2}u^2.$$

$$\left[\frac{\gamma}{\gamma - 1} C^{\frac{1}{\gamma}} p^{\frac{\gamma-1}{\gamma}}\right]_1^2 = [-\tfrac{1}{2}u^2]_1^2.$$

$$\text{Let } \frac{\gamma - 1}{\gamma} = n \quad \text{and } A = \frac{\gamma}{\gamma - 1} C^{\frac{1}{\gamma}}$$

$$A(p_2^n - p_1^n) = -\tfrac{1}{2}(u_2^2 - u_1^2)$$

$$\frac{1}{n} \cdot \frac{p_2}{\rho}\left[1 - \left(1 - \frac{p_2 - p_1}{p_2}\right)^n\right] = -\tfrac{1}{2}(u_2^2 - u_1^2)$$

Expanding and neglecting all powers above the first, since $\frac{p_2 - p_1}{p_2}$ is a very small quantity, the equation reduces to, $u_1^2 - u_2^2 = 2\frac{p_2 - p_1}{\rho}$.

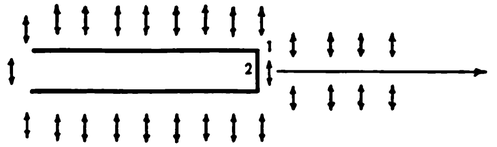

Fig. 7.

The cylinder is in the moving air as shown in Fig. 7. u_1 represents the velocity of the air outside the cylinder, and u_2 represents the velocity within the cylinder next to the closed end, while p_2 and p_1 are the pressures on the two sides of the closed end. If the air within the cylinder is at rest u_2 is zero, hence

$$u_1^2 = 2\frac{p_2 - p_1}{\rho} \qquad (1)$$

The constant of the torsion wire as previously given was .3° per dyne of force. The wire was 16 cm. long and the arm was 4.82 cm.

in length. The maximum deflection obtained was 1570°, hence the force was 68.26 dynes per square centimeter.

The force experienced by the cylinders is the mean of all the instantaneous values due to the velocity of the air at any instant. The force is proportional to the square of the velocity, hence the mean force will not directly give the mean velocity in equation (1).

The force F at any instant is equal to $\frac{1}{2}\rho u^2$, where u is the instantaneous velocity. Let A be the maximum displacement, and x any displacement at time t.

$$x = A \cos nt.$$

$$n = \frac{2\Pi}{T},$$

where T is the periodic time.

$$\frac{dx}{dt} = u = -An \sin nt.$$

Also

$$\frac{\int F dt}{\int dt} = p_2 - p_1,$$

the mean force experienced by the cylinders.

Therefore:

$$\frac{\int F dt}{\int dt} = \frac{\frac{1}{2}\rho \int n^2 A^2 \sin^2 nt\, dt}{\int dt}.$$

The limits of integration are 0 and $\frac{1}{2} T$; hence,

$$p_2 - p_1 = \rho A^2 \frac{\Pi^2}{T}$$

and

$$A = \frac{T}{\Pi} \sqrt{\frac{p_2 - p_1}{\rho}}.$$

The frequency of the pipe when blown at high pressure was 338 per second.

Therefore:

$$A = .216 \text{ cm}.$$

The total excursion of the particles was $2A = .432$ cm.

This must be a minimum value, that is less than the real value as the investigation above assumes that the air within the cylinder is at rest, and that there is no friction between the surface of the cylinder and the vibrating air. The effect of such friction would be to decrease the velocity and surround the cylinder with a skin of air of little greater density.

Among the methods that have been devised to determine the amplitude of vibration in a stationary wave, that of Raps[1] might be mentioned. He measured the change in density at the node of an organ-pipe by a rather complex mechanical method and deduced the amplitude from such measurements. He obtained an amplitude of .34 cm., the pipe speaking its fundamental of 184 vibrations per second. The force that might have been obtained had small cylinders been used would have been 50.25 dynes per sq. cm. This does not differ greatly from the maximum force obtained in the experiments above described, although from the very nature of the investigation accurate comparison of results is not possible.

A consideration of the curves of efficiency in Fig. 6 shows that the pressure of maximum efficiency for carbon dioxide, illuminating gas and air is nearly the same, also by Table VII. the force at this pressure is proportional to the density of the gas. This confirms experimentally the statement of Lord Kelvin[2] in his letters to Guthrie, that the change in density, the conditions being the same, would be proportional to the density.

At the blowing pressure of maximum efficiency the value of u in equation (1) is constant and as the frequency is the same, the amplitude of vibration is the same for all gases.

The energy of the sound produced would be proportional to $\frac{1}{2}\rho u^2$, and since the velocity u was constant, the energy of the sound should be proportional to the density of the gas. This was observed to be the case. The volume of sound emitted by the pipe was greater with the carbon dioxide, than with the air wave, while in the case of hydrogen it was very weak.

The curves in Fig. 5 show that at pressures from 9 cm. to 11 cm, of mercury the force was the same for the three gases, hence at these pressures the amplitudes were inversely as the square roots of the densities. The less the density the greater the velocity, so that the energy of the sound produced was the same for the three gases at these pressures.

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[1] Wied. Ann., New Series, XXXVI., p. 273.

[2] Phil. Mag., XLI., p. 405.

There are a number of natural phenomena that have their explanation in this relation between pressure, density and velocity of a fluid. Perhaps the best known one is the Clemént effect, in which a flat disc is held against an escaping jet of air. This is also illustrated by the ball-nozzle so extensively in use. The investigations of Hallock[1] on the Clemént disc show that the pressure is much decreased at the point where the velocity is greatest.

Another phenomenon is the acoustic attractions of Guthrie which Lord Kelvin[2] has shown to depend on this law also.

A very striking effect is the rotating resonators of Dvořák,[3] the action of which has been explained by Lord Rayleigh.[4] Still another effect is the setting of a disc across the stream-lines in a sound-wave obtained by Lord Rayleigh,[5] and which has been used by Boys[6] for the detection of faint sounds.

The magnitude of the force described in this paper is greater than the tendency of a disc to set across the stream-lines. An instrument designed on the plan of Boys' using small cylinders instead of a disc would probably be quite as sensitive as the one constructed by him.

PHYSICAL LABORATORY, COLUMBIA UNIVERSITY,
May 1, 1901.

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[1] Science, Aug. 2, 1895.
[2] Phil. Mag., XLI., p. 405.
[3] Pogg. Ann., CLVII., p. 42; Wied. Ann., III., p. 308.
[4] Theory of Sound, Vol. II., p. 42.
[5] Phil. Mag., 5th Series, Vol. XIV., p. 186.
[6] Nature, Vol. XLII., p. 604.

ON THE HEAT EVOLVED WHEN LIQUIDS ARE BROUGHT IN CONTACT WITH POWDERS.

BY C. E. LINEBARGER.

INTRODUCTION.

THE fact that there is a rise of temperature when liquids are mixed with fine powders was first noted by Pouillet,[1] and his results have been confirmed and extended by a number of subsequent investigators.[2] Nearly all of these investigations were purely thermometric, and the few that were calorimetric either employed rather crude methods or but few liquids, as water, alcohol and benzene. The object of this paper is to present the results of a number of determinations of the heat effect due to the bringing of different simple liquids in contact with silica powders of different degrees of fineness in order to ascertain the influence of the nature of the liquid as well as the fineness of the powder on the heat effect. The starting point of the investigation was an attempt to experimentally verify certain theoretical relationships between heat and surface energy, sketched by Gibbs;[3] as the experiments did not, however, corroborate the relationships, only the experimental work will be given here.

MATERIAL.

The powders were all of precipitated silica, several specimens of which from different sources were used. The method of preparation renders the assumption that they consist of approximately spherical granules of uniform size quite plausible. As a matter of fact, if a microscopic slide be dusted over with precipitated silica, and the powder examined under a high power, the particles are seen to be of fairly uniform size. By means of a stage micrometer the diameters of a large number of these granules were measured, from the average of which the surface and volume of an average granule could be calculated. Such an examination of four samples from different sources, one of which was furnished by Professor Martini[1] and was the same as that used by him in his investigations, resulted in an average diameter of 0.0005 cm. All four samples gave practically the same results. Two stage micrometers were employed, and measurements of several hundred different granules of each sample made. The silica having granules of this size will hereafter be designated as silica I.

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[1] Ann de chim. et de physique, XXII., 141 (1822).

[2] Junck, Pogg. Ann., CXXV., 292 (1865). Cantoni, Rendiconti del R. Istituto Lombardo, VIII., 135 (1866). Melsens, Mémoires del l'Academie de Belgique, XXIII. (1873). Chappuis, Wied. Ann., XIX., 21 (1883). Wiedemann and Lüdeking, Wied. Ann., XXV., 145 (1885). Meissner, Wied. Ann., XIX., 114 (1886). Gore, Phil. Mag., XXXVII., 306 (1894). Martini, Atti del R. Istituto Veneto, VIII. (1896); IX., (1897), XII. (1900).

[3] Thermodynamische Studien, page 316.

A fifth sample (a pound bottle of Koenig's) was found to consist of granules of an average diameter of 0.00096 cm. This sample will be called silica II. It is noteworthy that the granules of silica II. happened to have about twice the diameter of those of silica I.

The specific heat of precipitated silica does not seem to have been determined. From the determinations of the specific heat of various forms of quartz by Regnault,[2] Joly,[3] Bartoli,[4] and Pionchon,[5] the specific heat of precipitated silica is very near to 0.2, and that value has been used in this work. The specific heat of the glass container has also been taken as 0.2.

The methods of preparation of the liquids as well as some of their physical constants are given in what follows:

Benzene.—Eight pounds of "c. p." benzene were treated many times with small portions of strong sulphuric acid to remove traces of thiophene, and fractionally crystallized until about five pounds were obtained melting at 5°.38 to 5°.44 (corr.). It was then distilled, the first and last runnings being rejected. It boiled at 80°.1 (corr.) under a pressure of 757.1 mm. of mercury.

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[1] Loc. cit.

[2] Ann. de chim. et de physique (3), I., 129 (1841).

[3] Proc. Roy. Soc., XLI., 250 (1887).

[4] Nuovo Cimento (3), XV., 5 (1884).

[5] Comptes rendus, CVI., 1344 (1888).

Its specific heat was taken as 0.42 from Pickering,[1] DeHeen and Deruyt's,[2] Schiff's[3] and Regnault's[4] determinations.

Toluene.—Five pounds of toluene were fractionally distilled until about three pounds were obtained boiling at 109°.9 to 110°.0 (corr.) under a pressure of 757.5 mm. This portion was redistilled with a short column and the first and last runnings rejected.

Its specific heat was taken as 0.42 from the results of DeHeen and Deruyt,[10] and Schiff.[5]

Nitrobenzene.—Five pounds were fractionally crystallized until about three pounds resulted showing an almost constant melting point. This was distilled, the first and last runnings being rejected. From Regnault's[6] measurements its specific heat may be taken as 0.337.

Pyridine.—Five pounds of "c. p." pyridine after thorough drying were fractionally distilled with a long column. After three distillations about two pounds were obtained boiling at 114°.6 to 114°.8. Luginin[7] gives its specific heat as 0.4315.

PREPARATION OF THE POWDER.

Precipitated silica absorbs large amounts of air and other gases as well as the vapors of liquids. These can be removed only by heating to a high temperature in a vacuum. As the presence of absorbed substance has a considerable influence on the heat effect of wetting the silica with liquids, it has to be carefully removed to insure uniform results. This was done as follows: Large test-tubes about 2.5 cm. in diameter were drawn down to a narrow neck at about 6 cm. from their closed end, heated in an air-bath for some time to dry them thoroughly, cooled and weighed to tenths of a milligram. The silica was heated for an hour or so in an iron crucible at a red heat. While still hot, the proper amount was introduced into one of the tubes which was grasped with a test-tube holder and heated to incipient redness until no more gas or vapor was given off. It was then without delay connected with a Boltwood air pump, and the evacuation started. When the pressure had fallen to less than a millimeter, the tube was heated as hot as was possible without the distortion of the glass, and then sealed off with a blowpipe flame. Usually some gas was driven off by this last heating, but not in sufficient amount to affect the gauge. The bulb thus filled with silica and the remnant of the test-tube were then weighed separately in order to ascertain the weight of the silica as well as that of the container.

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[1] Proc. Roy. Soc., XLIX., 11 (1890).

[2] Bull. de l'Academie de Bruxelles (3), XV., 168 (1888).

[3] Liebig's Ann., 234, 300 (1886).

[4] Mémoires de l'Academie, XXVI., 262 (1862).

[5] Liebig's Ann., 234, 300 (1886).

[6] Ann. de chim. et de physique (3), IX., 322 (1843).

[7] Comptes rendus, CXXVIII., 366 (1899).

APPARATUS.

The *calorimeter* was a cylindrical vessel of thin, nickel-plated brass, 6 cm. high and 7 cm. wide, holding about 250 cc. This was held firmly inside a glass beaker by means of conical cork supports glued to the glass. In the same fashion this beaker was placed inside a larger beaker, which in turn stood on cork supports inside a battery jar. There were thus three layers of glass and of air separating the calorimeter laterally from the outside air. The battery jar was tightly closed with a thick piece of paraffined wood pierced with holes for the passage of the thermometer and the device for breaking the bulb. The whole was placed in a fume chamber where the temperature could be kept almost constant for several hours. The water-equivalent of the calorimeter was found by special experiment to be 3.99 cal.

The *thermometer* was a Beckmann instrument graduated in hundredths, and made by F. O. R. Götze in Leipzig. The readings were taken with a good hand lens so arranged as to reduce the error due to parallax to a minimum. The water-equivalent of the thermometer was found by special experiment to be 0.94 cal.

The "*crusher*." To break the bulb when immersed in the liquid in the calorimeter, after ineffectual efforts to employ to advantage a hammer with sharp points also immersed in the liquid, the following "crusher" was devised. A piece of thin brass tubing, 3.2 cm. in diameter and 2.8 cm. long was cut out and filed down so that it took the form of two rings connected by two narrow cross-bars. A small nut was soldered in the middle on the outside of one of these cross-bars and around it a tube of thin brass, 0.4 cm. in bore and 4 cm. long. A piece of glass tubing was fastened with cement over one end of this tube, thus serving as a handle of poor-conducting material. In the nut played a screw 6 cm. long, the upper end of which was flattened so as to engage in a split in the end of a brass rod a little longer and a little narrower than the glass tube. When a bulb containing the silica was placed in the cut-out brass tubing, the screw could be turned down so as to hold it securely. A turn or so more of the screw then sufficed to break the tube. This ordinarily broke in a few pieces so that the powder was almost instantaneously introduced into the liquid. The shape of the "crusher" was such as to make quite an efficient stirrer. The water-equivalent of the "crusher" was 1.35 cal.

MODE OF MAKING A DETERMINATION.

The calorimeter is filled about three-fourths full of the liquid (which had been kept standing in the fume chamber so as to be at the same temperature as the apparatus) and weighed to centigrams. It is unnecessary to weigh the liquid with a greater accuracy than this as the evaporation during the determination amounts to several centigrams. The calorimeter is then placed within the beaker, and the tube filled with the powder clamped in the "crusher" and immersed in the liquid which rises nearly to the glass handle. The cover to the jar is set on and the thermometer inserted.

Readings of the thermometer are taken at intervals of about ten minutes (care being taken that the temperature of the fume chamber be as constant as possible and that the liquid be stirred occasionally) until they do not differ by more than 0°.05. When this is the case the thermometer is read every minute, the liquid being stirred for about fifteen seconds after each reading and the thermometer being smartly tapped with a pencil. If the readings change uniformly by less than 0°.005, for at least ten minutes, the bulb is crushed, the brass rod removed, and gentle but thorough stirring kept up until the readings are again uniform. Commonly this is the case in less than two minutes. The thermometer is again read at intervals of one minute with stirring of fifteen seconds' duration as before for ten minutes. The change of temperature can then be calculated in the customary way, and is known to within 0°.005.

CALCULATIONS AND DATA.

It was assumed in the calculations that only the calorimeter, thermometer, "crusher," container, powder and liquid were to be taken into consideration. The water-equivalents of these were added together and multiplied by the temperature change. To refer all heat effects to that due to one gram of precipitated silica, this product was divided by the weight of the powder taken in each particular experiment. The data are given in the following table; the superscriptions render any detailed explanation unnecessary.

Silica.	Liquid.	Weight of Silica.	Weight of bulb.	Weight of liquid.	Rise of Temperature.	Cal. per gram of Silica.
I	H_2O	4.8467	4.8060	130.04	0.479	13.80
I	H_2O	4.3528	4.4411	129.98	0.435	13.81
II	H_2O	4.4308	4.5295	155.70	0.188	6.90
I	$C_6H_5NO_2$	4.1826	6.6047	174.49	0.744	11.21
I	$C_6H_5NO_2$	4.8915	6.1085	167.00	0.822	10.88
II	$C_6H_5NO_2$	4.5865	6.8934	179.51	0.443	6.67
II	$C_6H_5NO_2$	4.4446	4.2122	159.92	0.444	6.02
II	$C_6H_5NO_2$	5.1788	6.3000	170.99	0.482	6.13
I	C_7H_8	5.1135	6.8186	125.70	0.802	9.61
I	C_7H_8	5.4251	4.2354	107.41	0.750	7.55
II	C_7H_8	4.2034	6.0508	105.09	0.388	4.84
I	C_6H_6	6.4765	6.0427	126.30	0.428	4.09
II	C_6H_6	4.9773	5.8233	180.00	0.336	4.06
II	C_6H_6	5.9091	7.0179	130.08	0.383	4.11
I	C_6H_5N	5.1881	6.2174	142.71	0.876	11.84
I	C_6H_5N	4.5984	7.8442	146.40	0.797	12.46
II	C_6H_5N	5.3244	6.7236	140.00	0.382	4.89
II	C_6H_5N	5.4922	6.9446	140.00	0.409	5.07

DISCUSSION OF RESULTS.

The relative amount of liquid does not seem to influence the heat effect. A certain minimum quantity is necessary to produce the maximum heat change, but any excess of that amount has no perceptible influence. The heat effect per unit mass of the powder is the same for the same liquid at the same temperature, provided this minimum amount be present.

The finer the powder, the greater the heat effect, seems in general to be true. In the case of water, toluene, pyridine and nitrobenzene, the heat effect with silica I. is about twice that of silica II., and the granules of the first powder are about double the size of those of the second. But in the case of benzene no such differences are apparent. Here about the same heat effect is produced with either powder. While qualitatively it undoubtedly holds that the finer the powder, the greater the heat effect, it has not been shown quantitatively that such is the case. It is noteworthy that the different liquids do not vary much in their heat effects; water gives the largest and benzene the smallest. But pyridine and nitrobenzene, although very similar to benzene chemically, exhibit heat effects which approach that of water.

In conclusion, it is a pleasant duty to thank the members of the Board of Education of Chicago for their kindness in granting the writer the necessary means for carrying out this work. It is perhaps unique in the history of secondary schools that a board has voluntarily set aside a fund to aid in the prosecution of research in natural science.

CHICAGO, May 3, 1901.

ON THE DENSITY OF ICE.

By H. T. Barnes.

IN a very complete and comprehensive paper published in this Review by Professor E. L. Nichols,[1] some interesting new points were brought out in connection with the density of ice of different ages, besides an exhaustive résumé and criticism of our knowledge of the absolute value of the density of ice.

It appears that this is subject to variation, depending on the manner of formation, and according to Nichols' measurements a difference of two parts in one thousand is noted between old and new ice. The rapidity of formation apparently fixes, at least temporarily, the value of the density, but as is shown, the greater density, possessed by new natural ice or artificial ice which is formed rapidly at a low temperature, does not persist, but tends to pass to a lower value which is probably more nearly equal to the true density.

The many and varied phenomena connected with the formation of river ice in our northern climate at Montreal makes a study, such as this one of much interest.[2] Until the work of Nichols appeared, there were no decisive experiments involving the actual measurement of a variation in the density of ice, but since such has been unquestionably indicated, the possibility of the varying density of the ice crystals, which are formed exceedingly rapidly throughout the mass of a quickly-flowing river by the admixture of cold air, suggests itself. To account for certain phenomena, this seems to be a necessary premise, but until more decisive measurements have been attained little definite use can be made of it.

In regard to the influence of age on the density of ice, a matter of some interest, the author undertook during the past winter a few experiments, with the kind aid of Mr. H. Lester Cooke, B.A., to determine the difference, if any, between ice two years old and ice which had been recently formed during the winter. Blocks of beautifully clear ice, which had been cut during the winter of 1898–99 and during the winter of 1899–1900 from the St. Lawrence River, were kindly supplied to us by Mr. Beckets from the storage houses of the City Ice Co., together with several similar blocks taken from the river. These were kept intact, carefully labeled and buried in a pile of soft, newly fallen snow. The chief difficulty appeared to be to select a suitable method for the determination of the density, which would be easy of operation and sufficiently accurate to show the small differences to be expected. It was finally decided to weigh a small block of ice directly in water at 0° C., in preference to selecting any of the other methods hitherto used. At first sight this method appears to be open to every objection, but, by suitable arrangements of the experimental conditions, it has undoubtedly yielded us the most satisfactory results in the shortest time. It is, of course, essential to have both the ice and water absolutely at the zero point, which, with the great supply of ice and snow available during the winter for the production of freezing-point mixtures, offers no difficulties whatever. The method consists in surrounding a copper vessel (Fig. 1), by a mixture of pure snow and water, and thereby cooling the air in the vessel to zero. Through a narrow neck in the cover of the vessel, which is securely bolted down air-tight, passes a light suspension from a weighted grip, consisting of three fingers for clutching the ice, to one of the arms of a sensitive Oertling balance. Near the bottom of the vessel a side tube is fastened connected by rubber with a copper or glass tube passing up through the mixture. By pulling out the copper tube, the surrounding water at zero runs in and fills the vessel completely into the narrow neck. Small particles of ice carried in through the side

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[1] Physical Review, 8, 21 (1899).

[2] See the author's papers on river ice formation, Trans. Roy. Soc. Can., 2, 37 (1896) 3, 17 (1897), 5, 17 (1899).

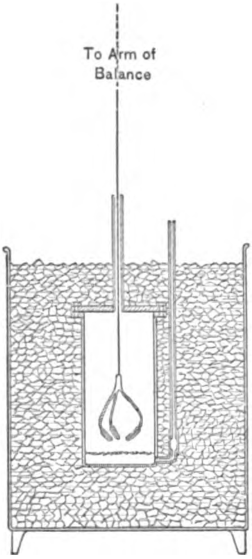

Fig. 1.

tube are filtered out by a fine wire gauze placed near the bottom of the vessel. The purity of the snow and water was a matter which demanded special attention, as well as thecleanliness of the copper vessels employed.

Careful determinations of the specific gravity of the suspension were made, under exactly similar conditions to those existing during an experiment, the water rising to the same point in the neck and including the same, or approximately the same, length of suspension wire in each case.

To carry out an experiment, blocks of ice were broken off from the main lump, in such a way as to include clear pieces free from cracks or air bubbles, and were then rounded off on the sharp edges by scraping and washing in cold water. Pieces of about 200 grams were thus prepared from the different lumps, and set aside under a protective cover surrounded by snow. The copper vessel having been carefully dried and partially packed in snow, the cover was removed and the whole taken outside the building into the air, which was not far from the freezing point during the days on which the measurements were made. The piece of ice selected was then carefully wiped dry with filter paper and a linen rag, and, without coming in direct contact with the hand, was introduced into the three-prong grip and quickly inserted in the copper vessel. The cover was then securely fastened down, and the complete vessel immersed in the snow and water. The mixture was then taken to the laboratory, where the weighings were made, and the suspension connected to the arm of the balance. A certain time was allowed for the ice to assume the temperature of the air in the vessel, which was not long because the temperature of the ice was nearly, if not always, zero when introduced. A slight evaporation from the ice was noticed when thus suspended, by its growing steadily lighter, but this never amounted to more than a milligram on the total weight of about 250 grams, during the time. As soon as the weight was accurately determined, the ice water was allowed to run in and fill the vessel. The loss in weight was then determined by the new weighing. Should the ice have been colder than the water, so as to have increased at the expense of the water in contact with it, the weight would not have been constant, but would have steadily diminished. On the other hand, if the water was not at zero but had been slightly above, some ice would have been melted and the weight steadily increased. As a matter of fact this happened after an immersion of about half an hour, but during the first twenty minutes remained very constant.

For a second measurement, the vessel was removed from the mixture, emptied and carefully dried inside. The ice being also removed, the suspension was dried and weighed.

Careful determinations had to be made of the density of the water used in the snow mixture, which was the river water supplied to the college mains. This, during the winter months, was very clear and quite pure. The observations were made on water emptied from the vessel after a measurement.

The following table gives the result of seven determinations which we made by this method for the different pieces of ice:

ON THE DENSITY OF ICE. METHOD OF WEIGHING IN WATER AT 0° C.

Date.	Year of Formation.	Density.	Difference from Mean.
March 9th.	1901	.91684	.00023
" "	1901	.91665	.00004
" "	1900	.91661	.00000
" 16th.	1900	.91642	.00019
" "	1899	.91650	.00011
" "	1899	.91648	.00013
" 23d.	1900	.91678	.00017

Mean = .916611 ± .000065.

The deviations are not larger than would have been obtained by other methods, and may be ascribed partly to variations in the density of the different pieces. We think that a check of the accuracy of the method is afforded by the two determinations of 1899 ice, the second measurement being made with the same piece of ice. This was done by removing the piece after the first measurement, drying it carefully outside in the cold air and repeating the observations. The two determinations thus obtained are .91650 and .91648. All the other measurements were for different pieces varying in size.

The mean value obtained is somewhat lower than the result given by Nichols for river ice, and higher than given by him for artificial ice, but agrees well with his measurements for old river ice.

It is evident that, according to the present determinations, the density of old and new river ice is the same. It appears that the density of the clear river ice may be taken as

$$.91661 \pm .00007$$

which agrees to two parts in 10,000 with the value given by Nichols for natural river ice not newly cut, which is .91644, and to the same order of accuracy with the measurements of Bunsen.

McGill University, April 27, 1901.

NOTE.

Henry Augustus Rowland, Physicist.—Professor Rowland was born at Honesdale, Pa., on November 27, 1848. He studied civil engineering at the Rensselaer Polytechnic Institute where he received his first degree in 1870. After two years, spent partly in the field as a surveyor and partly in teaching at Wooster College, Ohio, he returned to Troy as instructor in physics. In 1874, he was promoted to an assistant professorship.

In 1875 Rowland, whose extraordinary gifts were becoming known, was offered a place in the newly organized faculty of the Johns Hopkins University, and after a year abroad, a portion of which was spent in the laboratory of von Helmholtz, he entered upon his duties at Baltimore as head of the department of physics, which position he held until his death on April 16, 1901. Although his stay in Berlin was brief, the greater part of his year abroad being devoted to visiting the great centers of scientific activity and in collecting apparatus for his new laboratory, he found time to carry to completion a very important research on the Magnetic Action of a Moving Electrostatic Charge. The impression made upon his coworkers in the laboratory by this piece of work was a profound one, not less on account of the fundamental character of the experiment than of the extraordinary quickness with which the definitive result was reached. Such celerity of performance was quite beyond the experience of the German investigators who were accustomed to devote months and years to the establishment and verification of their results; but although there was much doubtful shaking of heads over the restless haste of the young American, his power as an investigator was fully recognized by the eminent director of the laboratory,[1] who paid Rowland the then very unusual compliment of making his work the subject of an immediate report to the Berlin Academy of Sciences.

The subsequent history of this research is an interesting one. Lecher[2] in 1884 repeated the experiment with seemingly unimportant modifications, but obtained wholly a negative result. Roentgen[3] in 1888 confirmed the existence of the effect. The following year, Rowland, in collaboration with Hutchinson, attacked the problem again with much more elaborate apparatus. They not only reëstablished the existence of the phenomenon to their own satisfaction, but were able to show that the observed results were in fair agreement with the computed values and with those obtained in the original experiment.[1] Himstedt[2] simultaneously and independently made measurements with an apparatus essentially similar to that of Rowland and Hutchinson. He likewise found an easily measurable effect; indeed his instrument was of sufficient sensitiveness in some instances to give deflections of more than 100 scale divisions. Himstedt hints that the failure of Lecher to obtain a positive result was probably due to a lack of sensitiveness in his apparatus. This seems hardly probable, since Lecher used an astatic needle in the axis of a vertical disk 200 centimeters in diameter, charged to 5,000 volts and driven at the rate of 200 revolutions per second. The charge was measured by means of an absolute electrometer. In 1889 Lippmann[3] called attention to the reciprocal of the Rowland effect and showed that variations in a magnetic field ought to produce a movement of electrified bodies placed therein. In 1897 Cremieu,[4] at his suggestion, undertook to test this conclusion experimentally by means of an apparatus with which he was led to expect, in accordance with his computations, an effect amounting to about one-thousandth of a dyne. He, however, obtained a purely negative result. In subsequent experiments[5] he obtained no deflection where the computed effect should have been 140 millimeters. A repetition of the original experiment with apparatus essentially similar to that employed by Rowland, and Hutchinson and by Himstedt likewise gave no deflection although the computed effect was 175 millimeters. Cremieu considers that he has discovered the source of the discrepancy between his results and those of his predecessors. He argues that the effects obtained with the earlier forms of apparatus are not due directly to the magnetic action of the moving electric charge, but to a rapid succession of instantaneous currents set up in the intervening sectors of metal under the action of the charged sectors of the moving disk.

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[1] Helmholtz: Berl. Berichte, 1876.

[2] Lecher: Rep. der Physik, XX., p. 151, 1884.

[3] Roentgen: Berl. Berichte, Jan. 19, 1888, p. 23.

In the fall of 1900 Professor Rowland returned for the second time to the attack of this problem and he had the satisfaction before his death of seeing his original conclusions reaffirmed at the hands of one of his own students. The singular conflict of evidence was indeed of a sort that could scarcely be settled without further experimental work, for as Rowland had himself pointed out in a brief letter to the editors of the *Philosophical Magazine*,[6] in which in vigorous and characteristic language he insisted upon his rights as the originator of every detail of the experiments described in the communication to the Berlin Academy in 1876, Helmholtz had before that time looked for the effect without result so that it was a question of Rowland and his coworkers and Himstedt, versus Helmholtz, Lecher and Cremieu.

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[1] Philosophical Magazine, 5, XXVII., p. 445, 1889.

[2] Wiedemann's Annalen, 38, p. 560, 1889.

[3] Lippmann: Comptes Rendus, 89, p. 151, 1889.

[4] Cremieu: Comptes Rendus, 130, p. 1544, 1900.

[5] Comptes Rendus, 131, pp. 578–797.

[6] Philosophical Magazine (3), viii, p. 442.

In this piece of work Rowland exhibited many of the qualities that have characterized his entire scientific career. His power of discriminating between the truly fundamental and important aspects of science and those which are secondary was no less remarkable than the directness of his method of attack, and these qualities were combined with three others which are equally important to the investigator; mathematical skill, mechanical insight and a wonderfully clear conception of what he regarded as the *modus operandi* of nature. His knowledge of machinery appears to have been instinctive rather than acquired and this was the quality of mind upon which he seems chiefly to have prided himself. Of his no less remarkable mathematical power he professed to have a poor opinion; but he said that he had never seen a machine every detail in the working of which he could follow with ease. His views concerning molecular mechanics and the mechanism of the ether were those of Faraday and Maxwell, and it is possibly because of the fixity of his conviction of the correctness of the Maxwellian conception that he took no part in the modifications and development of that great theory which has of recent years taken place at the hands of Hertz, Boltzmann, Lorentz, Planck and others.

It is in the realm of the definite rather than in the semi-speculative region beyond the boundary of the tangible that Rowland's contributions to science lie. He selected by preference problems the solutions of which must consist of precise quantitative results obtained by means of measurements with perfectly designed and accurately constructed apparatus, and in such work he had no superior. The clearness with which he saw the essential conditions of a problem and the completeness with which he succeeded in fulfilling them secured for his methods immediate and universal recognition and caused his results to be received as standards.

In his earliest publication of importance, in 1873, Rowland applied, for the first time, the analogue of Ohm's law to the magnetic circuit and laid the foundations of the precise study of magnetic permeability. Almost immediately upon his return from Europe he began his classical measurement of the mechanical equivalent of heat. The memoir describing these experiments, which was published in 1880 in the Proceedings of the American Academy, stands almost alone in completeness and precision and for the thoroughness with which every source of error was studied and accounted for. His values of Joule's equivalent have stood the severe test of comparison with all the more recent determinations and the slight discrepancy which for a time appeared to exist between them and the values obtained by electrical methods has recently been completely accounted for and eliminated by redeterminations of the electromotive force of the Clark cell at the hands of Jaeger, Kahle and of Carhart and Guthe. His determination of the variation of the specific heat of water with the temperature has likewise received substantial confirmation in the recent work of Barnes.

To the important subject of the determination of the value of the ohm, Rowland began to turn his attention soon after the opening of the Johns Hopkins Laboratory and measurements by modifications of the methods of Kirchhoff and of Lorenz were continued at his hand and under his direction at the hands of Kimball, Duncan, Wilkes and Hutchinson for a period of twelve years. In the meantime a variety of lesser topics such as the electric absorption of crystals, the diamagnetic constants of bismuth and calcite and the phenomenon discovered by E. H. Hall and subsequently known as the Hall effect received consideration. To the same period belongs the beginning of his greatest scientific achievement; the production of the concave diffraction grating and its application in the study of the spectrum.

In this work the genius of Rowland had ample scope; the investigation demanded theoretical insight, the ability to design machinery for the ruling of gratings of unapproachable perfection and an extraordinary degree of patience and skill in the use of them when made. The skill and intelligence of others was extensively drawn upon, but the central figure throughout the prolonged and arduous task was always that of Rowland. He invented the method of producing an approximately perfect screw and devised the method of eliminating its errors. The grinding of a perfect surface upon which to make the rulings called out the supreme skill of Brashear; but the idea of ruling upon the face of a concave mirror was Rowland's and he it was who better than any one else could set the diamond to produce the desired quality of line. The operation of the dividing engines was for many years the chief occupation of the mechanician Schneider who brought to this arduous labor almost infinite delicacy and patience; but the perfect system of temperature control, essential to the production of satisfactory gratings was due to Rowland and it was to his exacting scrutiny that every product of the machine was ultimately submitted.

In the long series of investigations made with gratings in the Johns Hopkins Laboratory, Rowland was assisted by Bell, Ames, Crew, Humphreys, Jewell and many others. The photographic mapping of the solar spectrum and of the arc spectra of metals occupied many years and a vast amount of data was collected, only a portion of which has as yet been published.

It is noteworthy that these greater researches were not taken up one after the other, each claiming the investigator's attention for a brief period, to be put aside as completed and replaced by another, but that they all appear to have had their origin at about the same time (in the period 1876–1881) and to have been carried simultaneously in mind for many years.

Outwardly Rowland's personality was a striking one. He was tall and erect, with a certain severity of countenance which betokened a high standard of performance and impatience of mediocrity, and with a directness and intensity of gaze that matched his extreme frankness of speech. He was fond of outdoor recreation, was an expert fisherman and sailor and a daring cross-country horseman.

For elementary routine teaching he had neither taste nor aptitude and even of the more advanced students only those who were able to brook severe and searching criticism reaped the full benefit of being under him; but he contributed that which in a university is above all teaching of routine, the spectacle of scientific work thoroughly done and the example of a lofty ideal.

That Rowland possessed the qualities of a great inventor, as well as of a man of science must be obvious to all who know his work. When in his later years he turned to engineering problems and to electrical inventing, it seemed to many that he was departing widely from the principles of his own fine *plea for pure science*, uttered in the American Association address of 1883; but when in 1899, as first president of the American Physical Society, he spoke again, and, as it proved, for the last time upon a general theme, it was in the same high and inspiring tone as of old.

The recognition of Rowland's genius was early and universal. He was honored by election to nearly all the important scientific bodies abroad and received numerous prizes and medals abroad and at home. In 1881, when he was only 33 years of age, he became a member of the National Academy of Sciences, the youngest man probably upon whom that honor had ever been conferred.

E. L. Nichols.

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THE PHYSICAL REVIEW

A Journal of Experimental and Theoretical Physics

CONDUCTED BY

EDWARD L. NICHOLS, ERNEST MERRITT and FREDERICK BEDELL

LXIII. May, 1901

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THE
PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

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Volume XIII. *August, 1901.* *Number 2.*

THE

PHYSICAL REVIEW.

THE VISIBLE RADIATION FROM CARBON.[1] I.

By Edward L. Nichols.

THE law of radiation has for a long time been considered by physicists as a subject of high interest, and numerous investigations looking to the establishment of a general relation between radiation and temperature have been made both from the theoretical and the experimental standpoint. The earliest attempts to determine incandescence in its relation to temperature were made with platinum. Draper[2] in 1847 made observations upon a wire of that metal heated by an electric current. The temperatures were from the expansion of the wire. Zöllner[3] in 1859 compared the light emitted by incandescent platinum with the heat evolved.

E. Becquerel,[4] who made an extensive study of visible radiation from various solids at high temperatures, used thermo-elements of platinum and palladium, calibrated by reference to melting points and with the air thermometer. A partial separation of the rays was effected by means of colored screens. He found that opaque bodies, such as lime, magnesia, platinum and carbon at the same temperature had very nearly equal emissive powers, a conclusion vigorously contested by his contemporaries, but explained in the light of later work, by the fact that the glowing bodies were en-

[1] An investigation carried on in part by means of an appropriation from the Rumford Fund for Research. Read at the meeting of the American Association for the Advancement of Science in New York, June 27, 1900.

[2] Draper, Philosophical Magazine, Vol. XXX., p. 345 (1847).

[3] Zöllner, Photometrische Untersuchungen (1859).

[4] Becquerel, Annales de Chimie et de Physique (3), 68, p. 47 (1863).

closed in a long earthen tube. The conditions for ideal blackness were thus approximately fulfilled. He likewise made photometric observations upon wires electrically heated and found the light to increase much more rapidly than the emitted heat.

Although some of Becquerel's results were at fault, particularly his estimation of temperature above the melting point of gold, his work is especially noteworthy in that he employed many of the methods to which, in the hands of later investigators, our knowledge of the laws of incandescence is due. He established the fact of the direct proportionality of the logarithm of the intensity of radiation to the temperature and pointed out the possibility of optical pyrometry.

In 1878 Crova[1] used the Glan spectrophotometer in the comparison of various sources of light such as candles, gas flames, the lime light, the arc light and sunlight, and proposed an optical method for the measurement of temperatures.

In 1879[2] I published the results of a series of measurements made in this manner upon the visible radiation from platinum at various temperatures. At that time, the measurement of high temperatures by means of thermo-elements, of platinum and platinum-rhodium, or platinum-iridium, had not been developed and the determination of the temperature from the change of resistance of the metal was, as has been previously pointed out by Siemens, a matter of great uncertainty on account of the varying performance of different samples of platinum. This difficulty, which was due to the impurities contained in the metal, has since been largely overcome and platinum thermometry has, through the study of Callendar and others, been advanced to the position of an operation of precision; but at that time I was forced to content myself in the investigation just referred to with an expression of temperature of the glowing platinum in terms of its increase of length.

Work upon the incandescence of carbon was first taken up in a serious manner after the development of the incandescent lamp.

Schneebeli,[3] in 1884, made some observations upon the total radiation and candle power of the Swan lamp. He made no estimation of temperatures.

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[1] Crova, Comptes Rendus, 57, p. 497 (1878).

[2] Nichols: Ueber das von glühendem Platin ausgestrahlte Licht. Göttingen, 1879; also American Journal of Science, Vol. 18, p. 446 (1879).

[3] Schneebeli, Wiedemann's Annalen, 22, p. 430 (1884).

In the same year Schumann[1] published his very complete spectrophotometric comparison of the various incandescent lamps in use in Germany. Lucas,[2] in 1885, heated arc-light carbons in vacuo, estimated their temperature from the current employed and measured the light given in carcels. I shall refer to his work in some detail later.

In 1887 H. F. Weber[3] began his studies of the spectrum of the incandescent lamp. He found that the first light to appear was not that of the region nearest the red end of the spectrum but corresponded in wave-length to the region of maximum luminosity and that at their low temperatures the spectrum was devoid of color. Stenger[4] in the same year corroborated Weber's observations and offered what has since been received as the proper explanation of the phenomenon.

In 1889 I published in colaboration with W. S. Franklin[5] a series of spectrometric comparisons of incandescent lamps maintained at various degrees of brightness. No attempt was made to determine temperatures. In 1891, H. F. Weber[6] read a paper at the Electrotechnical Congress in Frankfort on the general theory of the glow lamp. By means of numerous measurements through a wide range of incandescence, made upon lamps with treated and untreated filaments, constants were established for his empirical formula for the relation of radiation and temperature.

The infra-red spectrum of carbon has, since the appearance of the incandescent lamp, likewise been subjected to measurement. Abney and Festing[7] in 1883 published curves for the distribution of energy in the spectrum of such lamps from measurements made with the thermopile. In 1894 I compared with the help of the same instrument and a highly sensitive galvanometer the infra-red spectra of lamps with black and gray filaments.[1]

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[1] Schumann, Elektrotechnische Zeitschrift, 5, p. 220 (1884).

[2] Lucas, Comptes Rendus, 100, p. 1454 (1885).

[3] Weber, Wiedemann's Annalen, 32, p. 256 (1887).

[4] Stenger, Wiedemann's Annalen, 32, p. 271 (1887).

[5] Nichols and Franklin, Am. Jour. of Science, 38, p. 100 (1889).

[6] Weber, Bericht des internationalen Electrotechnikercongresses zu Frankfurt am Main, p. 49 (1891); also PHYSICAL REVIEW, Vol. 2, p. 112.

[7] Abney & Festing, Philosophical Magazine (5), 16, p. 224 (1883); also Proceedings of the Royal Society, 37, p. 157 (1884).

Of late years attention has been devoted especially to the problem of the law of radiation from an ideal black body and various formulæ have been proposed by means of which the rise of radiation of any single wave-length upon the one hand and of the total radiation on the other may be expressed as a function of the temperature. Interesting as this phase of the problem is from the point of view of theoretical physics, it is perhaps even more important to know the relation between temperature and radiation for actual surfaces.

Apparatus and Outline of Method.

I propose in the present paper to describe an attempt to measure the temperature of carbon rods rendered incandescent by the passage of an electric current, and to make spectrophotometric comparisons of the visible radiation from their surfaces with the corresponding wave-lengths in the spectrum of an acetylene flame.

The carbons used for this purpose were produced by the well-known process of squirting a semi-fluid carbonaceous paste through a cylindrical opening. They were straight cylindrical rods 10 cm. in length and 2 mm. in diameter. Still larger rods would have been preferable but I was unable to obtain any of greater diameter than the above that were capable of withstanding the temperatures to which it was necessary to heat them. The rods were mounted horizontally in a massive metal box 40 cm. in length, 20 cm. wide, and 20 cm. in height. This box, which was made especially for this investigation, had openings at the ends through which, by means of air-tight plugs, the terminals of the rod could be introduced. Through one of these plugs, likewise, the platinum and platinum-rhodium wires of the thermo-element, by means of which the temperature measurements were made, entered the box. In one of the vertical sides of the box was a row of five circular plate glass windows, which could be removed for cleaning, through which the carbon could be seen and the spectrophotometric observations could be made. Other openings in the top of the box and through the opposite sides served to connect it with a mercury air pump of the Geissler type and with manometers for the measurement of pressure. A vertical cross section of this part of the apparatus is shown in Fig. 1.

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[1] Nichols, Physical Review, Vol. 2, p. 260 (1894).

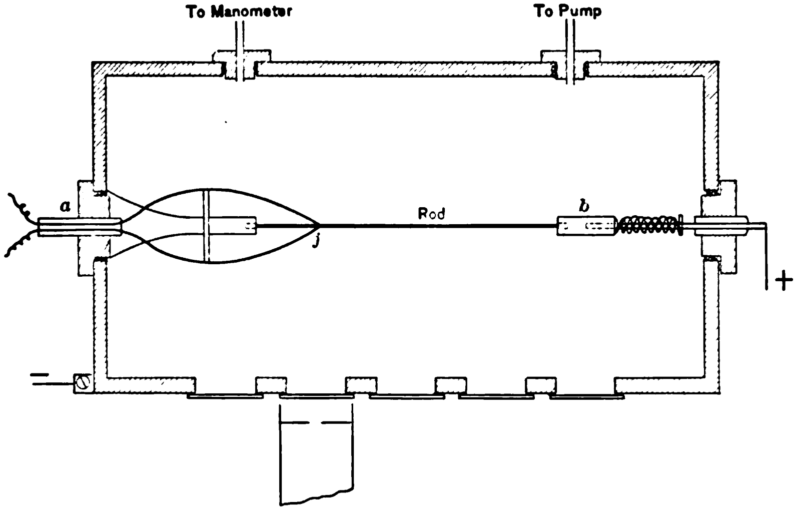

Fig. 1.

Attempts to locate by a variety of methods the hot junction of the thermo-element, by means of which the temperature of the surface of the rods was to be measured, in such manner that it would assume the temperature of that surface, made it only too clear that herein lay one of the chief difficulties of the investigation. It was found that any such junction, however small its size and however carefully it might be brought into contact with the surface of the rod, would not take even approximately the temperature of that surface; and recourse, after the failure of numerous other expedients, was had to the following plan, which although far from being free from objection, was found to be upon the whole the most reliable, and to give, when properly carried out, the most definite and satisfactory result.

By means of a drill made for the purpose from the smallest obtainable size of steel sewing needle, a minute hole was bored radially at the point upon the surface of the rod lying within the field of view of the spectrophotometer. This hole had an approximate diameter of 0.03 cm. It extended to a depth equal to about one-half the radius of the rod and was conical in form. Platinum and platinum-rhodium wires to be used for the thermo-element were drawn to a diameter of 0.016 cm., and their free ends having been laid together side by side, were fused in the flame of the oxyhydrogen blowpipe so as to form a junction. This junction, which after the action of the blowpipe took the shape of a small bead of the combined metals, was trimmed down into conical form, until it would just enter the hole in the side of the rod, care being taken that the entire junction was beneath the surface. The wires leading from this junction were next sealed into a glass tube of about 2 mm. bore through the interior of which they were carried from end to end; care being taken that they should be nowhere in contact. They were held in place by fusing the glass around them at either end of the tube. This tube was inserted through an opening in the plug *a* (Fig. 1), carrying one terminal of the rod and there made air-tight by means of cement. One end of the carbon rod was then inserted in a clamp attached to the inner face of the plug and the wires at a distance of about 1 cm. from the junction were bent downward at right angles so as to bring the junction into position for insertion into the hole in the rod and to hold it there when inserted by the slight but sufficient spring action of the wires themselves. This arrangement of the junction and rod is indicated in Fig. 2.

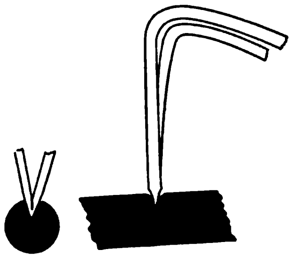
Fig. 2.

The introduction of the thermo-element having been successfully carried out by the method just described, it was possible to insert the plug, carrying the rod and thermo-junction with it, into the end of the box and to secure it in place; after which the free terminal of the rod was introduced between the jaws of a strong clip attached to the opposite plug (*b*, Fig. 1). This operation had to be performed through the open windows in the side of the box. These were then screwed tightly into place, and the box was ready for the exhaustion of the air.

This method of measuring the temperature of the surface, to be successful, involved the fulfilling of several rather difficult conditions and the application of an important correction. To bore into the material of a carbon rod carrying a current in the manner described, necessarily disturbs more or less the flow of the current; and the changes of resistance thus introduced were likely to bring about decided changes of temperature in that neighborhood. In some instances, this became obvious when the rod was heated, the temperature being higher near the hole than elsewhere. Indeed, it was often possible to note with the eye the increased incandescence of the region in question. In all such cases the mounting was rejected. It was found possible, however, to so nearly compensate for this loss of carbon by the introduction of the platinum junction that no difference in the incandescence of the surface could be detected by the closest observation; and since differences of temperature which cannot be detected by the eye will be negligible in spectrophotometric work, this was taken as the criterion of a satisfactory mounting of the thermo-junction. Measurements were attempted only when this condition was fulfilled. It is likewise obvious that there is danger from the contact of the two wires of the thermo-junction with the sides of the hole in the rod. A branch circuit for the passage of the current is thus formed which includes the galvanometer coils, thus imperiling the integrity of the readings of electromotive force. This could be obviated only by having the wires touch the rod at points in an equipotential surface, and the fulfillment of this condition was determined by the reversal of the current through the rod and the absence of any effect of such reversal upon the galvanometer.

Another and more serious objection to the method, and one which could only be met by the introduction of a correction, lay in the fact that even with the smallest wires which could be used for a thermo-element a certain amount of heat would be carried away by conduction through the metal; so that the junction would never reach the full temperature of the surfaces with which it was in contact. I was at first inclined to think that this correction would be a small one, but attempts to measure the temperature of the acetylene flame by means of a similar thermo-element indicated that the loss of heat from this source was by no means to be neglected. The numerical value of this correction was accordingly determined by direct experiment in the following manner. Thermo-elements drawn from the same pieces of wire but differing considerably in diameter were prepared. These were inserted two at a time in holes on opposite sides of a carbon rod and the rod was brought to incandescence by means of the current. The temperatures reached by these junctions were compared by means of the potentiometer and a curve was plotted showing the relation between the cross section of the wire in the thermo-element and the temperature of the junction. This curve, extended in the direction of decreasing cross section, served to indicate with at least a fair degree of accuracy the temperature which would have been reached by a thermo-element of *zero cross section* placed in contact with the surface to be measured. The difference between this temperature and that reached by a junction of any desired size gave the correction which was to be applied. The correction, as will be seen by inspection of the curve, Fig. 3, is a very large one, amounting even in the case of the smallest wires which it was found practicable to use to about 85°. The results of the calibration agreed, however, so well with similar experiments made by placing thermo-junctions of various sizes in the non-luminous outer envelopes of the acetylene flame, of the ordinary gas flame, and of the flame of the candle,[1] that I feel warranted in placing much dependence upon them.

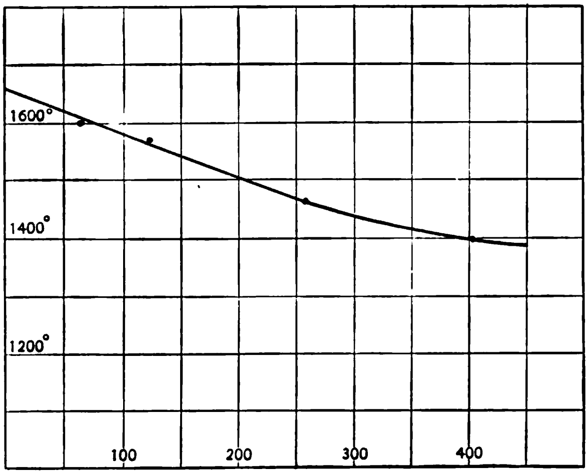

Fig. 3.

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[1] Nichols, PHYSICAL REVIEW, X., p. 234.

This method of correcting for the loss of heat in a thermo-junction was first employed by Waggener[1] in his investigation of the temperature of the flame of the Bunsen burner. I became acquainted with his research, however, only after the completion of my experiments.

Calibration of the Thermo-Elements.

All our modern estimates of very high temperatures may be said to rest in one way or another upon extrapolation. The upper limit of usefulness of the air thermometer has been found to lie in the neighborhood of 1500°. It was at this temperature that Erhardt and Schertel[2] in their admirable but little-known research upon the melting points of alloys of silver, gold and platinum were obliged to abandon direct determination; and very closely, at which Holborn and Wien[3] and Holborn and Day[4] in their latest studies upon thermo-electric thermometry found that the indications of the air thermometer, even when constructed of the most refractory of modern porcelain began to become erratic. We have, it is true, the investigations of Violle[5] upon the melting points of the metals of the platinum group, but these, it must not be forgotten, are based upon an assumed value for the specific heat and this assumption is equivalent to the extrapolation of the curve of the variation of the specific heat with temperature. The observed values, by means of which this value was determined, all lie far below those of the melting points of the metals in question. It is necessary therefore in spite of the accumulation of indirect evidence of their approximate accuracy, to hold in reserve the assignment of absolute values of these melting points until by some means as yet unthought of, we shall be able to obtain direct experimental data. In the meantime they afford us the best present available basis for a temporary scale, our confidence in the approximate accuracy of which must rest upon the fact that the melting points for palladium, platinum, etc., as given by Violle are found to lie upon what may reasonably be supposed to be an extension of the curves experimentally determined for lower temperatures by means of the air thermometer. As for the various formulæ for the variation of electromotive forces of thermo-elements with the temperature, we must not lose sight of the fact that they are simply analytical expressions for experimentally determined relations and that the extensions of them to temperatures lying far beyond the experimental range is not to be regarded as more trustworthy than the extension of a curve by graphical methods. Under these circumstances I decided to content myself with the provisional acceptance of the following values for the melting points of gold, palladium and platinum, namely:

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[1] Waggener, Wiedemann's Annalen, LVIII., p. 579 (1896).

[2] Erhardt and Schertel, Jahrbuch für das Hüttenwesen in Sachsen, 1879, p. 154.

[3] Holborn and Wien, Wiedemann's Annalen, XLVII., p. 107 (1892), and LVI., p. 360 (1895).

[4] Holborn and Day, American Journal of Science, VIII., p. 175 (1899).

[5] Violle, C. R., LXXXIX., p. 702 (1879).

Gold,[1] 1064° C.
Palladium, 1500° C.
Platinum, 1775° C.

and to ascertain as accurately as possible the electro-motive force given by the thermo-elements used at these points. It was thought that by drawing a curve through them, and reading intermediate temperatures from this curve the values thus obtained would be as close as our present knowledge of the subject will admit. The platinum-platinum-rhodium wire used for my elements was obtained, as has already been stated, from Heræus in Hanau and was supposed to be of the same stock as that employed by Holborn and Wien. The fact that the electromotive force given by these thermo-elements when exposed to the temperature of melting platinum agreed very closely indeed with that obtained by extrapolation of their data, seems to indicate that the metals were identical with those used by them.

For the purpose of calibrating the thermo-elements, I employed a very simple method, the results of which, however, proved highly satisfactory. This method, which consists in exposing the hot junction to the layers of heated gas surrounding the acetylene flame and lying just outside the luminous envelop of that flame, has been described in some detail in a separate communication.[2]

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[1] This value is that given by Holborn and Day in their second paper (American Journal of Science, X., p. 171 (1900).

[2] Nichols, On a New Method of Calibrating Thermoelectric Elements for Use in the Measurement of High Temperatures. [Livre Jubilaire dedié à H. A. Lorentz; La Haye, 1900, p. 339.]

The method is easily carried out and it is free from many of the most serious difficulties of the cumbersome operations usually employed, which involve the use of furnaces and of the porcelain bulb air thermometer. Fig. 4 contains the calibration curve of the thermoelements used in this investigation and likewise, for the purpose of comparison, a curve reproduced from Waggener's paper and extrapolated by him from the data given by Holborn and Wien. It will be seen that while the two curves are not identical, they are of the same general character and that the differences are not greater than experience would lead us to expect in the case of different thermoelements, even where these are of platinum and of platinum-rhodium of the same manufacture. It is not a question of absolute electromotive forces but rather of the form of the curves; since what we really need is a criterion by means of which we can determine whether temperature readings based upon Violle's values for palladium and platinum are in reasonable agreement with those obtained by the extension of the curve of Holborn and Wien.

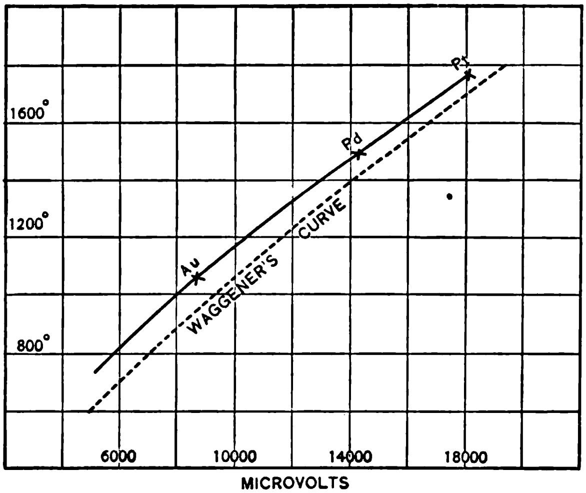

Fig. 4.

THE SPECTROPHOTOMETER.

The spectrophotometer used was a copy of the instrument designed by Lummer and Brodhun for the Imperial Institute in Charlottenburg. It consists of a one-prism spectroscope with two collimator tubes, placed at right angles to each other as shown in Fig. 5. Each of these tubes carries a slit the width of which is regulated by means of an accurate micrometer screw with a drum head divided into one hundred parts. By estimating tenths of a scale division, the width of the slits could be estimated to one one-thousandth of a revolution.

The essential feature of this photometer consists in the Lummer-Brodhun device (*D*), placed between the objective lenses of the two collimators and the prism, in such a position that the beam of light from one of the tubes is transmitted directly to the latter, while that from the other tube is bent through 90° by total reflection. The instrument was set up with collimator *A* in such a position that a portion of the surface of the incandescent rod lying nearest to the point at which the thermo-element had been inserted formed a field of illumination for the slit at a distance of about 25 cm. The region under observation was limited by means of a vertical diaphragm (*d*) 5 mm. in width which was mounted in a tube in front of a window of the metal vacuum box. The comparison source was the spectrum

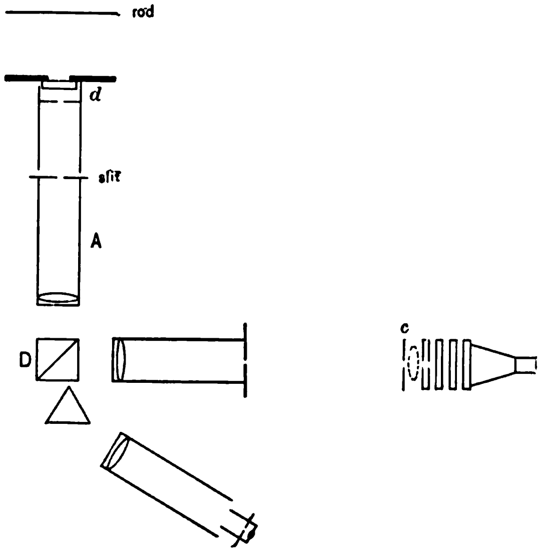

Fig. 5.

of the brightest part of an acetylene flame set up in the axis of the other collimator at a corresponding distance and viewed through a circular aperture (*c*) 5 mm. in diameter, cut in a metal screen interposed between the flame and the slit and as near the former as practicable.

The acetylene flame was adopted as a comparison standard for the following reasons :

1. It possesses a continuous spectrum brighter in the less refrangible regions than that of any other controllable source of light.
2. The radiating material is finely divided carbon, presumably of a character not unlike that of the surface of the untreated rod.
3. The acetylene flame is the result of the combustion of a definite fuel (C_2H_2) burning under reasonably constant conditions. It is preferable in this regard to any of the ordinary gas or candle flames in which the fuel is of an undetermined and more or less variable character.
4. When supplied with gas under constant pressure, an acetylene flame of the type used in these experiments, that, namely, obtained by means of a burner composed of a single block of steatite, is more nearly constant in its intensity and color than any other flame with which I am acquainted, with the exception of that of the Hefner lamp. It is indeed questionable whether the latter is superior to acetylene in this respect and its comparative weakness in the blue and violet renders it very undesirable as a comparison source in spectrophotometry.

METHOD OF CHECKING THE CONSTANCY OF THE ACETYLENE FLAME.

To secure as complete a check as possible upon the constancy of the flame the following method, based upon the assumption that so long as the radiation from the flame remained constant, its light-giving power would not vary, was employed. A diaphragm *d* (Fig. 6) similar to that interposed between the slit and the flame and having an aperture of the same size, was mounted on the opposite side of the latter and a thermopile *p* was placed at a distance of about 15 cm. from this opening. A second diaphragm *d'*, with an intervening air space served to cut off, in large part, the radiation from the heated metal. Two thin sheets of glass forming the sides of an empty glass cell *c* of the kind used in the study of absorption spectra, etc., were placed between the cone of the thermopile and the second diaphragm; so that only those rays from the flame which were transmitted by the glass fell upon the face of the pile.

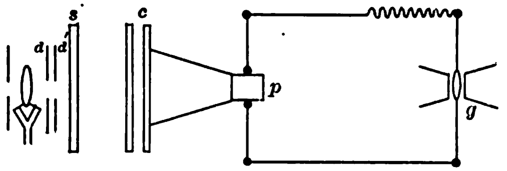

Fig. 6.

The thermopile was connected with a sensitive d'Arsonval galvanometer (*g*), the circuit being kept permanently closed, and a double metallic shutter *s*, which could be raised or lowered so as to open or close the opening in the diaphragm next to the flame was so mounted that it could be readily operated by an observer at the telescope of the galvanometer. When a reading of the radiation from the flame was to be made, the zero point of the galvanometer was noted and this shutter was raised during the short interval of time necessary to bring the needle, which was not strongly damped, to its first turning point. The shutter was immediately closed in order to prevent further heating of the face of the thermopile. This throw of the galvanometer was taken as an indication of the intensity of the flame. It was found that the thermopile would cool sufficiently within two minutes to admit of the repetition of the reading. These observations were taken by an assistant simultaneously with each setting of the spectrophotometer, the intention being to reject any spectrophotometric readings made at a time when the flame showed marked deviation from its standard intensity, and to reduce the readings to a uniform flame intensity under the assumption that for the small range of variation occurring from reading to reading, the changes in the brightness of the flame would be proportional to the variations of this galvanometer reading from the mean of the whole set. In point of fact it was found that the flame rarely varied by more than two per cent. from the mean in the course of a set of observations and remained most of the time within one per cent. From day to day its intensity was usually within the limits stated above although occasionally a larger variation was detected. None of these variations in the course of the present investigation reached values so great as to lead me to hesitate to apply the correction already referred to, and all the observations described in this paper have been reduced to a constant flame intensity by means of a correction factor obtained from the readings of the galvanometer.

CONTROL AND MEASUREMENT OF THE TEMPERATURE OF THE CARBON ROD.

The carbon rod having been brought to the desired degree of incandescence by means of the current from a storage battery, was held at a constant temperature by varying the resistance placed in the battery circuit. The indications of the thermo-element inserted in the rod were noted by means of a potentiometer consisting of an accurately adjusted resistance box of 100,000 ohms, the smallest coils in which contained one ohm each, a sensitive galvanometer of the d'Arsonval type and a large Clark cell of the old Feussner type, against which the electro-motive force of the thermo-element was balanced in the usual manner. The arrangement of this potentiometer has been described in my recent paper[1] on the temperature of the acetylene flame. The cells used in the measurement of the temperature of the carbon rod were the same as those employed in that investigation and the same precautions were taken to preserve constancy of temperature and to control and check their electro-motive force. It was found that at the temperature of 18°, which was very closely the mean temperature of the room, the electro-motive force of the Feussner cells was 1.430 volts.

The potentiometer having been balanced by looping the circuit containing the thermo-element around a sufficient portion of the resistance box to balance its current against that of the Clark cells, a condition which was indicated by the reduction of the galvanometer deflection to zero, the current was maintained at such a value as to hold the carbon at a constant temperature during the time necessary to complete measurements of the intensity of eight different portions of the spectrum, ranging from the extreme red to the violet, with the corresponding portions of the spectrum of the acetylene flame. In order to insure the maintenance of this constant temperature in the rod, an assistant made repeated observations with the potentiometer and readjusted the resistance in the battery circuit whenever necessary. Excepting at very high temperatures where the rod was subject to rapid disintegration, it was rarely necessary to make any adjustment during the progress of a single set of observations. Readings of the current flowing through the carbon and of the fall of potential between its ends were made at the beginning and end of each experiment.

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[1] Nichols, PHYSICAL REVIEW, 10, p. 237 (1900).

MAGNETIZATION OF IRON, STEEL, AND NICKEL WIRES BY INTERMITTENT CURRENT.

By K. Honda and S. Shimizu.

§ 1. Since Helmholtz first brought in question, about 50 years ago, whether the magnetization follows very rapid changes of magnetic force or not, several physicists have studied the same subject in different ways. Their results and criticisms are placed in a good order in the introduction to Wien's paper,[1] so that there is no need of repeating them.

In the paper above cited, Wien describes his own investigations on this point by elaborate experiments, and reconciles several apparent discrepancies, which exist in the results obtained by former observers. According to him, magnetic induction is always less in the alternate than in the constant field; the difference becomes greater and greater as the frequency increases. But the area of the hysteresis curve is always greater in the alternate than in the constant field, except when it is not very weak, in which the contrary is the case. The area also increases with the frequency. Whence he concludes that the magnetization cannot follow very rapid changes of the magnetic force.

Apart from the methods previously used, the following simple plan may conveniently be used to study the same question in its somewhat different aspect. Instead of using an alternate current, iron or nickel wire is magnetized by an intermittent current of definite frequency, and its mean intensity of magnetization is measured by the magnetometric method. The comparison of the intensity of magnetization corresponding to different frequencies will decide the proposed question.

§ 2. The apparatus for producing the intermittent current is the Pupin's interruptor. The copper or aluminum wire is stretched horizontally, as shown in Fig. a, one of the ends is fixed to a stout support S, while the other, passing over a small metallic pulley P, is attached to a weight Q. Near the center, a short initial wire R is dipped into a mercury cup, in which the circuit is made or broken in regular succession by the vibrating string. The string is maintained continuously oscillating by a magnet n, which may be either a permanent one or an electromagnet. A condenser C, whose capacity is one microfarad is connected to two sides of the mercury contact. The number of frequency is calculated from the formula

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[1] Wied. Ann., 66, 859, 1898.

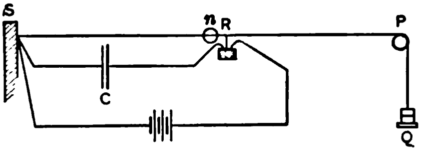

Fig. a.

$$n = \frac{1}{2l}\sqrt{\frac{T}{\rho}},$$

where l, T, ρ denote respectively the length, the tension and the linear density of the string.

The mean intensity of the current is measured by a Thomson graded galvanometer which is from time to time compared with a deciampere balance. The magnetometer consists of a small bell magnet suspended in a thick copper case by a fine quartz fiber, the period of vibration being about 3 seconds. It is placed due magnetic east of the magnetizing coil, and its deflection is read by means of scales and telescope. The magnetizing coil is 34.8 cm. long and its frame is made of wood. It is wound in 4 layers with copper wire of about 2 mm. diameter; the field at the center of the coil due to a current of one ampere is 19.38 C.G.S. units. The coefficient of self-induction of the whole circuit is 2.35×10^7 cm. and its resistance 19.3 ohms, so that the time of relaxation is 0.00122 seconds.

§ 3. The method of observation is as follows: In the first place, the magnetizing coil, the magnetometer and a compensating coil are carefully placed in a line perpendicular to the magnetic meridian. A constant or intermittent current is sent through these coils and the perfect compensation obtained by displacing the auxiliary coil in its axial line; then the current is reversed and the compensation tested, till there is no deflection by passing the current in either direction. The wire to be tested is then placed in the axial line of the coil, and magnetized in one direction and then in the other. If, on both sides, the corresponding deflections of the magnetometer are not numerically equal, the above process is repeated, till the compensation is perfect and the deflections of the magnetometer due to the magnetized wire alone are equal in two directions. Since the distance of the magnetometer from the nearer end of the magnetizing coil was only 21 cm., the adjustment was no easy task. When the adjustment is finished, the wire to be tested is demagnetized by reversals. The gradually increasing intermittent current of definite frequency is then passed through the magnetizing coil, and the corresponding deflections are noted, when they become stationary.

The deflection of the galvanometer or of the magnetometer was generally as steady as that due to steady current. It, therefore, represents the mean of the current or of the intensity of magnetization during the complete period of the intermittent current.

Owing to the smallness of the coefficient of self-induction compared with the resistance of the circuit, the deflection of the galvanometer due to stationary current was reduced to nearly half its original value, when the same current became intermittent with the frequency of 49 per second. When the frequency was increased, the spark gap remaining unaltered, the mean intensity of the current generally became a little greater. Hence in order that the deflection with the intermittent current may just be half that due to the steady one, the length of the spark gap had always to be increased slightly by an adjusting screw.

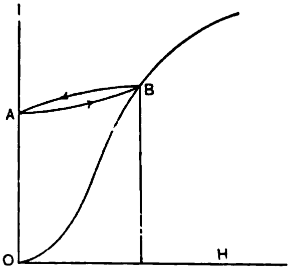

Fig. b.

The deflection of the magnetometer represents the mean intensity of magnetization during the complete period; but in the statical cycle between zero and a certain field, and perhaps in the cycle with intermittent current, the form of the hysteresis curve is a very elongated loop *AB*, as shown in Fig. b.

Let the extreme values of the intensities of magnetization be denoted by I_0 and I_1 respectively, and the intermediate value by I; the deflection of the magnetometer will be proportional to

$$\frac{1}{T}\int_0^T Idt.$$

In the evaluation of the integral, we may, with tolerable accuracy, put $I = I_0 + KH$, where K is a constant; hence

$$\frac{1}{T}\int_0^T Idt = I_0 + \frac{K}{T}\int_0^T Hdt.$$

But in our experiments, the mean intensity of the intermittent current was always so adjusted, that it is just half of the intensity of the steady current; hence if H_1 be the corresponding field due to the steady current,

$$\frac{1}{2}H_1 = \frac{1}{T}\int_0^T Hdt,$$

and therefore

$$\frac{1}{T}\int_0^T Idt = \frac{I_0 + I_1}{2}.$$

Hence the mean intensity of magnetization during the complete period is nearly the same as the arithmetical mean of the maximum intensity and the residual ones. This was also experimentally confirmed with the intermittent current of the frequency of 49 per second.

Though the increase of the mean intensity of magnetization does not necessarily imply that of the extreme intensity of magnetization for a given cycle, yet the experimental fact, hereafter to be described, teaches us that the change of the mean intensity of magnetization with the frequency is nearly the same amount as that of the actual intensity.

§ 4. The specimen tested had the following dimensions:

Metal.	Length.	Diameter.	Volume.	Density.
Wolfram steel.	21.00 cm.	0.135 cm.	0.302 ccm.	7.90
Soft iron.	21.00	0.135	0.302	7.80
Nickel.	21.07	0.134	0.299	8.95

The measurement of the magnetization of each of these ferromagnetics by steady current gave the following results expressed in *C.G.S.* units.

Wolfram-steel.		Soft iron.		Nickel.	
H	*I*	*H*	*I*	*H*	*I*
6.3	46	1.5	52	1.7	27
11.4	123	2.1	92	3.6	111
13.3	235	3.6	328	6.7	228
15.0	371	4.6	579	13.3	316
16.4	564	6.1	684	19.1	357
18.3	719	8.1	839	28.6	394
25.2	951	12.6	975	39.3	419
40.2	1121	23.5	1124	50.7	439
59.3	1212	51.5	1260	58.1	448

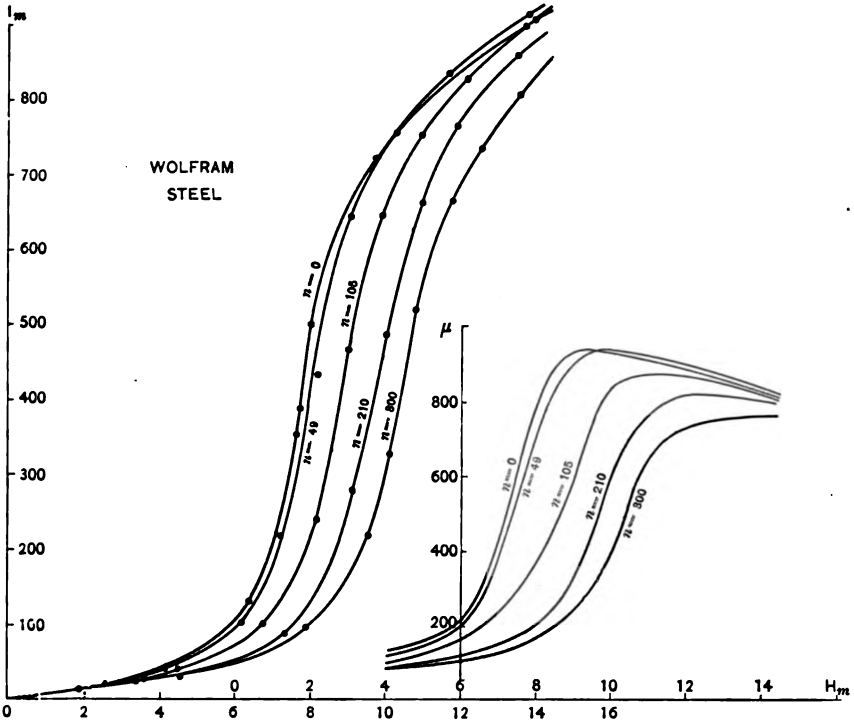

Fig. 1.

§5. The magnetization of Wolfram-steel with intermittent current of different frequencies is given in Fig. 1.

Here I_m denotes the apparent or mean intensity of magnetization, H_m the mean of the external field, the extreme field being always double its mean value, and n the frequency of the intermittent current. The curve corresponding to $n = 0$, which nearly coincides with the curve for $n = 49$, is one representing the arithmetical mean of the two intensities of magnetization, when the current is on and off.

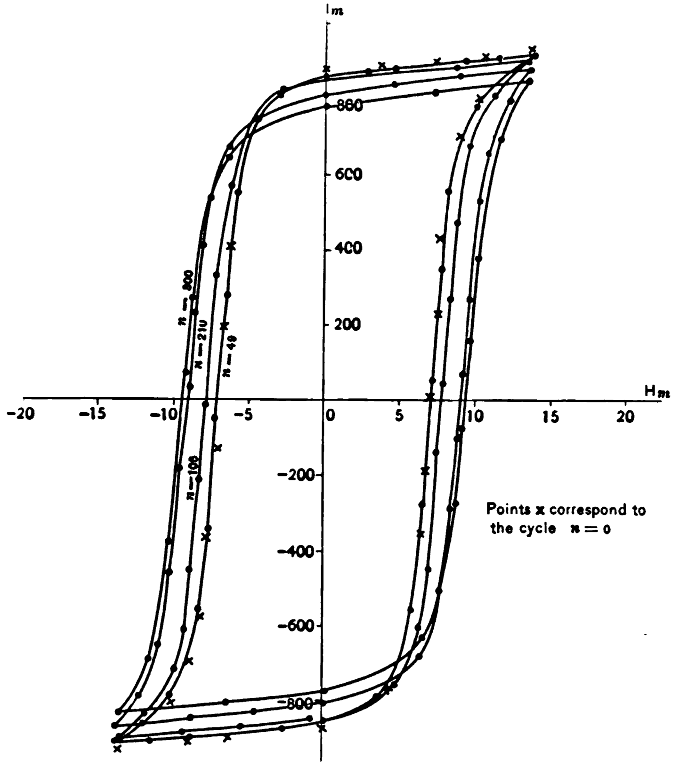

Fig. 2.

From these curves, we see that the mean intensity of magnetization decreases as the frequency increases. For a definite frequency, the difference becomes greater as the field increases, and attains its maximum value in a field near that of the maximum permeability. It then gradually decreases as the field is further increased.

The curves of the apparent value of permeability are also drawn in the same figure. It will be seen that the apparent value of permeability tends to a constant value as the frequency is increased, a fact which plays an important part in the theory of electric oscillation.

In the hysteresis cycle, the extreme value of magnetic induction becomes a little smaller, but the coercive force is greater than that for the statical cycle, so that the area of the hysteresis curve increases with the frequency, till it reaches a maximum and then decreases, as shown in Fig. 2.

The measurement of the areas of hysteresis curves with a planimeter gave the following results:

Frequency.	$n = 0$	$n = 105$	$n = 210$	$n = 300$
Apparent dissipated energy.	25600	27260	30120	29540

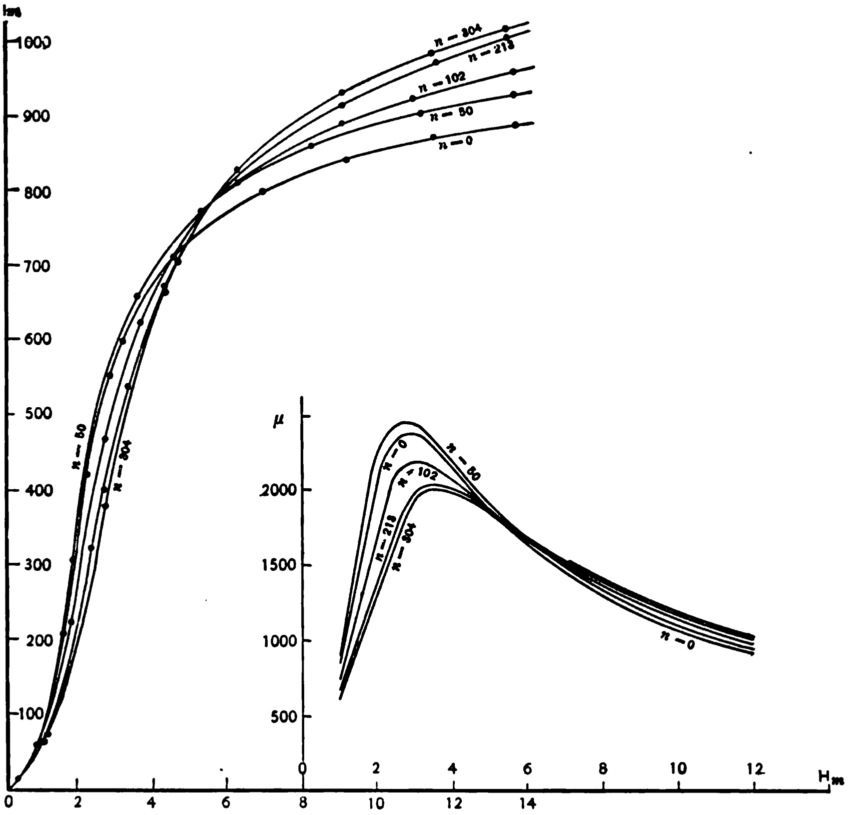

Fig. 3. Soft Iron.

In the form of the hysteresis curves, we notice that the portions of the curves lying between the maximum and zero fields are nearly parallel to each other; and as we have remarked, the apparent intensity of magnetization represents the mean of the extreme and residual intensities of magnetization. Hence it is evident from these two facts that the change in the intensity of magnetization by intermittent current is nearly of the same amount as that of the actual intensity of magnetization, as we have already remarked. Hence if we consider the apparent intensity to represent the extreme intensity of magnetization, the above results exactly coincide with those of Wien with alternate field.

§ 6. The results for soft iron and nickel are given in the following Figs. 3 and 4:

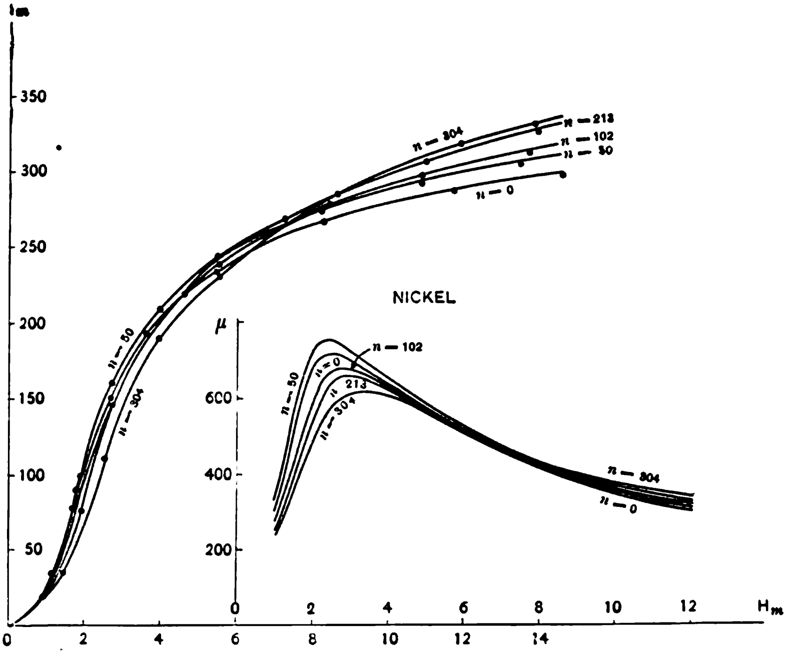

Fig. 4.

From these figures, we see that the nature of the change in low fields is exactly the same as for wolfram-steel, but the amount of the change is smaller than that for the steel. In stronger fields the increase of the intensity of magnetization with the frequency is observed. Wien did not notice this reversal of the change of magnetization. It will also be noticed that the curve $n = 50$ for iron and nickel cuts in very low fields the curve $n = 0$. In strict sense, however, the curve $n = 0$ can not be compared with the others, because in drawing the curve, the arithmetical mean of two intensities of magnetization, when the current is on and off, is taken as the intensity of magnetization, and one half of the corresponding field as the field of magnetization.

In the hysteresis cycle for iron and nickel, the extreme intensity of magnetization and the coercive force for intermittent current are both greater than those due to steady current, and the area of the hysteresis cycle increases with the frequency, a fact which is in accordance with Wien's results. This is shown in the following table:

Frequency.	$n = 0$	$n = 107$	$n = 213$	$n = 304$
Apparent dissipated energy for iron.	5325	5520	6340	6625
Apparent dissipated energy for nickel.	1931	2029	2183	2336

§ 7. The summary of these results runs as follows:

1. For iron, steel and nickel wires, the magnetization in weak fields decreases as the frequency of the intermittent current is increased. On the contrary, the magnetization in stronger fields increases with the frequency.

2. The diminution of magnetization by a definite frequency of the intermittent current becomes greater as the field is increased, till it reaches a maximum in a field near that of the maximum permeability; it then gradually goes on decreasing and continues in that state, till there is increase in the stronger intermittent field.

3. The field, in which the change of magnetization vanishes increases with the frequency.

4. The area of the hysteresis curve for the maximum field of 27.7 C.G.S. units increases with the frequency; it reaches a maxi-

mum and then gradually decreases. As regards the explanation of these facts, it only suffices to assume with Wien that the magnetization has inertia.

It is well known that, when the frequency is very small, the extremities of the ordinates of the maximum intensity of magnetization by a gradually increasing intermittent current lie almost on the statical hysteresis curve between the same extreme fields. It is highly probable that at least within the range of the frequency used in the present experiment, the above result holds good. If this be true, the present experiment with intermittent current measures the same thing as that of Wien with alternate current. The remarkable coincidence between his and our results seems to justify the supposition. If he pushed his experiments to a still stronger field, it seems quite likely that he would have reached such a place, where the change of magnetization begins to increase with the frequency of the alternate current.

PHYSICAL LABORATORY, IMPERIAL UNIVERSITY,
TOKYO, JAPAN. March 26, 1901.

THE SPECIFIC HEAT OF SOLUTIONS WHICH ARE NOT ELECTROLYTES. II.

By William Francis Magie.

1. This paper is a continuation of one published under the same title in this Review, Vol. IX., No. 2, August, 1899. It presents several series of specific heats of non-electrolytes in solution, which furnish further confirmation of the law developed in that paper, but it is especially concerned with the examination of the questions: (1) whether the molecular heats of isomers in solution are always equal to each other; and (2) whether the molecular heat of a substance dissolved in different solvents is invariable or differs in the different solvents.

2. The calorimeter employed was essentially the same as that described in connection with the earlier part of the work. Various sources of error or of inconvenience were detected in it during the progress of the investigation, and, to avoid these, certain changes were made in the details of its construction. The evaporation of the liquids in the calorimeter vessels was a most important cause of the irregularities which appeared in a series of determinations. The heat of vaporization of the solvent is so great in comparison with its specific heat that even very slight differences in the rate of evaporation from the two vessels are sufficient to introduce serious irregularities. To avoid this the calorimeter vessels were furnished with covers, to which the resistance coils were attached. The stirring was effected by rotating paddles borne by the covers. This arrangement considerably reduces the probable error of each determination, and further relieves the observer of the necessity of weighing and refilling the vessels after each experiment.

The paraffine coating which was used in the earlier work to protect the resistance coils against electrolytic action was often ineffective, and the results of many observations had to be rejected because of changes in the resistances. After many trials a satisfactory insulation was obtained by coating the coils first with asphalt varnish and then with ozokerite. Neither substance could be used alone, the varnish because it flaked off too readily and did not resist alcohol, the ozokerite, because it did not distribute itself well over the coils; but the double layer gave a tough and durable coating, which has protected the resistances perfectly since it was first applied.

Certain other sources of small errors were removed either by attention to the construction of the instruments or by the adoption of a definite procedure in making the observations. It is not necessary to go further into the consideration of this matter here. I am constructing a new instrument in a form which will embody such improvements as I have made, and shall take another opportunity to describe its operation and to exhibit the results which may be obtained with it.

3. The thermometers used were made for this work by Fuess. They are of Jena glass, No. 59, III., and are similar in size and shape. The graduation extends from 12° to about 34°, and each degree is divided directly into fiftieths. The reading microscopes are borne on the stems of the thermometers and are furnished with pointers which mark the center of the field of view, so that, by bringing the image of the mercury column between them, parallax is avoided, and a reading can be made by estimation to the thousandth of a degree. The thermometers were carefully compared with each other under the conditions of service.

4. *Cane Sugar in Water.*—By way of confirmation of the results for cane sugar obtained in my earlier work by a less accurate instrument, a test of this solute in a solution for which $N = 300$ was made, with the result that the value of ζ, the molecular heat, was determined as 153.6. Combining this value with those given in the former paper (§ 17), the mean value of ζ becomes 152.8.[1] This new value does not materially change the calculated specific heats.

5. *Dextrose with Water of Crystallization in Water.*—In order to deal with certain of the isomers of cane sugar, it is necessary to know

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[1] By a misprint, the value of ζ for $N = 150$ is given in the former paper as 150; it should be 152.

how the water of crystallization, which they contain, shall be reckoned in when the solution is made up. To test this matter I examined dextrose, which can be prepared either with or without a molecule of water of crystallization. A solution of dextrose containing water of crystallization was made up with $N = 250$, on the supposition that the water of crystallization leaves the dextrose when it dissolves and becomes a part of the solvent. The value of the molecular heat obtained with this solution was 78.9. This is in perfect agreement with the mean molecular heat given for dextrose without water of crystallization in my former paper (§ 18), and justifies the supposition on which the solution was made up.

6. *Maltose in Water.*—The maltose was obtained from Kahlbaum. The solution was made up on the supposition that the water of crystallization becomes part of the solvent.

Maltose ($C_{12}H_{22}O_{11}+H_2O$) *in Water. Molecular Weight*, 342.

N	s' Observed.	s' Calculated.	ς
300	0.9657	0.9653	145
400	0.9735	0.9735	142
445.4	0.9760	0.9762	141
Mean molecular heat.			142.7

7. *Milk Sugar in Water.*—The milk sugar was obtained from Merck. It was sufficiently dry and pure to be used directly. The solutions were made up on the supposition that the water of crystallization becomes a part of the solvent.

The first trials with the solution for which $N = 200$ showed an apparent progressive change in the specific heat of the solution and therefore in the molecular heat of the solute. This change could not be traced to any failure of the calorimeter, and repetitions of the test with fresh solutions proved that it was really due to the properties of the solution. Three sets of determinations were made, of which the results are given in the following table. The numbers given are the apparent molecular heats obtained at intervals of about 25 minutes. They are arranged so as to bring the numbers corresponding to the same condition of the solution in the same horizontal row.

Milk Sugar in Water. Successive Values of ζ at 25-minute Intervals.

I.	*II.*	*III.*	**Means.**
	147	148	147
135	139	139	138
143	143	147	144
146	148	148	147
149	151	148	149
		151	151
		153	153
		150	150
		146	
		150	
		151	

Solution II. was tested after 2 days and gave a constant molecular heat, 144; and after 13 days it gave again a constant molecular heat, 146. A solution of the same strength which was made up in boiling water and tested after 3 days gave also the constant molecular heat, 146.

A solution of milk sugar for which $N = 300$, made up with cold water, showed no changes in the molecular heats except such as might easily be due to the errors of observation. The molecular heat obtained for it was 155. A similar solution made up in boiling water gave the constant molecular heat, 154; but tested after 2 days it gave the constant molecular heat, 144. Apparently the molecular heat reaches its maximum value more speedily in the more dilute solution. The condition reached after the lapse of considerable time may be considered the true and final condition of the solution. For this condition the results are as follows:

Milk Sugar ($C_{12}H_{22}O_{11} + H_2O$) *in Water. Molecular Weight,* 342.

N.	s' Observed.	s' Calculated.	ζ
200	0.9501	0.9499	145
300	0.9656	0.9656	144
	Mean molecular heat.		144.5

It has been shown[1] that the rotatory power of a solution of milk sugar in water for polarized light changes gradually after the solution is made up until after about seven hours it attains a value which remains constant from that time on. This phenomenon is not accompanied by any perceptible temperature changes. Erdmann ascribes it to the gradual formation of an allotropic modification of the milk sugar in the solution, a modification which cannot exist in the dry state. In the light of this evidence for molecular changes in the solution we may ascribe the changes in the molecular heat to the changing form of the molecules and to the consequent changes in the restraints to which their various parts are subjected.

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[1] Erdmann, Berliner Ber., 13, p. 2180.

Erdmann found that when the solution is boiled for a short time it shows at once the final value of the rotatory power. To see if boiling would produce a similar effect on the molecular heats two trials were made with solutions in which $N = 200$, made up in boiling water and boiled for five minutes. In both trials the molecular heats showed variable values, beginning with about 145 and increasing gradually to 155 or more. One of these solutions when tested after 3 days gave the constant value 146.

8. *Levulose in Water.*—The levulose used in the two more dilute solutions was obtained from Kahlbaum, in the form of "large and beautiful formed crystals, which were transparent and exhibited the smallest trace of straw color. These crystals were permanent in the air." [1]

The powdered levulose had been kept for a year in a stoppered bottle, and when exposed over sulphuric acid for two days, it showed a slight loss in weight, less than one part in a thousand. It was then assumed to be dry. The most concentrated solution was made from the ordinary form of levulose, and its exact concentration was determined from its specific gravity.

Levulose ($C_6H_{12}O_6$) *in Water. Molecular Weight*, 180.

N	s' Observed.	s' Calculated.	ζ
201.5	0.9761	0.9763	89
300	0.9837	0.9838	88.8
400	0.9880	0.9878	91.2
Mean molecular heat.			89.6

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[1] Loomis, PHYS. REV., Vol. XII., No. 4, April, 1901

9. *Mannite in Water.*—In the former paper the values given for the molecular heat of mannite in water were not constant. It was there stated that the probable value for the molecular heat was about 108, the value obtained for the solution for which $N = 400$, and that the other values were in error. I have redetermined the specific heats for solutions of mannite, with the following results:

Mannite ($C_6H_{14}O_6$) *in Water. Molecular Weight,* 182.

N	s' Observed.	s' Calculated.	ς
107.8	0.9658	0.9651	109.2
200	0.9806	0.9804	108.6
400	0.9900	0.9900	108
500	0.9917	0.9919	106
Mean molecular heat.			108

In this table the result of the earlier work for $N = 400$ is included, which was not affected by the circumstances to which the erroneous values for the other concentrations were due.

10. *Dulcite in Water.*—The dulcite used was obtained from Kahlbaum, and was assumed to be pure. It dissolves with difficulty and only dilute solutions were used.

Dulcite ($C_6H_{14}O_6$) *in Water. Molecular Weight,* 182.

N	s' Observed.	s' Calculated.	ς
400	0.9883	0.9885	96
500	0.9910	0.9908	99
Mean molecular heat.			97.5

11. The isomers resorcin, hydroquinone and pyrocatechin were chosen for special investigation because they could be obtained from the chemists in reasonable purity, and because they dissolve in alcohol as well as in water. When their molecular heats are the same within the limits of probable error they will be combined to give the mean molecular heat and the calculated specific heats will be those determined from this mean.

12. *Resorcin and Hydroquinone in Water.*

Resorcin ($C_6H_6O_2$) *in Water. Molecular Weight,* 110.

N	s' Observed.	s' Calculated.	ς
300	0.9916	0.9915	64
400	0.9936	0.9936	63

Hydroquinone ($C_6H_6O_2$) *in Water.*

N	s' Observed.	s' Calculated.	ς
300	0.9915	0.9915	63.4
400	0.9936	0.9936	63.2
Mean molecular heat.			63.4

13. *Pyrocatechin in Water.*

Pyrocatechin ($C_6H_6O_2$) *in Water. Molecular Weight,* 110.

N	s' Observed.	s' Calculated.	ς
300	0.9937	0.9938	75
400	0.9953	0.9952	76
Mean molecular heat.			75.5

14. *Resorcin, Hydroquinone, and Pyrocatechin in Alcohol.*—The alcohol used in these solutions was an ordinary good commercial alcohol, with a specific gravity about 0.815, and containing about 10 per cent. of water. The specific heat of alcohol containing 10 per cent. of water is given by Dupré and Page[1] as 0.6576, and this value was accordingly taken as the specific heat of the solvent. This value cannot be so far wrong as to affect the conclusions which are drawn from the results.

In these cases, where the solvent has no definite constitution, the specific heats themselves have no interest. I shall therefore give only the molecular heats, determined for two concentrations, which nearly correspond, in respect to the volume occupied by one gram-molecule of the solute, to the solutions with water in which $N = 300$, and $N = 400$. These concentrations are designated respectively by I and II.

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[1] Phil. Trans., 1869; Pogg. Ann., Ergänzungsband V., p. 21.

Molecular Heats in Alcohol.

	I	II	Means.
Resorcin.	58.3	55.2	56.7
Hydroquinone.	56.1	57.3	56.7
Pyrocatechin.	57.6	56.6	57.1

15. *Phenol in Water and in Alcohol.*—The phenol used was obtained from Merck. It was sufficiently pure for the tests made upon it. Its molecular heat determined in water, in a solution for which $N = 300$, was 71.5. In an alcoholic solution, in which the volume occupied by the phenol was very nearly that which it occupied in the aqueous solution, the molecular heat was 51.4. In accordance with what might be expected from its formula, C_6H_6O, its molecular weight, 94, and its nature, it shows a marked resemblance to the three isomers just considered. Its behavior is analogous to that of pyrocatechin.

16. In commenting on these results, attention may be directed first to the general confirmation afforded by them of the theory developed in the earlier paper. The molecular heats are constant for different concentrations when the solutions are so dilute that the assumptions of the theory apply to them. This is true even in the case of the alcoholic solutions, although other alcoholic solutions do not obey this law. In all these cases, therefore, the osmotic pressure is directly proportional to the absolute temperature. In several of these solutions Loomis[1] has shown that the depressions of the freezing temperature indicate a certain degree of association among the molecules of the solute. The constancy of the molecular heats obtained with different concentrations for these solutions shows that this association is not noticeably changed by a change of temperature.

A word may be said about the range of concentration through which the molecular heats are constant. In my experiments, in different cases, this runs from about the concentration of a "normal" solution to that of a "tenth-normal," as used in the determination of electrical conductivities and of freezing temperatures. For such concentrations the study of the depression of the freezing temperature indicates that the osmotic pressure of the solution does not accurately conform to Boyle's law. On the other hand, the constancy of the molecular heats indicates that the osmotic pressure does conform to Gay-Lussac's law. These relations are perfectly comparable with those exhibited by gases under pressure, for which Boyle's law is never more than an approximate statement of the relations between pressure and volume, while the ratio between the pressure and the absolute temperature is very closely a constant.

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[1] PHYSICAL REVIEW, Vol. XII., No. 4, April, 1901.

17. It is evident, secondly, that the molecular heats of isomers in solution in the same solvent are not always the same. Thus the molecular heats of maltose and of milk sugar in its final condition are nearly the same, but differ from the molecular heat of cane sugar. The maximum molecular heat attained for milk sugar during its period of change is the same or nearly the same as that of cane sugar, but differs from that of maltose. The molecular heats of dextrose and levulose differ considerably, as do also those of mannite and dulcite. The molecular heats of resorcin and hydroquinone in water are the same, but differ from that of pyrocatechin. On the other hand, the molecular heats of all three of these substances in alcohol are the same. These solutions, therefore, furnish another illustration of the impossibility of representing the molecular heats of compounds by sums of constant atomic heats of their constituent atoms.

18. It is possible to obtain these molecular heats, at least to a fairly close approximation, in the way proposed by K. Puschl,[1] by assigning to each atom one of the numbers 12, 6, 3, 1.5, 0.75; but the perfectly arbitrary way in which the numbers assigned to any one element differ in the different compounds deprives them of any physical significance.

19. The only way, I think, to explain the values of these molecular heats is to refer them, in general terms, to the degrees of freedom introduced by the solute into the solution. A very ingenious, and in many respects successful attempt has been made by

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[1] Beiblätter, Bd. 24, No. 10, p. 1103.

Staigmüller[1] to calculate the specific heats of the gases from plausible hypotheses as to the degrees of freedom of their molecules; but no such calculation can be made with these substances, partly because of the complexity of their molecules, which makes an estimate of their degrees of freedom impossible, and partly because there is no reason to think that the molecular heats determined for them in solution are due entirely to their own molecular structure and not rather to a more complicated structure containing in one group a molecule of the solute and one or more molecules of the solvent. By supposing the molecular heats to be due only to the molecules of the solute, it is possible to calculate from them by Staigmüller's formulas the degrees of freedom of the molecules of the different substances examined. Certain curious and at first sight promising relations appear among the degrees of freedom thus calculated, and it is possible to reproduce them, in at least two different schemes, by assigning a certain number of degrees of freedom, not exceeding three, to the atoms of each element in the molecule. Notwithstanding a very considerable regularity in the numbers assigned to the elements in the different compounds and marked analogies which appear between the sets of numbers assigned to the isomers and to compounds of generally similar character, I am forced, for the reasons just adduced, to reject all such schemes for the present as illusory or at least as impossible of proof.

20. It is evident, thirdly, that the apparent molecular heats of the same solute differ in different solvents. It is, therefore, inadmissible to consider the process of solution as the mere passage of the molecules of solute into a state in every way equivalent to the gaseous state. The solvent must interact with the solute, so as either to change the structure and the number of degrees of freedom of the solute or to experience a change in its own structure and the number of its degrees of freedom.

21. The specific heats of electrolytic solutions confirm the conclusion that there is an interaction between the solute and the solvent, and indicate that for such solutions, at least, the solvent is itself affected by the presence of the solute. It was shown by Thomsen[2] that the specific heats of aqueous solutions of electro-

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[1] Wied. Ann., Vol. 65, 1898, p. 655.

[2] Pogg. Ann., 142, p. 337, 1871.

lytes are in general lower than those which would be expected if the heat capacities of such solutions were the sums of the heat capacities of their constituents. Indeed, in most cases the apparent molecular heats of the solutes, calculated on the hypothesis that the molecular heat of the solvent remains constant, become negative with increasing dilution. It is plainly inadmissible to assign negative specific heats to the solutes or to any part of them, and we are forced to assume that the specific heat of the solvent or of a part of it is diminished by the presence of the solute. I shall postpone a full discussion of this matter to some future time, when the experimental data are more complete. It will be sufficient at present to state that by proceeding on a particular form of this assumption, we are led to a formula for the heat capacity C of an electrolytic solution of the form

$$C = H + A - pB,$$

in which H is the original heat capacity of the solvent, p is the dissociation factor, or the ratio of the mass of the solute which is dissociated to the whole mass of the solute, and A and B are constants to be determined from the observed heat capacities. This formula applied to the solutions examined by Thomsen, gives a very exact reproduction of the heat capacities found by him for different concentrations. The agreement is so complete as to leave little doubt that the hypothesis upon which it is obtained is justified.[1]

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[1] Paper read before American Physical Society, April 27, 1901; Bulletin of Am. Phys. Society.

POLARIZATION AND INTERNAL RESISTANCE OF ELECTROLYTIC CELLS. I.

By Martin D. Atkins.

Introductory Note on the Polarization-Capacity of Electrolytic Cells.[1]

It is well known that an electrolytic cell behaves like a condenser, or better, two condensers in series, a difference of potential being produced by the passage of a quantity of electricity. In general

$$c\frac{dp}{dt}=i$$

c is called the polarization capacity and is obviously a variable quantity becoming infinitely large after a certain maximum limit of polarization has been reached. On the other hand c has a definite value when the time t, during which the current is flowing is decreased to zero. This value seems to be of importance and is called the "initial capacity" or short "the polarization capacity" of a cell.

Apparently the polarization-capacity may be approximately determined by alternating current methods. Varley,[2] M. Wien,[3] Gordon,[4] Neumann[5] and many others have worked on this problem and though their results are not in absolute agreement, it may be asserted that every metal has a definite "initial polarization capacity" with electrolytic solutions. Concentration and temperature of solution have some influence upon its value, and here the different experimenters arrive at widely different results, a great many disturbing factors entering into the process. So far only small polarization capacities, principally Pt and Hg in H_2SO_4 have been measured.

By knowing the exact formula for polarization as a function of the time and determining the constants, we will be able to calculate the capacity of a cell from these data. Wiedeburg has done so by making use of Bartoli's experiments on Pt in H_2SO_4 and found a result agreeing quite closely with Kohlrausch's direct results.

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[1] By K. E. Guthe.

[2] Proc. Roy. Soc., 1871.

[3] Wied. Ann., 58, p. 37, 1896.

[4] Wied. Ann., 61, p. 1, 1897.

[5] Wied. Ann., 1899.

In a former paper[1] I have been able to show that Bartoli's empirical formula, being the same as one derived by Wiedeburg from a few theoretical suppositions, explains the apparent change of the internal resistance with different current densities, very often attributed to a "resistance of transition."

This formula has the simplified form

$$p = P(1 - e^{-aq}) \tag{1}$$

where P is a constant, "the maximum polarization," a a constant depending on the size of the electrodes and q the quantity of electricity having passed through the cell.

By a simple arrangement of the apparatus, namely by using an E.M.F. very large in comparison with P and using a large non-inductive resistance in series with the cell we were able to have a constant current through the cell during one observation, such that we could employ the formula

$$\rho I = rI + P(1 - e^{-KI}) \tag{2}$$

where ρ is the apparent and r the real resistance of the cell. In the paper mentioned I have shown that this formula represents the resistance curve obtained for Cu in $CuSO_4$.

The same method enables us to determine the polarization capacity of the electrolytic cell and this method is also very well applicable to cells of very large polarization capacities, as those of the so-called unpolarizable electrodes.

It follows from equation (1) that the polarization capacity is

$$c' = \frac{e^{au}}{Pa}$$

and the initial capacity

$$c = \frac{1}{Pa}$$

or using formula (2)

$$c = \frac{t}{PK}.$$

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[1] PHYS. REV., 7, p. 193, 1898.

From the values of P and K calculated from the curves and knowing the time t and the area of the electrodes we can easily determine the capacity of unit area of the electrodes.

I wish to correct a slight error, which occurs in my former paper. From equation (2) we see that for very large currents the apparent resistance equals the real resistance of the cell. For zero current $\rho - r = PK$, or the curve will cut the Y axis, on which the ρ's are plotted at a distance equal to PK above the real resistance.

We have therefore here *a constant current method* for finding the polarization capacity of an electrolytic cell, applicable also to large capacities, which cannot be determined by alternating current methods.

On the other hand all the disturbing influences, which have been found by other observers, will be noticeable here. This method may therefore be used to a great advantage in the study of the changes going on in an electrolytic cell.

It will also enable us to throw some light on peculiarities, observed in researches on the resistance of transition.

MICH. UNIV. PHYSICAL LABORATORY, June, 1899.

PART I. THEORY AND FORMULÆ.

Of the various theories, advanced in explanation of the phenomena observed in electrolytic cells, that of Wiedeburg[1] is deserving of attention in that it is expressed in mathematical formulæ, and that it explains, by polarization alone, those various changes, attending the passage of an electric current through a cell, which, by other theories, have been ascribed to causes as various as the theories themselves. An investigation of this theory becomes then one of extreme interest; since, should it be found to accord with the results of experiment, the question of what occurs within an electrolytic cell would be much simplified. An added interest has attached to this subject, since this investigation was begun, owing to the fact that Dr. Guthe has derived a formula, reducible to the exact form of that of Wiedeburg, from considering the polarization capacity alone. By making use of this derived formula and the hypothesis that the polarization capacity, for any given solution and electrodes, is constant, the method of this research becomes applicable to the measurement of the polarization capacity of a given cell.[1] This assumption is in agreement with the experimental results of several investigators, among whom may be mentioned: Herwig, 1871; Blondlot, 1881; Kohlrausch, 1878; and others.

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[1] Wiedeburg's Habilitationschrift, Leipzig, 1893. Wied. Ann., 51, 302, 1894.

It was for the purpose of determining, if possible, the value of Wiedeburg's theory, and the application of the formulæ he derives, that this investigation was undertaken. It may be said to have had as its objective points the answering of these two questions: (*a*) "Is the change in the resistance of an electrolytic cell with varying current a real or an apparent change?" (*b*) "Does the Wiedeburg theory, with its derived formulæ, satisfy the known conditions, and the characteristic curves, of this change?"

In a preliminary paper,[2] making use of some of the first results of this research, Dr. Guthe has described the method employed and given a brief statement of the theory of Wiedeburg. In the interest of unity, however, these points may well be briefly treated here.

A statement of his fundamental hypotheses is given by Wiedeburg[3] as follows: "I assume that under all circumstances, even with extremely small electromotive forces, electrolysis occurs, viz: of such a nature that in general a part of the ions of the electrolyte, accumulating at the electrodes, is neutralized; *i. e.*, undergoes chemical transformation with loss of its electric charge. This neutralization advances with a variable rapidity, in accordance with a law very similar to that of ordinary chemical reaction. The more numerous are the unneutralized, polarizing ions, accumulated before the electrodes, the more rapidly the further accumulation diminishes with the time; and the more rapidly, on the other hand, the neutralization (electrolysis) proceeds. The increasing polarization would soon produce a cessation of the migration of the ions, for every E.M.F. below its maximum, were it not that other processes, not electric in their nature, operate constantly to decrease it; removing the ions, partly by transference to the interior of the electrodes

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[1] Guthe and Atkins, Polarization and Polarization Capacity of Elect. Cells, Proceedings A. A. A. S., 1899, p. 109.

[2] Polarization and Internal Resistance of Electrolytic Cells: Guthe, PHYS. REV., Vol. VII., No. 38.

[3] "Zur Frage nach dem Grundgesetz der Elektrolyse," O. Wiedeburg, Zeit. Phys. Chem., XIV., I., 1894.

—occlusion—partly by union with neutral dissolved substances, *e. g.*, atmospheric oxygen. It is evident then that, even with an E.M.F. far less than a Daniell, by suitable arrangement of the apparatus, an actual electrolysis of 'acidulated water' may be brought about, increasing even to the point of the evolution of gas."

Let us now apply this theory to the simplest possible case, that of a metallic electrode in a solution of its own salt. If we pass through such a cell any current, of even the slightest E.M.F., the ions are swept to the respective electrodes, carrying their electric charges. Some of these ions pass at once to the electrodes, giving up to them their electricity, becoming themselves neutral and contributing nothing to the counter-electromotive force of polarization. Not all the charged ions however thus give up their electricity, some remaining heaped up before the electrodes. The difference in concentration thus produced results in the counter E.M.F. of polarization. The number of ions free at any instant, with any given solution and electrode, is proportional to the difference between the maximum concentration possible and that obtaining at the moment under consideration.

The formulæ derived from these suppositions by the author of this theory are the same as those of Bartoli[1] upon whose experiments they are founded, viz:

$$\text{(I.)} \qquad p = \frac{P_1}{2}\left(1 - e^{-\frac{\beta_1 q}{S_1}}\right) + \frac{P_2}{2}\left(1 - e^{-\frac{\beta_2 q}{S_2}}\right).$$

Here p is the momentary value of the polarization, whose maximum values are denoted by P_1 and P_2, for the respective electrodes; β_1 and β_2 are constants depending upon the electrodes. S_1 and S_2 denote the areas of the electrodes; and q the quantity of electricity that has passed through the cell.

In the case under consideration, since the electrodes are identical in kind and of equal areas, $\beta_1 = \beta_2$ and $S_1 = S_2$; and the formula simplifies to

$$\text{(II.)} \qquad p = P\left(1 - e^{-\frac{\beta q}{\;}}\right).$$

If now we make the times of each discharge through the cell very short, and the same for all currents, since $q = It$ (II.) reduces to

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[1] Wied. Elektricitat, Vol. II., 883.

$$p = P(1 - e^{-\frac{\beta tI}{S}}).$$

Then for $\left(\frac{BtI}{S}\right)$ we may write KI, and equation (II.) becomes

(III.) $$p = P(1 - e^{-KI}).$$

Deriving the expression for the current in accordance with Ohm's law we obtain

(IV.) $$I = \frac{E' - P(1 - e^{-KI})}{r}.$$

Where E' is the applied E.M.F. at the terminals of the cell, and r its internal resistance. Upon the assumption that no polarization has occurred in the time of passing the current, the apparent internal resistance ρ is usually computed from the formula

(V.) $$\rho = \frac{E'}{I}$$

Substracting (IV.) from (V.) gives equation (VI.), another form of equation (IV.)

(VI.) $$\rho - r = \frac{P(1 - e^{-KI})}{I}.$$

Inspection of this equation shows that the apparent resistance, as usually found by methods taking no account of the polarization, is larger than the real resistance r, found by this method in accordance with Wiedeburg's theory. In every case where comparison could be made with results obtained by the Kohlrausch method, this condition has been fulfilled; while the method has the advantage of applicability to those cases in which the latter method could not be used. Further inspection of equation (VI.) shows that for very large currents, and small polarization, ρ, the apparent, becomes equal to r, the real, internal resistance; while, for zero current, $\rho - r = KP$.

If the apparatus be of such sort that the times for several series of observations can be maintained the same, we may form the following series of equations:

(VII.) $$\rho_1 - r_1 = \frac{P(1 - e^{-KI_1})}{I_1}$$

(VIII.) $$\rho_2 - r_2 = \frac{P(1 - e^{-KI_2})}{I_2}, \text{ etc.}$$

If now we assume $r_1 = r_2$ we obtain, by combining three observations, the following equations

(IX.) $$P = \frac{E_1I_2 - E_2I_1}{I_2(1 - e^{-KI_1}) - I_1(1 - e^{-KI_2})}$$

(X.) $$\frac{E_1I_2 - E_2I_1}{E_2I_3 - E_3I_2} = \frac{I_2(1 - e^{-KI_1}) - I_1(1 - e^{-KI_2})}{I_3(1 - e^{-KI_2}) - I_2(1 - e^{-KI_3})} = Q.$$

The values of E and I can be computed from the observed data and the quotient Q thus determined. K is then computed by successive substitutions until the assigned value satisfies the second member of the equation. In practice, usually, values were found for K which gave values for Q, lying near, but both above and below, the required value. By plotting a curve then, with the assigned value of K and resulting quotients, Q, the value required, could be taken from the curve. Equation (IX.) could then be solved for P. Evidently, were there nothing to indicate the region to be explored for the value of K, such a method, involving a large expenditure of time and a wearisome amount of computation, would be of little practical value. Fortunately it is possible to obtain at once, in most cases, approximate values for both K and P.

Inspection of equation (IX.) shows us that, in case of large currents, with K large, as it is when the electrodes are small, the factor $(1 - e^{-KI})$ is approximately equal to 1. If then we make use of observations taken with large currents, we may use equation (IX.) in this modified form.

(IX.^{a}) $$P = \frac{E_1I_2 - E_2I_1}{I_2 - I_1}, \text{ which gives us } P \text{ at once.}$$

There are then two methods for obtaining approximate values for K. (1) We may compute r for all currrents, beginning with the largest used, and taking an average from such values as do not largely differ. With this mean value, and the observations made with smallest currents, we may compute K from equation (VI.).

(VI.) $$\rho - r = \frac{P(1 - e^{-KI})}{1}$$

(2) From an observation with large current combined with one when the current is small we obtain

$$r = \frac{E_1 - P}{I_1} = \frac{E_2 - P(1 - e^{-KI_2})}{I_2} \text{ which solved for } K \text{ gives}$$

$$\text{(XI.)} \qquad K = \frac{\log\left\{\dfrac{P}{\dfrac{I_2}{I_1}(E_1 - P) - (E_2 - P)}\right\}}{I_2 \log e}.$$

Having, by either of these methods, approximate values for *K* and *P*, the solution, by successive substitutions and the use of the curve, becomes less difficult. The latter of these methods was the one employed almost exclusively, in the second part of this investigation, in every case where *K* was sufficiently large to render it applicable.

It may be said in further explanation of the formulæ, that, with the high applied E.M.F.—from 66 to 82 volts—employed in these experiments, and the small values obtained for the polarization—in only one case exceeding 0.2 of a volt—the assumption that *I* in any case does not vary while the current is flowing is certainly correct to a degree far within the limits of negligible error.

THE APPARATUS AND METHOD.

The apparatus used was the pendulum apparatus designed by Professor Henry L. Carhart[1] and the arrangement was that shown in the accompanying diagram (Fig. 1). The pendulum when released struck and overturned in order keys I. to IV., causing a current from the storage battery *B*, with an E.M.F. of from 66 to 82 volts, to flow through the electrolytic cell *E* during the passage of the pendulum between these points. When key II. was thrown by the pendulum in its passage the condenser *K* was charged at will—as determined by the commutator *C*—with either the difference of potential between the electrodes, or that between the terminals of the known resistance *R*. This resistance was made nearly equal to that of the cell under investigation, to eliminate possible errors which might arise from widely diverging deflections of the galvanometer. The condenser was discharged through the galvanometer *G* when key III. was overthrown. The condenser used was one which careful tests had found approximately free from leakage and absorption. The constant (102.1) for the ballistic galvanometer, had been many times carefully determined. Deflections were read from the image of an aperture and crosswire upon a ground-glass scale; and, by the use of reading lenses, could be easily determined to tenths of a scale division. Later in the investigation, after the eye had become accustomed to the work, it was found possible to dispense with these lenses.

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[1] PHYS. REV., II., 392, 1895; Carhart & Patterson, Elect. Measurements, p. 106.

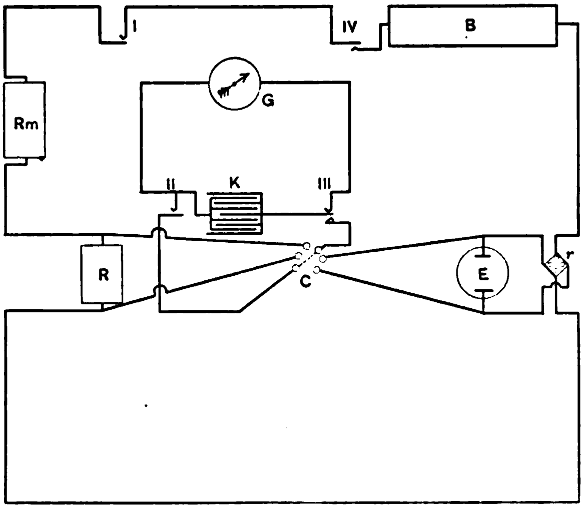

Fig. 1.

Several points of possible error, which suggested themselves, were the subjects of preliminary investigation. It was thought possible that, with the smallest currents used, there might be a loss in charging the condenser; or that it might not fully charge in the short interval allowed for this purpose. To determine the first of these points a key was placed in the galvanometer circuit. The pendulum was then allowed to swing, charging the condenser, but without discharging it. It was then charged again and allowed to discharge as usual through the galvanometer. Readings, thus obtained, did not differ from those obtained by the single charging of the condenser. To determine the second point, the time of charging the condenser was varied largely, key II. being even placed before key I., so that the condenser was in circuit before the current began to flow. The results did not change. It was then evident that the condenser was fully charged, even by the smallest currents, and that there was no loss upon these currents within the condenser and its circuit.

To eliminate adventitious potential differences, either within the cell or condenser, keys, not shown in the diagram, were introduced, by means of which both were kept short-circuited until the instant of releasing the pendulum.

Although the current was allowed to flow for such short intervals, it was found that, with some cells, a marked polarization gradually established itself. To eliminate this, together with possible errors in the galvanometer, a reversing key, *r*, was introduced, by means of which the cell could be depolarized, or the readings tested, if desired.

A somewhat extended, though unsatisfactory, investigation, made early in the work, seemed to indicate that temperature changes had very little influence upon the nature of the curves obtained. An example of this is seen in the two curves for bright platinum electrodes (in H_2SO_4), shown in Plate III., No. 1. It will be seen that, with so great a difference in temperature as 17 degrees, the curves, reduced to the same resistance, are nearly identical. Although a further investigation of this question is intended, and necessary, before an authoritative statement can be made as to the effect of temperature upon the constants, it is quite certain that such small changes as may have occurred in the course of an investigation, rarely reaching 1.5 degrees, could have had no appreciable influence upon the results. Precautionary measures were always taken, by using large amounts of the solutions (from 2 to 10 liters), and by placing the cells in a constant temperature bath. By this means, also, irregularities due to variations in density, convection currents, etc., were reduced to a minimum.

PRELIMINARY INVESTIGATION OF $CuSO_4$ BETWEEN COPPER ELECTRODES.

Since in the preliminary investigation, already reported by Dr. Guthe, copper electrodes in a solution of $CuSO_4$ had been employed, with results in such seeming accord with the theory, the first step in this investigation was a repetition of the observations with these materials. Copper electrodes 10 x 13 cm. were used, insulated—as were all plates investigated—upon the back with a coating of beeswax. The solution was prepared by dissolving 25 grammes of C. P. copper sulphate in 1 liter of distilled water at 20°. It became very soon evident that the results obtained depended upon how long the electrodes had been in the solution. So an experiment was made to determine, if possible, the reasons for the irregularities observed. Successive observations were taken; one, immediately after placing the plates in the solution; another, after two and one-half hours; and a third, after twenty-four hours. The three curves, for the change in apparent resistance (ρ) with the current (I), are shown in Plate II., Fig. 1, Nos. I., II., III. Since it seemed probable that the great increase in apparent resistance was due to oxidation, it is of interest to note that, despite the great disparity in range of the three curves, they start at the same point; *i. e.*, the actual resistance r is the same in each case. This fact indicates that it is not really an increase in resistance, due to a badly conducting film of copper oxide, as has been said, but, rather, an effect due to polarization and its conditions. A possible explanation of this change will be given later, in the general discussion of the results. From the tables of results which follow, it may be seen that, while the first curve is well satisfied by our formulæ, the second is less, and the third still less, in agreement therewith.

The notation employed in these, as in all tables to be given hereafter, is the following: Cap., in the first column, denotes the capacity of the condenser in Micro-Farads; dC, the galvanometer deflection upon the cell; dR, the same upon the resistance. I signifies the current in ampères; ρ, the apparent resistance in ohms; r, the internal resistance computed according to the Wiedeburg formula; P, the maximum value of the polarization in volts; $(1 - e^{-KI})$, the correction factor.

To avoid the errors arising from taking small differences of values from observations lying adjacent, it has been the custom in all these computations to divide the observations into two groups, as has been done here, *e. g.*, P is computed from 1, 4; 2, 5; 3, 6 and 4, 7 respectively.

TABLE I.

I. Oxidation Curve, Cu in $CuSO_4$. *Nov.* 19. $T = 25.5°$; $R = 5$ *ohms. Electrodes* 10×13 *cm. Solution* 25 *G. in* 1 *L.* $K = 20$; $P = .056$; $KP = 1.12$.

Cap.	dC	dR	I	ρ	$(1-e^{-KI})$	r	P
0.1	176.3	160.3	.6200	5.50	1.0000	5.41	
0.2	187.5	168.0	.3291	5.58	.9983	5.41	
0.6	245.7	212.0	.1385	5.80	.9019	5.42	
1.0	212.8	177.0	.0693	6.01	.7395	5.42	.0564
"	108.5	87.0	.0341	6.24	.4829	5.44	.0589
"	44.5	35.0	.0137	6.36	.2335	5.41	.0547
"	28.0	21.9	.0086	6.39	.1583	5.40	.0530

TABLE II.

II. Oxidation Curve, after $2\frac{1}{2}$ *hours. Nov.* 19. $T = 26.1°$; $R = 5$ *ohms. Electrodes and solution undisturbed.* $K = 30.56$; $P = .101$; $KP = 3.09$.

Cap.	dC	dR	I	ρ	$(1-e^{-KI})$	r	P
0.1	178.0	161.0	.6307	5.53	1.0000	5.37	
0.2	194.7	169.5	.3320	5.74	1.0000	5.44	
0.6	265.5	213.0	.1391	6.23	.9853	5.41	
1.0	238.3	177.7	.0696	6.70	.8808	5.44	.1067
	127.8	87.5	.0343	7.30	.6492	5.39	.0975
	54.8	34.7	.0136	7.90	.3399	5.38	.0979
3.2	109.0	67.0	.0082	8.13	.2219	5.40	

It is unnecessary to give the data for the third oxidation curve, as the changes there have become so great that the formula no longer gives any satisfactory results. While, in general, the changes are in the same direction—K having increased to about 46 and P to .21—the values obtained for r vary much more widely than the allowable amount from the constant value demanded by the formula.

Another observation, with the same solution and another pair of electrodes, is given here, for purposes of comparison.[1]

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[1] See also Tables XXVII.-XXVIII. of this paper.

TABLE III.

Copper Electrodes in $CuSO_4$ *(25 G. in 1 L.). Nov. 26.* $T = 23.0°$; $R = 5.0$ *ohms ;* $K = 30$; $P = .054$; $KP = 1.62$.

Cap.	dC	dR	t	ρ	$(1-e^{-Kt})$	r	P
.10	187	164	.6425	5.70	1.0000	5.62	
.25	251	216	.3385	5.81	1.0000	.65	
.50	210	173	.1356	6.08	.9283	.67	
1.00	220	174	.0682	6.32	.8708	.63	.0549
3.20	370	278	.0341	6.67	.6400	.63	.0531
	156	111	.0136	7.03	.3350	.65	.0549
	95	66.5	.0082	7.14	.2178	.65	

The curve, for observed and calculated values for ρ, for a set of observations with copper in $CuSO_4$, is given on Plate III., No. 2. It will be seen that again, as in the former investigation with these materials,[1] there is good agreement between the values thus obtained. While it was evident, then, that with copper electrodes, freshly introduced into a solution of copper sulphate, results in agreement with the theory of Wiedeburg could be obtained, it was desirable, in order that extended observations might be made, that a metal be used less subject to the disturbing influence here encountered. Upon the supposition that this was due to oxidation, the next metal tried was silver, in a solution of silver nitrate.

OBSERVATIONS WITH SILVER IN SILVER NITRATE.

Into a solution of $AgNO_3$, 100 g. in 1 liter at 20°, the electrodes, silver plates, 7.9 x 12 cm., were introduced and successive observations made, as had been done with the copper. These observations extended through a period of five days ; and showed a similar increase in the range of apparent resistance. The longer the plates remained in the solution, the greater this range became, as will be seen by the following table.

TABLE IV.

Silver in $AgNO_3$. *Time change in* ρ.

No. of Observation.	Interval.	Range of Apparent Resistance.
1	53 min.	1.11 to 3.40 *i. e.*, 2.29 ohms.
2	4 hours.	1.12 to 4.88 " 3.76 "
3	60 "	1.19 to 5.45 " 4.26 "
4	74 "	1.11 to 6.42 " 5.31 "

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[1] Guthe, K.E., PHYS. REV., Vol. VII., p. 197.

It is to be noted that here again, as with the copper, there is no increase in the real resistance. Correcting for temperature the curves all start at the same point; while the range through which the apparent resistance changes is more than doubled. That this phenomenon is due to oxidation is impossible; for, as far as could be detected, the plates were as clean and bright at the end of the test as when placed in the solution. Further, the plates were then removed from the cell and burnished; then replaced in the cell; and in the next five days, showed only the following change.

TABLE V.

No. of Observation.	Interval.	Range of Apparent Resistance.
1	50 min.	1.27 to 9.00 *i. e.*, 7.73 ohms.
2	5 days.	1.38 to 9.48 " 8.10 "

An investigation, as to whether this change is one of the electrodes or the solution, while of great interest, would, it was felt, carry us too far from the purpose of the work in hand, especially as a difficulty, not encountered with the copper, was met with here. It consisted in the difficulty of obtaining constant readings. The deflections would slowly increase, as they were repeated, becoming constant at last for a moment, and falling then to a lower point. If now the current were reversed, the resulting reading was at first much the lowest yet obtained. After several readings had been taken, it would pass through the same changes observed before reversing. It was thus constant at three points; giving a minimum, a mean, and a maximum curve. Such a group of curves is given on Plate II., Fig. 3, Nos. VII., VIII., IX.

An explanation of the phenomena just described, in conformity to the theory of Wiedeburg, is quite possible. We need only assume that absorption of the ions by the electrodes occurs, to be in a position to understand these changes. When the first currents pass, absorption is large, and the resulting polarization is cut down. Repeated passing of the current in one direction fills the kathode, practically, and the polarization rises to the fixed value possible for the time and current employed. Meanwhile the anode is destitute of absorbed Ag ions, and hence in the best condition to absorb them. When, on reversing, it becomes the kathode, absorption is at its maximum value, and the resulting point on the p curve has its lowest position. The process repeats itself indefinitely. In accordance with this supposition we should expect, if the supposition be correct, the best agreement with the formulæ in case of those observations taken where absorption is the least. That this is the case, reference to our data shows. I give below in Table VI. the values computed from a set of observations taken immediately upon putting the electrodes in the solution; and in Table VII. those taken from the maximum curve after the electrodes have been thirty days in the solution. Comparison will show that while the errors in r in the former reach 20 per cent., in the latter they are less than 3 per cent.

TABLE VI.

Ag in $AgNO_3$. *Nov.* 26. $T = 19.2°$; $R = 3.0$ *ohms.* *Electrodes,* 7.9 × 12 *cm.*
Solution, 100 *G. in* 1 *L. at* 17°. $K = 79.94$; $P = .0466$; $KP = 3.72$.

Cap.	dC	dR	I	ρ	$(1-e^{-Kt})$	r	P
0.2	73	197.5	.6488	1.11	1.0000	1.038	
0.6	122	313.0	.3405	1.17	1.0000	1.033	.0431
1.0	96	208.0	.1358	1.38	1.0000	1.037	.0493
3.2	191	331.0	.06745	1.73	.9954	1.043	.0474
	124.5	162.6	.0333	2.30	.9302	1.000	
	72.5	64.0	.01306	3.40	.6479	1.089	
	52.5	40.0	.00816	3.93	.4792	1.195	

TABLE VII.

Ag in $AgNO_3$. *Jan.* 2. $T = 16.0°$; $R = 3.0$ *ohms.* *Electrodes,* 7.9 × 12 *cm.,* 30 *days in solution.* *Solution* 100 *G. in* 1 *L.* H_2O *at* 17.0°. $K = 167.4$; $P = .0796$; $KP = 13.4$.

Cap.	dC	dR	I	ρ	$(1-e^{-Kt})$	r	P
0.2	92	193.5	.6317	1.43	1	1.304	
0.4	103	200.0	.3265	1.55	1	.306	
1.0	130	205.0	.1339	1.90	1	.305	.07962
3.2	273.5	327.0	.0666	2.50	1	.304	.07961
	201.5	160.0	.0327	3.78	1	.339	.07355
	158	63.0	.0129	7.52	1	.324	.08471

While the errors in the final values are far above those of observation, they are in accord with the assumption we have made. The observations were always begun with the large currents. If absorption occurs, with consequent depolarization, we should expect the greater effect with large currents; *i. e.*, the values of K and P computed for the lower part will be too small to satisfy the upper part of the curve. While it is possible that a different arrangement in conditions, size of electrodes, concentration of solution, etc., might have given more concordant results, the difficulties in the way of using this metal for the purpose desired seemed too great to warrant its further trial at this time. The fine agreement of the values for the large currents, however, suggests the possibility of making this combination the basis of a later investigation as to the cause of the time change which occurs upon the standing of the electrodes in solution.

PLATINUM ELECTRODES IN SULPHURIC ACID.

Some observations, made in the earlier investigations by Dr. Guthe, with platinized platinum electrodes, showed a sufficient agreement with the formulæ to lead to the thought that, possibly, with bright electrodes, results might be obtained in good agreement with the theory. A number of observations with such electrodes were accordingly made at this point in the work, with the hope of getting rid of the time change in the range of apparent resistance. The results conform even less than had those with the platinized electrodes, to the requirement of a constant internal resistance.

The observed values for the internal resistance show changes from 5 to 160 ohms, with current changes from .6 to .0069 of an ampère respectively, in a single set of observations. While these reduce, by computation of r, to values ranging from 1.43 to 2.02, the variations from a constant value are such as cannot be occasioned by experimental error. The same difficulty encountered with the silver was present here to a much greater degree, *i. e.*, the creeping up of the deflections.[1] Moreover, the time change in the range of apparent resistance was also present, as with copper and silver; a further indication that oxidation cannot be assigned as its cause. Table VIII. shows these changes.

This solution had been boiled to exclude air, and was covered with a layer of paraffin oil.

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[1] See also Guthe, l. c.

TABLE VIII.

Bright Platinum in H_2SO_4. Time change in ρ.

Observation.	Temperature.	Interval.	Range.	Difference.
1	20.9	1 hour.	5.29–165.0	159.7
2	22.5	12 hours.	5.27–203.3	198.0
3	40.5	48 hours.	5.16–211.5	206.3

The readiness with which platinum absorbs hydrogen, and the high value found for the polarization, are possibly chargeable with the variation here found; although the conditions are much more complicated than with a metal in a solution of its own salt. It is very interesting to note that, in spite of the ill agreement in the values for the internal resistance, the mean value obtained for the polarization, 1.88 volts, is in good accord with that found by other investigators. From the numerous data at hand upon this point, only the results from a single set will be given, showing the successive values for *P*.

TABLE IX.

Bright Platinum in H_2SO_4. $T = 38.3°$; $R_1 = 10.0$ *ohms* ; R_2, 30.0 ; $K = 134$; $P = 1.88$.

	Cap.	dC	dR	I	ρ	$(1-e^{-KI})$	r	P
R_1	.05	71.5	162.5	.6366	4.40	1.0000	1.43	2.12
	.10	126.7	171.0	.3350	7.41	1.0000	1.88	1.97
	.20	227.0	173.0	.1695	13.10	1.0000	2.11	1.88
	.20	210.0	87.0	.0852	24.14	1.0000	2.02	1.81
R_2	.30	298.0	211.0	.0459	42.37	.9979	1.53	1.56
	.30	257.6	105.2	.0229	73.46	.9535		1.60
	.30	221.2	211.5	.0138	104.58	.8428		2.26
	.30	173.5	105.6	.0069	164.30	.6030		

ZINC IN $ZnSO_4$.

A. Unamalgamated Plates.—It was with considerable interest that this part of the investigation was approached, owing to the conflicting testimony as to the polarization of this combination. For the sake of comparison only, a series of observations was first taken with the zinc unamalgamated. The plates were of the same size as the copper ones previously employed, 10 x 13 cm. ; and the solution contained 25 G. c. p. zinc sulphate in 1 L. of distilled water. An observation was attempted as soon as the cell was set up; but the rapid time change made it impossible to complete it. To obtain an idea of the rate of this change, observations were taken

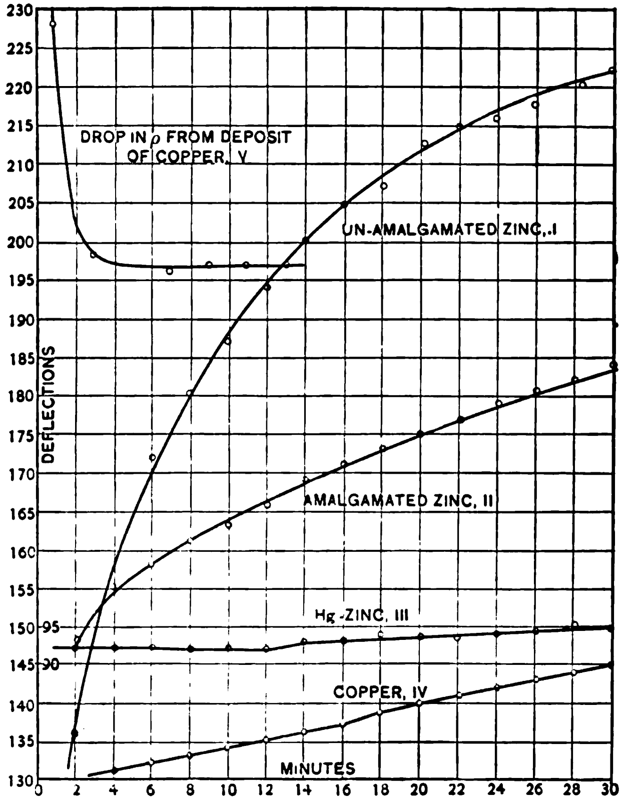

Plate I.

with large sensitiveness of the apparatus, every 20 seconds for half an hour. The results were plotted in the form of a curve, vide Plate I., No. 1.

After the cell had stood undisturbed for two hours and a half it was found possible to take observations, although repeated swings of the pendulum would cut the deflections down from 3 to 5 scale divisions. The curves for these observations, Plate II., Fig. IV., Nos. X. and XI., are very instructive. The observations began with the smallest current used—at the point where the time curve had been taken 2.5 hours before. The currents were gradually increased to the largest used, and then returned to the original point. This order is indicated by the arrow heads upon the curves. These facts are evident: (*a*) the time curve shows an increase of 130 scale divisions in the first half hour, to which corresponds a change of

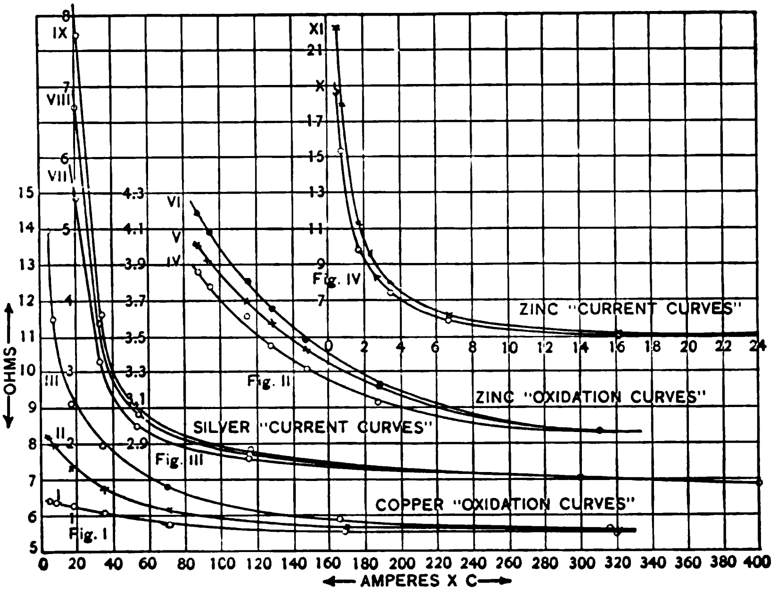

Plate II.

only 17 divisions in the next 2.5 hours; (*b*) the use of large currents cuts down the apparent resistance to a marked degree.

The behavior of these electrodes, upon further standing in the solution, is shown in the following table.

TABLE X.

Time and Current Effect on ρ. Unamalgamated Zn in $ZnSO_4$.

Hours in the Cell.	Starting Point, with Small Current.	Minimum, Largest Current.	End Point, Smallest Current.	Range.	Temp.
2.5	22.3	4.72	18.5	17.6	19.6
24	32.0	3.97	23.3	28.03	17.4
48	32.0	4.10	22.0	27.90	16.6

Consideration of the results here shown makes it clear that, whatever the cause operating to increase the apparent resistance may be, it was in this case practically completed in the 24 hours after setting up the cell. Further, as with the platinum, silver and copper, we cannot conceive it to be an actual increase in resistance, due to an oxidation film. For here we have a cell in which the actual resistance, after 24 and 48 hours, is less than at the beginning; while the apparent resistance ρ has increased its range 60 per cent.

While it was not to be expected that a combination, so susceptible as this, would give very accurate agreement with the formulæ, yet the computation of a set of observations is not without value for comparison with those of silver and platinum. Such a completed set follows in Table XI.

Table XI.

Zn in $ZnSO_4$. *Jan.* 7. *Temp.* 19.6°. $R = 5.0$ *ohms. Unamalgamated Electrodes,* 10 × 13 *cm.,* 2½ *hours in solution. Solution,* 26 *G. C. P.* $ZnSO$, *in* 1 *Liter* H_2O *at* 20.0 *degrees.* $K = 88.8$; $P = .1983$; $KP = 17.7$.

Cap.	dC	dR	I	ρ	$(1 - e^{-KI})$	r	P
0.2	295	312	.6612	4.72	1.0000	4.40	
0.2	162	160.5	.3164	5.04	1.0000	4.41	.1883
0.5	198	168	.1317	5.89	1.0000	4.39	.2083
0.9	222	151	.0657	7.41	.9971	4.40	
1.0	169	84	.0329	10.05	.9462	4.36	
1.0	103	33.6	.0132	15.33	.6896	4.81	

It will be seen that, as with the silver, the resistance r is constant until the smallest currents are reached. The same assumptions there made would suffice to explain these variations also.

B. Amalgamated plates.—It was at this point in the work that plates of varying areas were procured and observations upon them made. It was the intention to determine thus, at the same time, the conformity of the amalgamated plates to the theory, and the relation of the constants to the areas of the electrodes. The discussion of the latter question having been left for the second part of this paper, it is as well to defer to the same place the description of the preliminary work done with these plates, as it mainly was intended to bear especially upon that phase of the determination. It will be sufficient to give here the results bearing directly upon the question: Does Wiedeburg's formula satisfy the curves obtained with amalgamated zinc?

Zinc plates, especially prepared for battery use, were procured, cleaned with some care, amalgamated, and investigated as follows: First, as with the unamalgamated zinc, a time curve was taken—vide Plate I., No. II. It was found that, even with amalgamated zinc, the same time effect is active: the apparent resistance increases merely from the standing of the plates in the solution.

Successive observations were then taken, as with the copper electrodes. The resulting curves are shown in Plate II., Fig. II., Nos. IV., V., VI., where they may be compared directly with those obtained with the copper. There was a very slight increase here in the real resistance, only .15 of an ohm from the first to last observation. Part of this, if not all, is due to a fall in temperature of 0.2°. That better comparison may be possible, the curves have all been referred to the same starting point on the largest current used. The plates were only one eighth the area of those used, unamalgamated, and the solution was one containing 150 grammes in 1 liter, at 17.0°. The time consumed in taking the observations was one and a half hours. The range of increase of ρ is shown in Table XII.

Table XII.

Amalgamated Zinc in $ZnSO_4$. *Time and Cnrrent Change in* ρ.

Observation.	Interval.	Range.	Difference.	Temperature.
1	30 min.	3.86–2.97	0.89 ohms.	19.0
2	"	4.13–3.07	1.06 "	18.9
3	"	4.33–3.12	1.21 "	18.8

Table XIII.

Amalgamated Zinc in $ZnSO_4$. *Feb.* 2. $T = 19.8°$. $R = 2.0$ *ohms. Zinc Plates,* 1. *Solution* (*for all observations*), 150 *G. in* 1 *L.* $K = 6.0$; $P = .0166$; $KP = .0996$.

Cap.	dC	dR	I	ρ	$(1 - e^{-KI})$	r	P
0.5	281	350	.6835	1.606	.9837	1.584	
0.5	142	174	.3408	1.632	.8706	1.593	.0179
0.5	953	116	.2275	1.643	.7446	1.590	.0152
0.7	100.8	122	.1707	1.651	.6409	1.593	.0118
1.0	72	86.7	.0849	1.661	.3972	1.585	

It will be seen that, even with well amalgamated zinc, the difficulties encountered with metals formerly used, are not avoided.

While not so rapid in the first stages of the changes due to standing, the cell seems more susceptible to current effects; and gave often very indifferent curves. From the large number of observations, those given in tables XIII.–XVIII. are selected; showing to what degree the formula fits.

TABLE XIV.

Zinc Plates, 8. $K = 17.4$; $P = .1162$; $KP = 2.08$. *Jan.* 31. T, 17.0; R, 3.0 *ohms*.

Cap.	dC	dR	I	ρ	$(1 - e^{-KI})$	r	P
.2	186.5	191	.6236	2.93	1.0000	2.745	
.2	100.8	97	.3170	3.12	.9960	2.749	
.4	115.0	100	.1632	3.45	.9416	2.780	
.7	146.0	119	.1110	3.68	.8551	2.777	.1231
.9	146.0	114.8	.0833	3.82	.7652	2.749	.1159
1.0	133.5	101.4	.0662	3.95	.6840	2.749	.1096

TABLE XV.

Zinc Plates, 16. $K = 18.0$; $P = .1634$; $KP = 2.88$. *Feb.* 2. $T = 19.4$; $R = 4.0$ *ohms*.

Cap.	dC	dR	I	ρ	$(1 - e^{-KI})$	r	P
.1	105.5	129	.6317	3.721	1.0000	3.014	
.2	116.0	132	.3208	3.515	.9969	3.033	
.3	127.8	136	.2220	3.774	.9816	3.037	
.3	101.0	102	.1665	3.961	.9501	3.028	.1669
.4	112.0	108.8	.1334	4.147	.9094	3.027	.1619
.5	124.0	115.8	.1134	4.283	.8701	3.030	.1613

TABLE XVI.[1]

Amalgamated Zinc, 10 × 13 *cm.*, *in solution* $ZnSO_4$, 25 *G. in* 1 *Liter*. *Jan.* 14. T, 24.0; $R. = 1.0$ *ohm*. $K = 9.7$; $P = .03369$; $KP = .33$.

Cap.	dC	dR	I	ρ	$(1 - e^{-KI})$	r	P
0.5	259	165	.6464	1.57	.9983	1.517	
0.5	135	83.3	.3263	1.62	.9605	1.521	.0333
1.0	114	67	.1312	1.70	.7212	1.515	.0334
3.2	185.5	105	.0643	1.77	.4708	1.520	.0343
3.2	94	52	.0318	1.81	.2703	1.521	

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[1] The curve showing agreement between observed and computed values for these observations, will be found on Plate III., No. III.

TABLE XVII.

Amalgamated Zinc, 5 × 6.5 *cm. Jan.* 19. T, 23.0 ; R, 3.0 *ohms.* $K = 20$; $P = .052$; $KP = 1.04$.

Cap.	dC	dR	I	ρ	$(1-e^{-KI})$	r	P
0.2	104.5	202	.6594	1.55	1.0000	1.489	
0.2	56	102	.3330	1.65	.9987	1.491	.05331
0.9	116	191	.1386	1.84	.9381	1.488	.05070
1.0	132	214	.1397				
1.0	42	106	.0686	2.04	.7462	1.492	.05146
3.2	123.5	166	.0339	2.23	.4921	1.479	
3.2	52.5	65	.0134	2.42	.2356	1.484	

TABLE XVIII.

Amalgamated Zinc Plate 2.5 × 6.5 *cm. Jan.* 21. T, 23.0 ; R, 5.0 *ohms.* $K = 20$; $P = .0676$; $KP = 13.5$.

Cap.	dC	dR	I	ρ	$(1-e^{-KI})$	r	P
0.2	276	312	.6112	4.437	1.0000	4.311	
0.2	145	161	.3154	4.503	.9982	4.290	.0691
0.5	160	168	.1324	4.762	.9292	4.260	.0683
1.0	171	170	.0666	5.029	.7329	4.326	.0702
3.2	281	265.3	.03248	5.296	.4777	4.301	.0627
	114	104	.01273	5.481	.2249	4.288	
	72	65	.00796	5.538	.1471	4.290	

NOTE.

Franz Emil Melde.—Franz Emil Melde was born on the 11th of March, 1832, at Grossenlueder near Fulda. He was the son of the village apothecary. He attended school in his native place until twelve years of age, when he entered the Gymnasium at Fulda; from which institution he went to the University at Marburg in order to devote himself to the study of physics and mathematics. After the completion of his course, he taught for a short time at Hanau and in 1860 accepted a position as assistant in the Mathematical and Physical Institute of his alma mater under Professor Gerling.

Melde's first published work which was accepted as a thesis for the doctorate was entitled, *Ueber einige krumme Flächen, welche von Ebenen parallel einer bestimmten Ebene durchschnitten, als Durchschmittsfigur einen Kegelschnitt liefern.*

A year later followed his paper on *The Production of Standing Waves in Strings.* In 1864, Melde was appointed extraordinary professor and on the 19th of April, 1866, he became ordinary Professor of Physics and Astronomy at the University of Marburg. For nearly 35 years he was the director of the institute which he had entered as assistant. After a long illness he died on the 17th of March, 1901.

Melde devoted himself especially to acoustics to which subject he has contributed a series of important investigations. Aided by his rare skill as a musician and by his unusually accurate ear, he introduced new methods of experimentation which are of lasting value and opened out new fields for study. Among the best known of his contributions to science are his recently perfected methods for the determination of the frequency of tones of high pitch and his investigations upon the vibration of strings, plates and bells. He was the inventor likewise of many beautiful forms of apparatus for the investigation and demonstration of the phenomena of vibration the best known of which is the kaleidophon. One of his most important researches was upon the *Estimation of the Upper Limit of Audibility.* In 1883 Melde published a text-book on acoustics. In 1888 appeared his well-known monograph entitled *Chladni's Leben und Wirken.*

Melde's activity showed itself likewise in other departments of physics. His researches upon the flow of liquids and upon the formation of bubbles in cylindrical tubes are well known. Astronomy is indebted to him for a treatise on the *Measurement of Time*, meteorology for his memoir on the *Cloudless Days observed at the Meteorological Station at Marburg during the years 1886–1894.* This station, which was established through Melde's influence in 1865 by the Hessian government, was patterned after the existing stations in Prussia.

An entire generation of physicists, chemists, medical students and pharmacists have been educated in Melde's lecture room and laboratory. As an experimenter he possessed extraordinary skill and he succeeded with the simplest means to produce from wood, glass, cork and sealing wax, apparatus with which to carry out the most precise investigations. Outwardly he was not infrequently rough and curt, but he possessed a strong sense of humor which cropped out continually in the course of his lectures.

KARL SCHAUM.

NEW BOOKS.

The Elements of Physics for use in High Schools (2d Edition). By HENRY CREW, Ph.D. The Macmillan Co., New York.

In the revised edition of Crew's Physics an already good book is distinctly improved for the high school teacher by the modification of some of its more mathematical parts.

The more important changes are as follows: All use of the method of limits and of trigonometric functions has been abandoned, English letters have been used as symbols in place of Greek, except π, and the number of problems has been increased.

We believe all of these changes are in the right direction. Symbols afford an invaluable shorthand to the student who is conversant with them, and but for the memory of Faraday one would be tempted to say that no advance beyond the most elementary stage is possible without them, but still the more directly the simple facts of physics are presented to the beginner the better. The student should have the facts so presented that he may have a clear mental picture of them unencumbered by symbols or formulas. Of course formulas have an important place and must be used, but in an elementary book they should be kept strictly subordinate.

The disposition to teach physics as though it were a branch of mathematics is illustrated in a definition found in a recent text-book of physics, where the *thermal capacity* of a body is said to be "the quotient of the heat received by the rise of temperature produced." While such a statement may express the process by which the quantity defined is calculated, it is vicious in that it fails to bring out clearly the physical concept. The student should think of thermal capacity not as a quotient, but as a certain quantity of heat.

The book under consideration is certainly entirely free from this defect. It develops the subject in a remarkably clear and exact way, the relations between phenomena being well brought out. It is a book well adapted to lay a sound foundation for more advanced study.

A. L. KIMBALL.

One Thousand Problems in Physics. By W. H. SNYDER and I. O. PALMER. Boston, Ginn & Co., 1900. Pp. 142.

This book contains over a thousand problems intended to supplement laboratory and lecture courses in preparatory and high schools. Problems of this kind are of great value provided they are of a practical character, clearly stated, and selected to illustrate physical principles rather than as examples in applied mathematics. While the greater part of these problems are of the desirable kind, the number is greater than the average pupil can use to advantage, and it is believed the book as a whole would be improved by the elimination of at least a hundred of the thousand problems.

GEO. A. HOADLEY.

Notes and Questions for the Dynamo Laboratory. By F. C. CALDWELL and F. A. FISH. Columbus, O., Fred J. Heer, 1900. Pp. 67.

This little volume consists of a brief chapter of instructions to students in the dynamo laboratory followed by sets of questions upon nearly all the topics in direct and alternating current work usually included in the laboratory courses of our technical schools. There is likewise a set of tables of units, symbols and the physical constants of metals, a page of conventional forms to be used in the diagrams of laboratory reports and a four-place logarithm table.

While planned with special reference to the work in electrical engineering in the Ohio State University, the questions will doubtless be found useful to instructors in other institutions. E. L. N.

PORTRAITS

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THE PHYSICAL REVIEW

A Journal of Experimental and Theoretical Physics

CONDUCTED BY

EDWARD L. NICHOLS, ERNEST MERRITT and FREDERICK BEDELL

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EDWARD L. NICHOLS
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THE

PHYSICAL REVIEW.

THE VISIBLE RADIATION FROM CARBON. II.

By Edward L. Nichols.

Spectro-photometric Observations.

It was my expectation, in planning this research that whatever might prove true as to the character of the radiation from gray carbon, the distribution of energy in the spectrum black carbon would change in such a manner with increasing incandescence as to become nearly or quite identical with that of the various luminous gas flames at temperatures corresponding to the temperature of the glowing carbon in those flames. I had also hoped, among other things, to be able to bring about a degree of incandescence approaching that of the acetylene flame itself, before the usefulness of the thermo-element as a means of measuring the temperature failed because of the melting of the platinum wire, and in this way to obtain a check upon my previous measurements of that flame, and at the same time to be able to determine the temperature of any given luminous flame in which the incandescent material consists of carbon particles by ascertaining the temperature of the carbon rod for which its surface had a spectrum corresponding in distribution of energy to that of the flame.

It will be seen from inspection of the curves to be discussed in a subsequent paragraph that this expectation was far from being realized, and that the distribution of energy in the spectrum of the carbon rod instead of approaching that of the acetylene flame as the temperature of the rod increased took on an entirely unexpected character. Even at low temperatures, that is to say up to about 1100° the change in the spectrum was not of the comparatively simple character which had been anticipated, and shortly after passing the temperature of 1100° a very surprising complication in the results arose. The energy in the yellow of the spectrum, which from the beginning had been increasing at a relatively more rapid rate than either in the red or at the blue end, became so great as to give to the distribution curve a form entirely contrary to expectation.

I was very slow to believe in the integrity of these results and nearly a year was spent in repetitions of the measurements before I could convince myself that the phenomenon was a genuine one. Measurements taken upon a great number of different rods and at different times showed the same result, however, and I was finally forced to the conclusion that the radiation from the carbon rods showed a much more complicated law of distribution than had been anticipated, and that a sort of selective radiation occurred such as to render the establishing of any simple relationship between the curve of distribution and temperature out of the question.

The hope of being able to make direct temperature measurements up to the melting point of platinum was also disappointed. While the carbon rods at comparatively low temperatures showed a fair degree of stability under the action of the current, they appeared to undergo a decided change of behavior at about 1500° and before that temperature a rather rapid disintegration, showing itself by a change of resistance, manifested itself. This effect appeared to be similar to that which shortens the life of the filaments of incandescent lamps. It appeared also that at these high temperatures, the carbon tends to combine with the metals of the thermo-element affecting the electromotive force very much as the vapors in a furnace have been found to do. The thermo-elements inserted in the rod begin, in consequence of this action, to fail of their purpose. It was found that after exposure to temperatures much above 1400°, the electromotive force corresponding to even lower temperatures was considerably below the normal. I was consequently compelled to abandon the attempt to measure directly temperatures above this point, although it was possible to bring the rods to a higher degree of incandescence for a length of time sufficient to perform the spectro-photometric observations. In order to obtain at least an approximate estimate of these high temperatures, I made use of the fall of potential between the terminals of the rod, and also of the current of the heating circuit, and by extending these curves which, throughout the range of measured temperatures were found to be nearly straight, to the higher temperatures which I wished to estimate, to obtain some idea, even if not an exact one, of the latter.

In expressing the results of the photometric measurements already described, I have made use of two forms of curve. One set of curves, in accordance with the nomenclature proposed in my original paper on the visible radiation from platinum, and later adopted by Paschen[1] and other writers, I may designate *isotherms*. These curves give, in terms of the corresponding wave-lengths of the comparison source (in this case the acetylene flame), the relative distribution of energy in the visible radiation from carbon rods. The other curves, which I have termed *isochroms*, indicate the rise in the energy of any particular wave-length of the visible spectrum, with increase of temperature. Each of these curves, taken by itself, is entirely independent of the nature of the light of the comparison source but the absolute relation of such curves to one another can only be obtained when we know the distribution of energy in the spectrum of that source. By means of the isochroms, it is, however, possible even without this knowledge, to compare the rise in intensity of various wave-lengths of the spectrum with increasing temperature.

The set of curves shown in Fig. 7 are plotted directly from observations upon a black (untreated) carbon at temperatures ranging between 795° C. and 1055° C. In this diagram abscissæ are wave-lengths and ordinates are ratios of the brightness of the spectrum of the carbon rod in each region to that of the corresponding region in the spectrum of the acetylene flame. The data are given in Table I.

A noteworthy fact exhibited by means of these curves is the relatively rapid increase of intensity in the middle of the spectrum. In

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[1] Paschen, Wiedemann's Annalen, LVIII., p. 455, 1896.

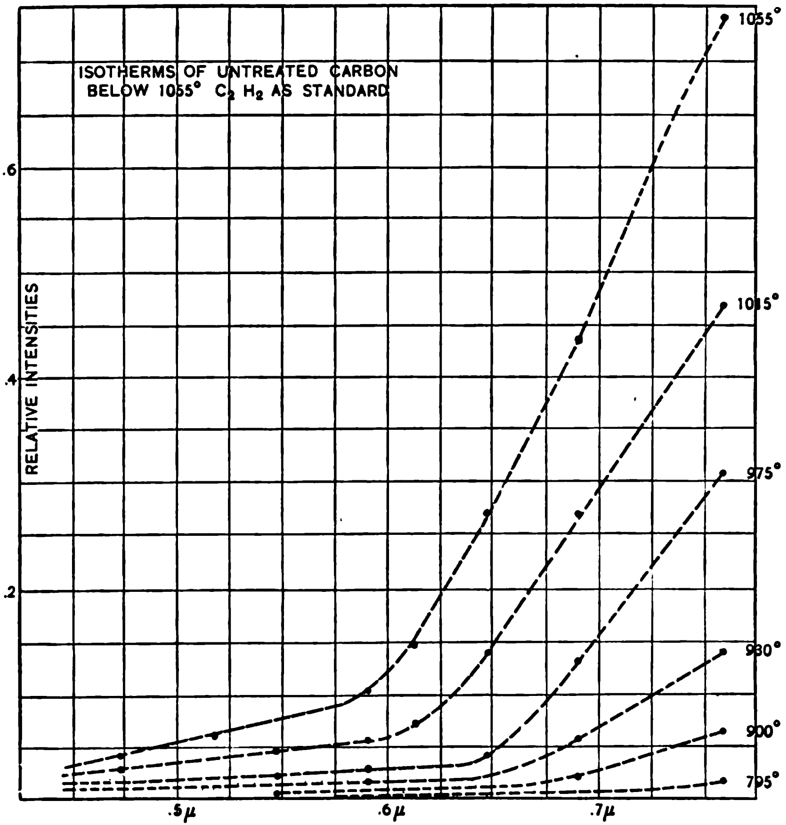

Fig. 7.

TABLE I.

Relative intensities of various regions in the spectrum of an untreated carbon rod; the intensity of the corresponding region in the spectrum of the acetylene flame being in such case taken as unity.

	Value of $\frac{\text{Intensity of Carbon}}{\text{Intensity of } C_2H_2}$ at Temperatures Given Below.					
λ	795°C.	9000C.	930°C.	975°C.	1015°C.	1055°C.
.760μ	.018	.064	.140	.310	.466	.742
.690μ	.007	.022	.057	.131	.270	.436
.647μ	.005	.012	.022	.042	.128	.272
.613μ					.072	.148
.589μ	.003	.008	.017	.028	.067	.103
.548μ			.013	.022	.046	
.518μ			.011	.017		.058
.470μ			.008	.014	.027	.041
.440μ			.006	.012	.022	.030

passing from 930° to 1055° the brightness of wave-length .76 μ increases 5.3 times; that of .70 μ, 7.2 times; that of .60 μ, 13.5 times *and that of .50 μ only 9 times.* We have here the beginnings of a process which becomes more marked in its effects as higher temperatures are attained.

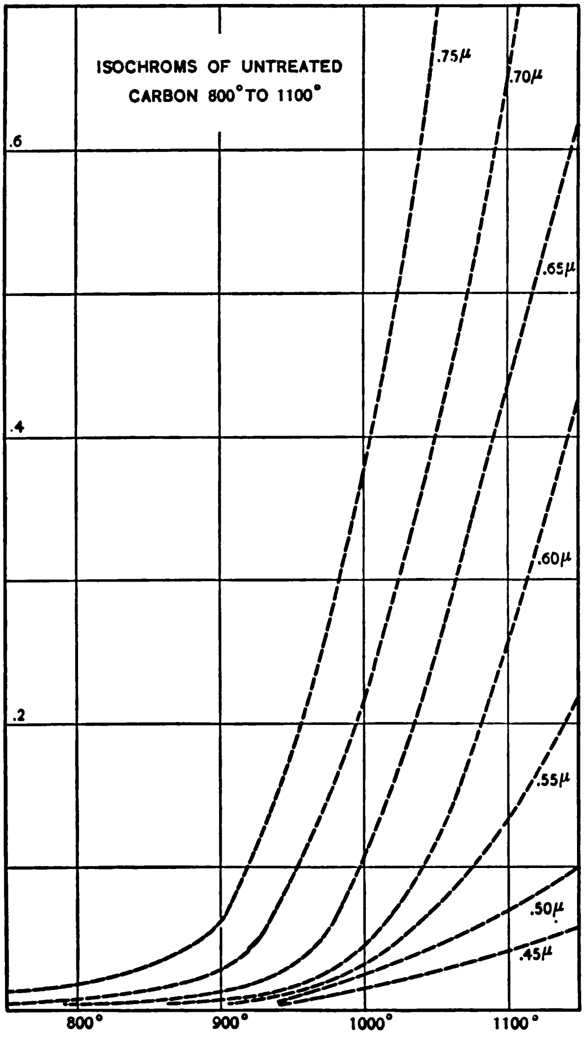

Fig. 8.

From 1100° upwards it was found much more difficult to obtain satisfactory readings. The carbon rods which I had brought from Paris for this investigation would not stand prolonged heating and it was necessary to replace them frequently. In order to bring the observations upon the various rods to a common scale, isochroms from the readings for each rod were plotted. The general character of these curves is shown in Fig. 8 in which the isochroms corresponding to the isotherms of Fig. 7 are given. From the ordinate at 1000° of the isochrom for .6μ, which for convenience was taken as unity for the entire set a reduction factor was obtained by means of which all the curves for all the carbons were brought to the same scale. A new set of isochroms were then plotted for each of the

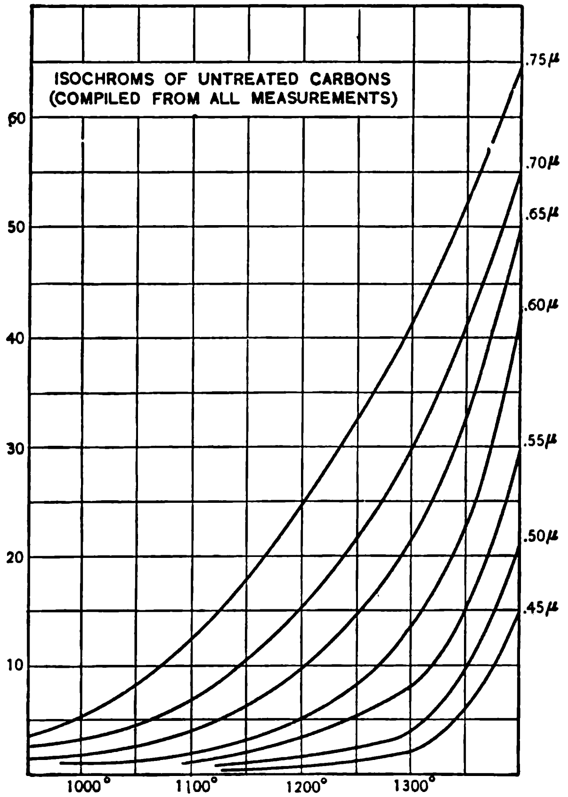

Fig. 9.

wave-lengths .75 μ, .70 μ, .65 μ, .60 μ, .55 μ, .50 μ and .45 μ in the drawings of which all the observations upon the rods were used. While this method did not bring the various sets of observations into perfect agreement, the results were sufficiently definite to indicate with a close degree of approximation the trend of these curves for temperatures up to 1400°. The result of this compilation for the wave-lengths just mentioned is shown graphically in Fig. 9. From these curves in turn, isotherms for the temperatures 900°, 1000°, 1100°, 1200°, 1300° and 1400° were plotted. These

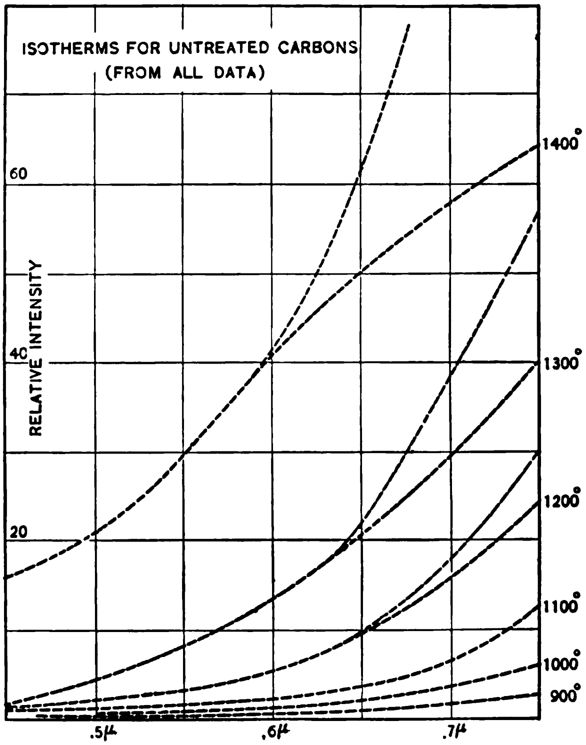

Fig. 10.

curves are given in Fig. 10. The numerical data are contained in Table II.

Had the law of increasing intensities throughout the spectrum with rising temperature been that anticipated at the beginning of

TABLE II.

Relative intensities of various regions in the spectrum of untreated carbon, compiled from all data; the brightness of the spectrum of the acetylene flame being taken as unity throughout and the intensity of the region .6μ in the spectrum of carbon being taken as equal in brightness to that of the corresponding region in the flame spectrum when the temperature of the rod is 1000° *C.*

	Value of $\frac{\text{Intensity of Carbon}}{\text{Intensity of } C_2H_2}$ at Temperatures Given Below.					
λ	1400°C.	1300°C.	1200°C.	1100°C.	1000°C.	900°C.
.750μ	64.4	40.4	24.2	11.9	5.2	2.3
.700μ	58.2	29.7	15.6	6.0	3.2	1.6
.650μ	50.3	21.0	9.8	3.4	1.7	0.8
.600μ	40.7	13.6	5.3	2.1	1.0	0.5
.550μ	29.7	8.2	3.3	1.5	0.5	0.3
.500μ	21.3	4.2	2.2	1.2	0.3	
.450μ	16.3	1.8	1.3	0.7		

this investigation, the trend of the isochroms would necessarily have been such as to bring all the curves together at a common point corresponding to the temperature of the acetylene flame. In other words, if the spectrum of the acetylene flame were identical throughout with that of the carbon rod at the same temperature, the isotherm of the spectrum of the rod at that temperature would be a horizontal line. It is obvious, however, that if the wave-lengths of the middle of the spectrum should continue to increase faster than the red and the violet, a condition would presently be attained in which the ordinate of the isotherm would be greater in the yellow or green than at either end of the spectrum. We see indications of the approach of this condition in the diagram of isochroms (Fig. 9) from which it is evident that the curves for .65 μ and .60 μ would cut each other and would cut the curve for .70 μ at some temperature not far above 1400°, whereas the isochroms for the shorter wave-lengths would not be likely to cut the curves for the red until some much higher temperature had been reached. The curves in Fig. 10 show the nature of this unexpected development of the spectrum in a somewhat different aspect. It will be seen from this figure that the growth in the extreme red so far lags behind that the regions of the full red and this in turn behind that of the orange and this in turn behind that of the wave-length .6 μ, that at 1400° the isotherm instead of being concave as seen throughout, actually becomes convex. I have indicated by means of lighter lines *a*, *b* and *c* the form of curve which might have been expected had the type of isotherm which exists at lower temperatures been maintained.

Above 1400° it was found impossible to obtain consistent readings on account of the rapid disintegration of the carbon rods, but I was able to satisfy myself after repeated trials that at temperatures not far above 1500° this change in the character of the isotherms had progressed to the point at which the yellow regions of the spectrum possess an ordinate greater than that of the extreme red or violet. At a temperature of about 300° below that of the acetylene flame the spectrum of the carbon rod was relatively weaker in the red, stronger in the yellow and weaker again in the shorter wave-lengths than the spectrum of the acetylene flame. There is no reason to suppose that had it been possible to heat the rods to the temperature of the flame itself the law of increase of intensity for the various wave-lengths would have undergone such radical modification to bring the two spectra at that temperature into identity.

Spectro-Photometric Measurements upon Rods with Treated Surfaces.

In order to compare the radiation of rods of black surface with those the surface of which have acquired a gray coating by treatment in hydrocarbon vapor, rods were mounted in the usual manner and after the exhaustion of the air from the metal box, gasoline vapor was allowed to enter until the atmosphere surrounding the rod was saturated. The rod was then brought several times to a high state of incandescence for a few seconds at a time, by which means the entire surface became coated with a gray deposit of carbon similar to that obtained by the treatment of incandescent lamp filaments. The metal box was then again pumped out and spectro-photometric measurements similar to those already described were made upon the radiation from the treated surface. It was thought that as the result of this treatment the carbon rods would stand a more prolonged exposure at high temperatures and thus it might be possible to extend the measurements beyond the points reached with the rods of black surface. This was found to be the case. As has already been indicated in a previous paragraph, the indications of a thermo junction at these high temperatures was subject to serious suspicion. I was obliged to content myself therefore with estimations of the temperature based upon the difference of potential between the terminals of the rod. Fortunately the relation between the electromotive force and the temperature up to 1400° was of such a character that but little error was to be feared in extrapolating. The relation between electromotive force in volts and temperature is shown in Fig. 11. From this curve temperatures above 1400° were determined.

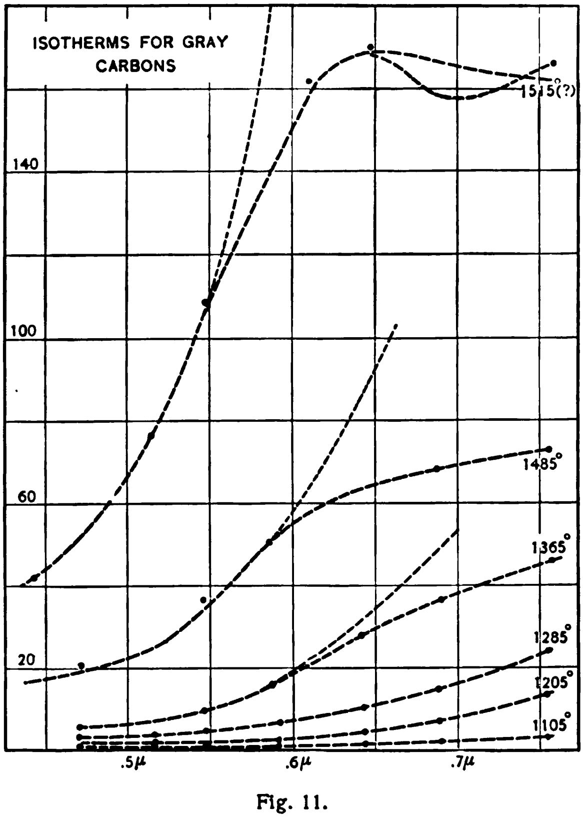

Fig. 11.

The work upon treated carbons was confined chiefly to high temperatures; a sufficient number of readings within the range already explored with the untreated carbons being taken to show that the distribution of intensities at the lower temperatures did not differ materially from that in the spectrum of the former. The set of isotherms shown in Fig. 12, the data for which are given in Table III., will suffice to indicate the general character of the results. It will be seen that in this case as in that of the untreated carbon the convexity of the curve between .6μ and the red end of the spectrum is well marked at 1365°; and that at 1515° there was a well pronounced maximum at about 65μ. The greater stability of the

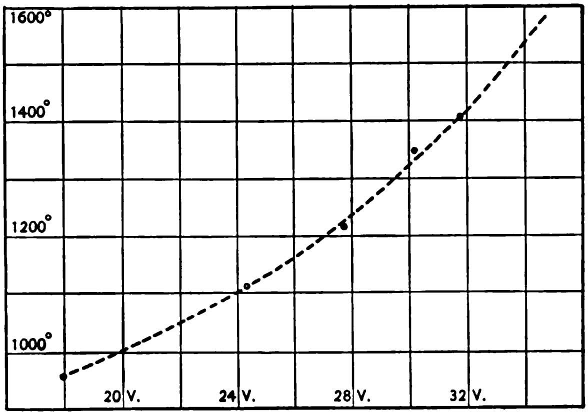

Fig. 12.

Table III.

Relative intensities of various regions in the spectrum of gray (treated) carbon, the brightness of the spectrum of the acetylene flame being taken as unity throughout and the intensity of the region .6μ in the spectrum of carbon being taken as equal in brightness to that of corresponding region in the flame spectrum when the temperature of the rod is 1000° *C.*

	Value of $\frac{\text{Intensity of Carbon}}{\text{Intensity of } C_2H_2}$ at Temperatures Given Below.					
λ	1515°(?)	1485°	1365°	1285°	1205°	1105°
.760μ	166.0	72.7	45.5	23.8	13.3	2.4
.690μ	158.0	67.0	36.3	15.6	6.0	1.7
.647μ	170.0	52.5	28.3	10.7	3.6	1.5
.613μ	161.8					
.589μ	140.0	36.6	17.2	6.9	2.8	1.3
.548μ	108.8		10.2	5.4		0.7
.518μ	76.6			4.6	2.3	0.5
.470μ		20.5	5.6	3.5		0.3
.440μ	41.3					0.2

treated carbon made it possible to obtain consistent measurements on a number of rods at temperatures above 1500° and to establish beyond doubt the form of the curves.

It is obvious that for the study of the spectrum of incandescent carbon at this and higher temperatures the conditions would be much more favorable in the case of the incandescent lamp than with rods mounted in a large vacuum chamber like that used in the present investigation. Lamp filaments in the process of manufacture are brought by thorough carbonization into a condition to withstand permanently much higher temperatures than the rods at my disposal were capable of doing. There is as yet it is true no direct means of determining the temperature of the lamp filament; but the curve for the relation of electromotive force of temperature (Fig. 11) is of such a character as to lead us to expect that comparisons of the spectra of incandescent lamps, in which electromotive forces were used as a criterion of the degree of incandescence would at least enable us to confirm the existence of the remarkable phenomenon brought out by the present experiments and to extend observations of it to still higher temperatures.

Mr. Ernest Blaker has since the completion of the measurements described in this paper made comparisons of the spectrum of incandescent lamps with treated filaments and of lamps the filaments of which before exhaustion had been coated with lamp black with the spectrum of the acetylene flame. The results of his measurements confirm very completely those which I have described in this paper and contribute important evidence in favor of the existence of this anomaly from the law of distribution of intensities in the spectrum of glowing carbon.

THEORETICAL ASPECTS OF THE FOREGOING DATA.

The efforts of students of radiation have of late years been directed particularly to the testing of the various formulæ by means of which the mathematical physicists have attempted to express the intensity of radiation as a function of wave-length and temperature. The equation reached from quite different points of view by Wien[1] and Planck,[2]

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[1] Wien, Wiedemann's Annalen, 58, p. 662, 1896.

[2] Planck, Drude's Annalen, 1, p. 69, 1900.

$$I = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}},$$

in particular, has been the subject of exhaustive discussion and of experimental tests. To this end Paschen[1] determined with the bolometer the distribution of energy in the infra-red spectra of various bodies from 15° C. to 1300°. The materials thus subjected to measurement were oxide of copper, platinum, lamp black, and graphitic carbon. The range of wave-lengths explored extended from 9.2 μ to 0.7 μ. Lummer and Pringsheim[2] and still more recently Mendenhall and Saunders[3] have made similar determinations with the ideal black body; and Lummer and Jahnke[4] have compared the infra-red spectrum of such a body with that of platinum. Wanner[5] working with Paschen made careful spectro-photo metric measurements of the visible radiation from the ideal black body. To test the applicability of the Wien-Planck formula to these measurements of the radiation the equation is given the form,

$$\log I = \gamma_1 - \gamma_2 \frac{1}{T},$$

in which

$$\gamma_1 = \log (c_1 \lambda^{-5}),$$

$$\gamma_2 = \frac{c_2}{\lambda} \log e.$$

The isochromatic curves are then plotted with the logarithm of the intensities as ordinates and the reciprocal of the absolute temperature as abscissæ. The agreement of he equattion with the observations is found in the fact that isochroms thus plotted, at least as far as the work of Paschen and Wanner is concerned, always take the form of straight lines and that the quantity, c_2, computed for various wave-lengths is found to be a constant. Lummer and Pringsheim on the contrary find in the discussion of their measurements that the constant c_2 increases steadily with a wave-length from 13500 at 1.2μ to 16500 at 5μ and 18500 at 8.3μ. The value of c_2 computed by measurements from Beckman at wave length 24 was found to be 24250. Lummer and Pringsheim find moreover that the logarithmic isochroms, especially when extended to higher temperatures, are not straight lines, but show a slight convexity towards the $\frac{1}{T}$ axis.

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[1] Paschen, Wiedemann's Annalen, 58, p. 445, 1896; also, Wiedemann's Annalen, Vol. 60, p. 662, 1897.

[2] Lummer and Pringsheim, Deutschen Phys. Gesellschaft, I., p. 23; II., p. 163, 1900.

[3] Mendenhall and Saunders, Astrophysical Journal, Vol. 13, p. 25, 1901.

[4] Lummer and Jahnke, Drude's Annalen, Vol. 3, p. 283, 1900.

[5] Wanner, Drude's Annalen, 2, p. 141, 1900.

Exception has also been taken to the Wien-Planck formula on the ground that it gives for infinite temperatures a finite limit to the value of the intensity; a result which Rayleigh[1] in a recent paper has characterized as *physically improbable.*

Rayleigh proposes form

$$I = c_1, T \lambda^{-4} e^{-\frac{c_2}{\lambda T}}$$

but Lummer and Pringsheim find that this likewise fails to properly express their experimental results. Lummer and Jahnke propose in view of these discrepancies to give the equation the general form

$$I = cT^5 (\lambda T)^{-\mu} e^{-\frac{c}{(\lambda T)^\nu}}$$

an expression which coincides with Wien's formula for $\mu = 5$ and with Rayleigh's for $\mu = 4$. They find the measurements of Lummer and Pringsheim satisfied when μ lies between 4.5 and 5 and ✓ lies between .9 and 1.0. If we accept the value $\mu = 5$ and ✓ = 0.9 this equation always leads to a finite value of intensity for infinite temperature. All other values of these quantities give infinity as the limit of intensity.

Whether logarithmic isochroms or the value of the quantity c_2, computed from measurements upon carbon rods, would aid in deciding between the various equations under discussion is a question. The data given in this paper would not lead us to class the carbon rod studied as *black bodies.* The emissive power of various forms of carbon is well known to be smaller than that of the ideal black body and there is no reason to suppose that it is independent of the temperature. The relative lagging behind of the intensities in the red might perhaps be taken as an indication of a tendency to approach the finite maximum demanded by the Wien-Planck formula; but the isochrom for .76 shows that the effect if it exists must be looked for at some much higher temperature than that covered by these measurements. In spite of these doubts as to the applicability as to the measurements on carbon rods to the problem of the law of radiation of the ideal black body, I have plotted the various iso-

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[1] Philosophical Mag., Vol. 49, p. 539, 1900.

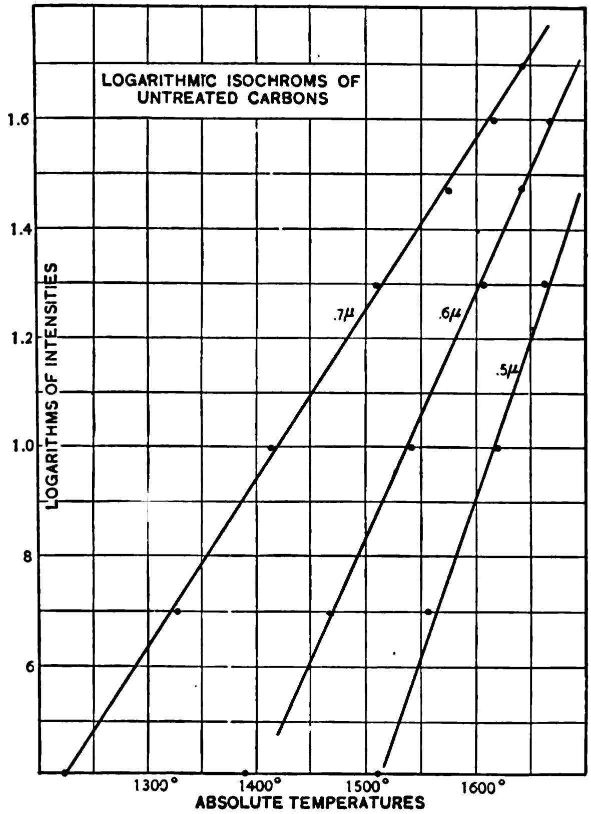

Fig. 13.

chroms obtained in the course of this investigation in logarithmic form; absolute temperatures being taken as abscissæ and the logarithm of the intensity as ordinates. As will be seen from Fig. 13, in which three curves from Fig. 9 are reproduced, these logarithmic isochroms are straight lines. The range of temperatures is doubtless too small to bring out the curvature found by Lummer and Pringsheim; but they show clearly the change of direction with the wave-length mentioned by them on page 222 of their paper before the German Physical Society.[1]

In carrying on portions of the work described in these papers I have had valuable help from Drs. C. H. Sharp and Leopold Kann and from Mr. L. W. Hartman, to all of whom I desire to express my obligations and to extend my hearty thanks.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY, April 24, 1901.

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[1] Lummer and Pringsheim, Verhandl. d. Deutschen. physikal. Gesellsch., 1899, p. 222.

ON THE PERIOD OF A ROD VIBRATING IN A LIQUID.

By Mary I. Northway and A. Stanley Mackenzie.

As early as 1786 Buat announced in his "Principes d'Hydraulique" that the period of a pendulum was greater when vibrating in a fluid than when vibrating in a vacuum, not only because of the loss in weight due to buoyancy but also because of the mass of fluid which must be considered as participating in the motion of the pendulum. The latter is loaded by, or drags with it, a certain amount of the fluid. Buat determined this added mass for the case of a sphere and for bodies of other forms. Little attention was paid to this work until Bessel[1] about forty years later, in determining the length of the seconds pendulum, concluded from theoretical reasons that it was necessary to take into account the inertia of the air as well as its weight. When the dimensions of the bob were small in comparison with the length of the suspending wire, Bessel represented the apparent increase in the weight of the pendulum by a constant, k, times the mass of the displaced fluid. For a sphere 2 in. in diameter vibrating in water he found for k the value 0.9459; later he changed this to 0.956.

Sabine[2] tried to find the effect of air on a pendulum by making it vibrate in air and then in a vacuum. He found that the old correction for buoyancy should be multiplied by a factor m, whose value was 1.655, in order to obtain the total correction.

In 1832 Baily[3] published the results of some similar experiments. Four spheres of about 1.5 in. in diameter gave for m the value 1.84; and three spheres of about 2 in. in diameter gave 1.748. For spheres of about the same size Bessel had found the value 1.956. For small cylinders Baily found the value of m to increase regularly with decrease of diameter, but according to no apparent law.

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[1] Abh. d. Akad. d. Wiss., Berlin, 1826; Coll. de Mem. Soc. Fr. de Phys., 5, 1.

[2] Phil. Trans., 1829; Coll. de Mem. Soc. Fr. de Phys., 5, 134.

[3] Phil. Trans., 1832, p. 399; Coll. de Mem. Soc. Fr. de Phys., 5, 185.

Poisson[1] determined analytically the value of m for spheres and found it to be 1.5; which agrees very well with the results of Buat's experiments made fifty years earlier.

In 1843 Stokes[2] found from theoretical considerations that for a long cylinder oscillating in a direction perpendicular to its length, the effect of the inertia of the fluid was the same as if a mass equal to that of the displaced fluid were distributed along the axis of the cylinder. This agrees very well with the experimental results obtained by Baily for a long tube 1.5 in. in diameter.

Up to this time no law was known connecting the variation of m with the dimensions of the pendulum, and the only property of the fluid taken into consideration was its density. In 1848 Stokes[3] treated the problem of a body oscillating in an infinite liquid medium. He dealt with the cases of a disk rotating about its axis, of a sphere oscillating in the direction of a diameter, and of an infinitely long cylinder vibrating in a direction perpendicular to its length. He considered the effect of the viscosity as well as of the density of the fluid. This paper is of especial interest here because the experiments about to be described approximate more nearly to this problem than to any other that has been treated mathematically. The main results of this important piece of work will be referred to later.

In 1880 Montigny[4] published an account of some experiments which he had made in 1859 in order to find the effect of the density and of the compressibility of various liquids on the pitch of bells. (He intended these experiments to be preliminary only and to repeat them with electrically driven forks. He evidently did not do so, although in a second paper[5] he describes his method of maintaining forks in vibration in a liquid, and states that the pitch is lower the denser the liquid.) He found the pitch of the bell first in air, and then when submerged mouth upward in the liquid; also when in air and filled to the brim with the liquid; and finally when empty and immersed to the brim in the liquid. The pitch was the same for the last two cases, and the lowering of pitch was less than when the bell was entirely submerged; from this he concluded that the lowering was due to that part of the liquid which was in contact with the bell. The lowering was found to depend also on the shape and material of the bell. The pitch was determined by comparing the note by ear with that of a monochord. He found that the lowering of pitch was greater the denser the liquid and the graver the pitch. He gives the following intervals of lowering[1] for four of the bells used.

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[1] Mem. de l'Acad. de Paris, 11, 1831.

[2] Trans. Camb. Phil. Soc., 8, 105, 1843; B. A. Report, 1848.

[3] Trans. Camb. Phil. Soc., 9, pt. 2, p. 35; Coll. de Mem. Soc. Fr. de Phys., 5, 277.

[4] Bull. de l'Acad. de Belg., [2], 50, 159, 1880.

[5] Bull. de l'Acad. de Belg., [2], 50, 300, 1880.

	Sol_4	Ut_5	Sol_5	Ut_6
When filled with water . .	1.204	1.125	1.127	1.118
When submerged in water .	1.411	1.327	1.286	1.261

Auerbach,[2] in a paper written in 1878, attempted to deduce a simple theoretical relation between the pitch of a tuning fork in air and that in a liquid as a consequence of the different way in which kinetic energy is dissipated in the two cases. The pitch, he contends, always depends on the square root of the coefficient of elasticity, and this can have two entirely different values according as the vibration takes place isentropically or isothermally. The former he considers to be the case for air and the latter for liquids. He then takes the coefficients of elasticity in the two cases to be as the ratio of the specific heats, 1.4 : 1, and the interval of lowering of pitch to be as $\sqrt{1.4} : 1$; that is, as 1.18 : 1. He tried four forks in air and in water, observing the pitch by ear, and obtained the following intervals:

Frequency of fork in air,	132	264	396	528
Interval of lowering,	1.11	1.12	1.13	1.15

He concluded from these experiments that the ideal interval, 1.18, would be approached more and more nearly the higher the pitch. He adds that experiments with other liquids show that the density and viscosity of the liquid have no influence on the lowering of pitch.

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[1] The ratio of the pitch in air to that in the liquid is commonly called the interval of lowering.

[2] Wied. Ann., 3, 157, 1878.

Koláček,[1] referring to this work by Auerbach, suggests that an explanation of this lowering of pitch can be given from simple mechanical principles, and that the effect of the liquid is mainly to load the fork by a certain quantity which is the product of the density, ρ, of the liquid and a factor, c, depending on the size and shape of the vibrating body. Accordingly he finds the interval of lowering to be $\frac{T'}{T} = \sqrt{1 + c\rho}$. Taking the mean, 1.13, of the intervals found by Auerbach, he determined the value for c to be 0.277. Using this value of c it follows that Auerbach's forks should give $\frac{T'}{T}$ equal to 2.18 when made to vibrate in mercury, and 1.21 when in sulphuric acid. On plunging a 435 fork into various liquids and noting its changed pitch by an ear comparison with a monochord, Koláček found the value of $\frac{T'}{T}$ to be 2.1 for mercury, 1.21 for sulphuric acid, and about the same for ether and alcohol as for water. He proceeds to treat analytically the simple cases of the radial dilatational and linear translational oscillations of a sphere in a liquid, and finds expressions for $\frac{T'}{T}$ in the two cases. When applied to a solid iron sphere vibrating in water these expressions give for $\frac{T'}{T}$ the values 1.03 and 1.17 respectively. Neither of these cases is that of a tuning fork, but they give limiting values for it and the mean of the two agrees roughly with Auerbach's results. In a later paper Koláček[2] gives a more rigorous hydrodynamical discussion of the general problem of the vibration of a solid in a liquid.

In 1882 Auerbach[3] experimented with glass cylinders filled with liquid. Assuming that the effect of the liquid was to load the cylinder with a mass, c, depending on the nature of the liquid and on the shape and dimensions of the cylinder, he deduced the equation, $\frac{T'}{T} = \sqrt{\frac{m + c}{m}}$, for the lowering of pitch, where m is the mass of the cylinder. He obtained the following results: (1) the interval was smaller the higher the pitch ; (2) the interval was independent of the height of the cylinder ; (3) the interval was greater in proportion to the narrowness of the cylinder ; (4) using different liquids whose densities varied from 0.729 to 1.364, and in addition mercury, he found that as a first approximation the specific lowering of pitch (the ratio of the lowering for any liquid to that for water) depended on the density only and increased with it, but more slowly, and as a second approximation that the specific lowering increased as the compressibility of the liquid decreased. He determined the pitch by Kolácek's method.

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[1] Wied. Ann., 7, 23, 1879.

[2] Sitzb. math.-naturw. Cl. Wien, 87, Abth. 2, 1147, 1883.

[3] Wied. Ann., 17, 964, 1882.

Miss L. R. Laird,[1] working in this laboratory, made some experiments on the change in the period of a pianoforte wire 0.446 mm. in diameter and 107 cm. long, vibrating in the following liquids : water, solutions of potassium carbonate, and mercury. The wire was made to vibrate by means of an electro-magnet and its record was taken on a revolving drum along with that of a standard tuning-fork. The results for the lowering of pitch agreed very well with the values obtained analytically by Stokes in his 1848 paper for the case of an infinite cylinder. The change of pitch gave the following intervals : 1.06 for water ; 1.13 for potassium carbonate of density 1.47 ; 1.095 for potassium carbonate of density 1.22 ; 1.72 for mercury. In the case of water the observed values agreed still more closely with numbers calculated on the simple supposition that the wire was loaded with the mass of the displaced liquid, which was the conclusion arrived at by Stokes in his 1843 paper. Some experiments on a similar wire 0.933 mm. in diameter and 37 cm. long gave the interval 1.065 for water, 1.18 for oil of density about 0.9, and 1.12 for a sodium nitrate solution of density 1.38. It will be noticed that the lowering for water was the same for both wires.

The following experiments were made in continuation of those of Miss Laird, and were intended to separate, if possible, the effects of the density and the viscosity of the liquid. The change in pitch of a tuning fork is of most direct interest, and, as the nearest simple approach to this, a rod clamped at one end was studied. As has been stated, this problem is a very rough approximation to the case of the infinite cylinder discussed by Stokes. The present problem differs from his in that the rod is of a different cross section and has edges, which greatly changes the stream lines; that it has not the same amplitude at all points, and is of finite length. Nevertheless, a consideration of Stokes' work will be of great service, and will throw at least some light upon the way in which the constants of the liquid must enter.

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[1] Phys. Rev., 7, 102, 1898.

If we suppose that a mass m has a simple oscillatory motion as a whole in the x direction, and is in a vacuum, its equation of motion is

$$m\frac{d^2x}{dt^2}+px=0; \tag{1}$$

and its period

$$T=2\pi\sqrt{\frac{m}{p}}. \tag{2}$$

Stokes found that the effect of a liquid medium is to introduce two terms into the above equation of motion. The first of these is of the nature of an acceleration, the second of a velocity. The former is equivalent to an increase of the mass of the vibrating body by a certain fraction of the mass of the displaced fluid; m becomes $m+km'$ where m' is the mass of the displaced fluid. This is the same as saying with Buat that the body drags along with it a definite quantity of the liquid. The quantity k will evidently be determined by the form of the stream lines and must accordingly be a function of the shape and size of the cross-section of the vibrating body, of the density of the liquid, and of the coherence of its separate layers, or of its viscosity; and this Stokes found to be the case. The second of the terms which he found to enter into the equation of motion is a retarding force proportional to the velocity. The form of this term can be assigned from general considerations; for the energy absorbed by the friction will be determined by the amount of the fluid displaced, and will depend not only on the velocity but also on the number of times its direction is changed. Stokes found it to be of the form $k'm'n$, where n is the pitch of the body in vacuum, and k' a quantity depending on the dimensions of the body and on the density and viscosity of the liquid. For the infinitely long cylinder he found, when $\frac{2}{a}\sqrt{\frac{1}{\pi n}\cdot\frac{\mu}{\rho}}$ was small,

$$(3) \qquad k = 1 + \frac{2}{a}\sqrt{\frac{1}{\pi n}\cdot\frac{\mu}{\rho}}, \quad \text{and} \quad k' = \frac{2}{a}\sqrt{\frac{1}{\pi n}\cdot\frac{\mu}{\rho}},$$

and hence $k = 1 + k'$ where a is the radius of the cylinder.

The quantity $\sqrt{\frac{1}{\pi n}\cdot\frac{\mu}{\rho}}$ enters also into the values of k and k' found for the disk and the sphere.

Our equation of motion (1) can now be written

$$(4) \qquad (m + km')\frac{d^2x}{dt^2} + k'm'n\frac{dx}{dt} + px = 0.$$

The well-known solution of this equation gives for the period T' the value

$$T' = 2\pi\left[\frac{p}{m + km'} - \frac{1}{4}\left(\frac{k'm'n}{m + km'}\right)^2\right]^{-\frac{1}{2}}$$

$$= 2\pi\left(\frac{m + km'}{p}\right)^{\frac{1}{2}}\left[1 + \frac{1}{8}\frac{k'^2m'^2n^2}{p(m + km')}\right]$$

approximately, assuming k'^2 to be small. This can be put in the form

$$T' = 2\pi\left(\frac{m}{p}\right)^{\frac{1}{2}}\left[1 + k\frac{\rho}{\Delta}\right]^{\frac{1}{2}}\left\{1 + \frac{1}{8}k'^2\left(\frac{\rho}{\Delta}\right)^2\frac{1}{1 + k\frac{\rho}{\Delta}}\right\}$$

where Δ is the density of the body. Using equation (2) this becomes

$$\frac{T'}{T} = \left(1 + k\frac{\rho}{\Delta}\right)^{\frac{1}{2}}\left\{1 + \frac{1}{8}k'^2\left(\frac{\rho}{\Delta}\right)^2\frac{1}{1 + k\frac{\rho}{\Delta}}\right\}$$

If k' is at all small we may neglect the second factor, especially for liquids of small density; in this case the interval of lowering of the pitch becomes

$$(5) \qquad \frac{T'}{T} = \sqrt{1 + k\frac{\rho}{\Delta}}.$$

This is equivalent to neglecting in our equation of motion the term which depends on the velocity. The following experiments were made to test this equation and to find the form and value of the quantity k. One could approximate more closely to Stokes' theoretical problem than was done in the present investigation by using rods of circular cross-section; but the mechanical difficulties of maintaining such rods in vibration in a plane are too great, and those used had a rectangular cross-section, and so corresponded to one-half of a tuning-fork. Although in the theoretical discussion T is the period in a vacuum, it is a sufficiently good approximation for this investigation to take it as the period in air.

The rod to be experimented on was held in a heavy clamp screwed to the inside of a wooden box, which held, when required, the liquid under consideration. The rod was maintained in vibration by an electro-magnet which could be suspended at any height from a movable bar across the top of the box. The circuit was completed through the rod and a piece of platinum soldered to its free end dipping into a mercury cup fitted with a plunger and projecting up from the bottom of the box. When the rod was of brass a thin piece of soft iron 9 mm. wide and 32 mm. long was soldered to it just beneath the electro-magnet. The vibrations were recorded on a revolving drum, upon which a marker also recorded seconds as given by a standard clock. The pitch of a given length of the rod used was found first with the vibrations taking place in air; then without disturbing anything the box was filled with the liquid under consideration, and the lowering of pitch calculated. The box was large in order that the nearness of its walls might not influence appreciably the pitch of the rod. Three boxes were used, as follows:

	With Water.	With Oil.	With Sodium Nitrate Solution.
Internal length.	57 cm.	55 cm.	55 cm.
" breadth	26 "	21 "	25 "
" depth	30 "	22 "	22 "

To show that the size of the box is of great importance it may be stated that the pitch of the rod A (described below) in water was increased nearly five per cent. when it was changed from the large box to one whose dimensions were 52 x 4.5 x 4.5 cm. In order to prevent as far as possible the absorption of energy by the box, etc., the clamp was made of a mass of metal weighing fifteen pounds; the whole apparatus was placed on a stone pier; and the box, the cross-bar supporting the electro-magnet, and the projecting end of the rod, if any, were heavily weighted down with masses of iron. Unless this were done the pitch was sensibly affected.

The duration of contact of the platinum wire with the mercury, the distance of the electro-magnet and piece of soft iron from the clamp, the shape of the electro-magnet and its height above the rod, and the strength of the driving current were all found to affect the pitch of the rod. The time of contact used was always the shortest that would maintain the rod in vibration. The distance of the electro-magnet from the clamp affected the pitch in the following way: the shorter the distance the greater was the interval of lowering, no matter what was the vibrating length of the rod. Table I. gives a specimen of the readings taken for the purpose of studying this question; they were made on the rod *A* with half of its length (21.1 cm.) vibrating.

TABLE I.

Distance of Electro-magnet from Clamp.	Frequency in Air.	Frequency in Water.	Height of Electro-magnet above Soft Iron.	Interval of Lowering.	Average Interval.
2.1 cm.	44.67	38.45	3 mm.	1.162	1.162
" "	44.63	38.40	3 "	1.162	
" "	44.62	38.37	3 "	1.163	
11.0 "	38.89	33.63	8 "	1.153	1.154
" "	39.09	33.72	8 "	1.156	
" "	39.09	33.91	8 "	1.153	
17.5 "	36.92	32.21	8 "	1.146	1.147
" "	36.95	32.22	8 "	1.147	
19.5 "	36.64	32.09	8 "	1.142	1.142
" "	36.63	32.08	8 "	1.142	

The final average intervals, found as above, for various lengths of the rod were:

Average distance of electro-magnet,	6 cm.	11 cm.	19 cm.
Average interval,	1.160	1.153	1.148

The interval, therefore, diminishes as the driving force is placed nearer to the free end of the rod. In order to get results undisturbed by this possible source of variation, the electro-magnet was placed at a constant distance from the clamp throughout the whole series of experiments, the piece of soft iron being taken off the rod and soldered on in a new place whenever the vibrating length of the rod was changed. The distance from the clamp chosen was such that with the shortest length of rod used the soft iron was at the tip end. For the rod *A* this length was 10.55 cm. and for the other rods it was 21.1 cm. A set of observations was taken to find also the effect of the height of the electro-magnet above the rod. The electro-magnet first used was of the *U* shape. Its cores did not project beyond the bobbins, whose ends were each about 2 sq. in. in area. The effect of the height was not very noticeable for air, but for water a change of height from 7 to 23 mm. made the rod vibrate more slowly by more than 1 per cent. There was no doubt that with this electro-magnet near the rod, the coils acted partly as a wall, and they were replaced by a single coil whose core projected 15 mm. and was 6 mm. in diameter. With this electro-magnet it was found that a diminution of height increased the pitch, no matter how far the electro-magnet was from the clamp, but in no case by so much as 1 per cent. and usually by much less. It was evident, therefore, that the height must be kept constant throughout the experiments; but as the height of the electro-magnet was increased, the current also had to be increased, and it seemed desirable to keep the current also constant. The current strength chosen was the smallest that would drive the shortest length of rod used, and the height chosen was the greatest possible for that current and length.

Five rods were used during the investigation. Four of them, called *A*, *B*, *C* and *D*, were of brass, and one, *E*, of steel. *B* had approximately twice the thickness of *A* and the same width; *C* twice the width and the same thickness; and *D* twice the width and twice the thickness. *E* had approximately the same dimensions as *A*. The dimensions and densities of the rods were:

	A	*B*	*C*	*D*	*E*
Width. . . .	0.94 cm.	0.97 cm.	1.91 cm.	1.95 cm.	0.97 cm.
Thickness. .	0.32 "	0.65 "	0.34 "	0.67 "	0.33 "
Density. . .	8.56	8.56	8.70	8.56	7.79

The liquids used were water, cotton-seed oil, and a solution of sodium nitrate saturated at 15° C. The first two had approximately the same density but very different viscosities, while the first and last had approximately the same viscosity but quite different densities. The viscosities of the oil and the solution of sodium nitrate were found by the method used by Gartenmeister.[1] The temperature throughout was kept as nearly as possible at 17°.6 C. At this temperature the liquids have the following constants :

	Water.	Cotton-Seed Oil.	Sodium Nitrate Solution.
Density, ρ	1.000	0.921	1.355
Viscosity, μ	0.011	0.781	0.030

The tables which follow give the results obtained. All of the separate readings are not given, but the average frequency in air and in the liquid for each length, and the corresponding interval of lowering. In no case were less than two distinct sets of observations made on each length of the rod used. Ordinarily 1,000 vibrations for each observation were counted on the drum and the corresponding number of clock seconds, and the pitch calculated therefrom.

Table II.

Rod A. Dimensions, 42.2 × 0.94 × 0.32 *cm. In water.*

Vibrating Length.	Frequency in Air.	Number of Readings.	Frequency in Water.	Number of Readings.	Interval of Lowering.
Full length.	10.33	4	9.04	3	1.148
¾ "	18.16	2	15.78	2	1.151
⅝ "	25.58	2	22.27	2	1.149
½ "	39.10	2	33.95	2	1.152
⅜ "	66.73	2	58.09	3	1.149
¼ "	136.44	4	119.45	6	1.143
Average Lowering.					1.149

From this table it is seen that for a range of about 3½ octaves the lowering is independent of the pitch within the limits of accuracy attained. The same deduction can be made from the numbers in each of the other tables. The lowering here is a little less than a minor whole tone plus a small semitone $\left(\frac{10}{9}\times\frac{25}{24}=1.157\right)$.

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[1] Zeitschr. f. physikal. Chem., **6**, 524, 1890.

Table III.

Rod E. Dimensions, 42.2 × 0.97 × 0.33 *cm. In water.*

Vibrating Length.	Frequency in Air.	Number of Readings.	Frequency in Water.	Number of Readings.	Interval of Lowering.
¾ length.	24.86	2	21.48	2	1.157
⅝ "	35.62	2	30.71	2	1.159
½ "	53.54	2	46.20	2	1.158
Average Interval.					1.158

The lowering here again is independent of the pitch. The interval is exactly a minor whole tone plus a small semitone, and differs by less than 1 per cent. from the interval for the brass rod of approximately the same dimensions. It seems safe to conclude that no great importance is to be attached to the materials used for the rods, at least for such materials as are ordinarily used for tuning forks, bells and kindred musical instruments.

Table IV.

Rod B. Dimensions, 42.0 × 0.97 × 0.65 *cm. In water.*

Vibrating Length.	Frequency in Air.	Number of Readings.	Frequency in Water.	Number of Readings.	Interval of Lowering.
Full length.	19.80	2	18.29	2	1.083
¾ "	34.56	2	31.65	2	1.094
⅝ "	47.50	2	43.60	2	1.089
½ "	72.14	3	66.51	3	1.085
Average Interval.					1.088

The lowering is nearly a semitone plus a comma, or a great limma $(\frac{16}{15}\times\frac{81}{80}=1.080)$, and is not far from being one-half of that for rod *A* of one-half the thickness.

TABLE V.

Rod C. Dimensions, 42.2 × 1.91 × 0.34 *cm. In water.*

Vibrating Length.	Frequency in air.	Number of Readings.	Frequency in Water.	Number of Readings.	Interval of Lowering.
Full length.	10.12	2	7.95	2	1.273
¾ "	17.81	2	14.01	2	1.274
⅝ "	25.05	2	19.78	2	1.267
½ "	37.86	5	30.32	5	1.244
Average Interval.					1.265

The lowering is exactly a major third plus a comma ($\frac{5}{4} \times \frac{81}{80} = 1.265$) and is nearly twice as much as that for rod *A* of one-half the width.

TABLE VI.

Rod D. Dimensions, 42.0 × 1.95 × 0.67 *cm. In water.*

Vibrating Length.	Frequency in Air.	Number of Readings.	Frequency in Water.	Number of Readings.	Interval of Lowering.
Full length.	19.89	3	17.19	3	1.157
¾ "	35.13	8	30.29	7	1.160
⅝ "	48.23	2	42.34	2	1.140
Average Interval.					1.152

The lowering here is a little less than a minor whole tone plus a small semitone, and is almost exactly the same as that for rod *A* of one-half the width and one-half the thickness, as we should expect from the results shown in Tables IV. and V.

GENERAL APPROXIMATE RESULTS.

1. The interval of lowering for a rod of given cross section is independent of the length.
2. The interval of lowering for a rod of given cross section is approximately the same for brass and steel and is probably independent of the material within the range of substances ordinarily used.
3. The interval of lowering for a rod of given width is approximately inversely proportional to the thickness.
4. The interval of lowering for a rod of given thickness is approximately directly proportional to the width. More definite relations will be given later.

The above experiments were then repeated with cotton-seed oil instead of water. This oil was used because it is inexpensive, and consequently can be used in large quantities, and has its coefficient of viscosity about seventy times that of water. With this liquid it was hoped that the importance of the viscosity in the lowering of pitch could be determined. Whereas in the case of water the contact breaker was under the liquid, it had to be outside for the oil and the nitrate solution. The platinum wire was replaced by a platinum-tipped thin steel wire which reached above the surface of the liquid where it was bent over and dipped into the mercury cup. No special difficulty was experienced in maintaining the rod in vibration in the viscous liquid. The results of these experiments were as follows:

TABLE VII.

In Cotton-seed Oil.

Rod.	Vibrating Length.	Frequency in Air.	Number of Readings.	Frequency in Oil.	Number of Readings.	Interval of Lowering.	Av. Interval of Lowering.
A	Full length.	10.00	10	8.67	10	1.140	
"	¾ "	17.32	2	14.81	2	1.169	
"	⅝ "	24.57	2	20.91	2	1.175	
"	½ "	37.15	4	31.90	4	1.164	1.162
B	Full length.	20.26	3	18.42	4	1.105	
"	¾ "	36.11	3	32.22	3	1.121	
"	⅝ "	49.67	4	44.72	3	1.111	
"	½ "	65.18	3	59.65	3	1.093	1.108
C	Full length.	10.21	4	8.13	4	1.256	
"	¾ "	18.38	3	14.33	3	1.283	
"	⅝ "	25.76	3	19.90	3	1.294	
"	½ "	38.60	2	30.58	2	1.262	1.272
D	Full length.	19.67	3	16.89	3	1.166	
"	¾ "	35.30	2	29.54	5	1.195	
"	⅝ "	48.43	3	41.44	3	1.169	1.176

We shall discuss these numbers more carefully later, but we can say at once in a general way that the effect of viscosity is not very marked. In order to throw still more light on the effect of viscosity as compared with that of density, the four brass rods were made to vibrate in a solution of sodium nitrate, which had a viscosity of only 0.0299, and so not very much greater than that of water, and whose density was 1.355, about a third as much again as that of water. These determinations resulted as follows:

TABLE VIII.

In Sodium Nitrate Solution.

Rod.	Vibrating Length.	Frequency in Air.	Number of Readings.	Frequency in Nitrate Solution.	Number of Readings.	Interval of Lowering.	Av. Interval of Lowering.
A	Full length.	10.00	2	8.47	3	1.181	
"	¾ "	17.25	2	14.48	3	1.191	
"	⅝ "	24.70	2	20.46	3	1.207	
"	½ "	37.58	2	31.51	2	1.193	1.198
B	Full length.	20.35	2	18.10	2	1.123	
"	¾ "	35.75	2	31.84	2	1.123	
"	⅝ "	50.99	2	44.96	2	1.134	1.127
C	Full length.	10.21	2	7.61	2	1.343	
"	¾ "	18.51	2	13.76	2	1.347	
"	⅝ "	25.98	2	19.26	2	1.349	1.346
D	Full length.	19.97	2	16.57	2	1.206	
"	¾ "	35.30	3	29.51	3	1.196	
"	⅝ "	49.15	3	40.88	3	1.202	1.201

Comparing these numbers with those obtained for water it is seen that the lowering is increased very decidedly, and that the increase is not far from being directly proportional to the added density. It is evident therefore that the main factor in the lowering of pitch is the density of the medium, and that the effect of viscosity is relatively small.

Thus far the conclusions drawn from the investigation have been of an approximate and general nature; it remains to determine whether an expression for $\frac{T'}{T}$ can be found which will contain the results of every case investigated. Stokes' work can afford little direct help since he found the value of $\frac{T'}{T}$ to depend on n, as will be seen by reference to equation (3); whereas in the present investigation the interval of lowering is independent of the pitch within the limits of accuracy attained. Indirectly his work is of great service, and his results have been used already in the formation of the equation of motion (4). Since for the case he discussed, the value of k'

is considerably smaller than that of k, and since the results here obtained for oil show that the change of pitch due to friction is relatively small, it is allowable as a first approximation to neglect k', and consider the lowering of pitch to be expressed by equation (5),

$$\frac{T'}{T} = \sqrt{1 + k\frac{\rho}{\Delta}}.$$

The following table contains the values of k found from this equation.

Table IX.

Values for k.

Medium.	**Vibrating Length.**	**Rod A.**	**Rod B.**	**Rod C.**	**Rod D.**	**Rod E.**
Water.	Full length.	2.697	1.478	5.396	2.895	
	¾ "	2.776	1.682	5.418	2.954	2.641
	⅝ "	2.737	1.589	5.263	2.561	2.678
	½ "	2.796	1.515	4.761		2.660
Average.		2.752	1.566	5.210	2.803	2.660
Oil.	Full length.	2.785	2.054	5.454	3.342	
	¾ "	3.407	2.385	6.101	3.978	
	⅝ "	3.537	2.178	6.369	3.407	
	½ "	3.298	1.809	5.596		
Average.		3.257	2.107	5.880	3.576	
$NaNO_3$	Full length.	2.483	1.642	5.152	2.858	
	¾ "	2.632	1.643	5.220	2.707	
	⅝ "	2.873	1.799	5.255	2.797	
	½ "	2.662				
Average.		2.663	1.698	5.209	2.787	

The quantity k can be a function of the constants of the liquid and the dimensions of the rod. Stokes has shown that μ and ρ enter always as a ratio, and further that in the equation for the lowering of pitch they enter as $\sqrt{\frac{\mu}{\rho}}$; an attempt was accordingly made to express k as a function of $\sqrt{\frac{\mu}{\rho}}$ and the cross section of the rod (the length has been shown not to enter). This it has been found can be done in several ways of about the same degree of satisfactoriness. The following four equations give values for $\frac{T'}{T}$

with various forms of the quantity k; w denotes the width of the rod, and t the thickness.

$$(a)\qquad \frac{T'}{T} = \sqrt{1 + 2.63\frac{w^{\frac{7}{8}}}{t^{\frac{3}{4}}}\left(1 + \frac{1}{4}\sqrt{\frac{\mu}{\rho}}\right)\frac{\rho}{\Delta}}.$$

$$(b)\qquad \frac{T'}{T} = \sqrt{1 + 1.12\frac{w}{t^{\frac{3}{4}}}\left(1 + \frac{1}{4}\sqrt{\frac{\mu}{\rho}}\right)\frac{\rho}{\Delta}}.$$

$$(c)\qquad \frac{T'}{T} = \sqrt{1 + \frac{w}{t^{\frac{7}{8}}}\left(1 + \frac{1}{4}\sqrt{\frac{\mu}{\rho}}\right)\frac{\rho}{\Delta}}.$$

$$(d)\qquad \frac{T'}{T} = \sqrt{1 + 0.91\frac{w}{t}\left(1 + \frac{1}{4}\sqrt{\frac{\mu}{\rho}}\right)\frac{\rho}{\Delta}}.$$

The last of these equations is preferred on account of its simplicity, but its concordance with the observed values is not so good as is the case with the first, where the greatest deviation is 1 part in 110. The observed average values for $\frac{T'}{T}$ and those calculated from equation (d) are given in Table X., along with their differences.

TABLE X.

Rod.	Medium.	Water.	Oil.	NaNO$_3$.
A	Obs. Cal. Diff	1.149 1.150 + .001	1.162 1.164 + .002	1.198 1.200 + .002
B	Obs. Cal. Diff.	1.088 1.078 − .010	1.108 1.086 − .022	1.127 1.106 − .021
C	Obs. Cal. Diff.	1.265 1.271 + .006	1.272 1.295 + .023	1.346 1.358 + .012
D	Obs. Cal. Diff.	1.152 1.148 − .004	1.176 1.163 − .013	1.201 1.198 − .003
E	Obs. Cal. Diff.	1.158 1.162 + .004		

It will be seen that the agreement is as close as that of the separate values of $\frac{T'}{T}$ in any of the tables II. to VIII.

The above equations (*a*), (*b*), (*c*) and (*d*) were found from a close consideration of Stokes' work. It is worthy of remark, however, that an entirely empirical formula can be found which will give an equally good agreement with the results. The following table contains the observed values of $\frac{T'}{T}$, and those calculated from the equation

$$(e) \qquad \frac{T'}{T} = 1 + 0.45\,\frac{w^{\frac{1}{2}}}{t}\,\frac{\rho}{\Delta} + 0.03\,\mu.$$

TABLE XI.

Rod.	Medium.	Water.	Oil.	$NaNO_3$.
A	Obs. Cal. Diff.	1.149 1.157 + .008	1.162 1.168 + .006	1.198 1.212 + .014
B	Obs. Cal. Diff.	1.088 1.079 − .009	1.108 1.096 − .012	1.127 1.107 − .020
C	Obs. Cal. Diff.	1.265 1.273 + .008	1.272 1.275 + .003	1.346 1.368 + .022
D	Obs. Cal. Diff.	1.152 1.180 + .028	1.176 1.196 + .020	1.201 1.254 + .053
E	Obs. Cal. Diff.	1.158 1.155 − .003		

In the work which has been done on this subject there is only one case where sufficient details are given to enable us to apply our equations to the values found for the lowering of pitch. Koláček[1] states that for a horseshoe magnet 340 mm. long, 12.5 mm. wide and 46 mm. thick vibrating in water the value of $\frac{T'}{T}$ was 1.03. Calculated from equations (*d*) and (*e*) the interval is 1.02.

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[1] Loc. cit.

It was thought worth while to test equation (*d*) by applying it to the numbers found by Miss Laird, taking $\frac{w}{t}$ as equal to 1. It was found that the values thus determined for the interval of lowering in water agree with the observed values even better than do the values calculated on the simple supposition that the wire is loaded with the mass of the displaced fluid. For denser liquids, however, equation (*d*) does not agree so well as the equation of Stokes, since the lowering was found to depend on the pitch. The following table contains the frequencies in air and in liquid observed by Miss Laird, and the differences between the observed frequency in liquid and numbers calculated (1) from Stokes' equation, (2) from equation (*d*), and (3) on the assumption that the wire is loaded with the mass of the displaced liquid. These differences are called Δ_1, Δ_2 and Δ_3 respectively.

TABLE XII.

	Pitch in Air.	Pitch in Liquid.	Δ_1	Δ_2	Δ_3
Water.	73.8	70.1	− 2.9	− .4	− .6
	92.3	87.9	− 3.6	− .7	− 1.0
	105.5	99.4	− 2.8	+ .3	− .1
	113.4	107.7	− 3.8	− .5	− .9
	126.5	117.8	− 1.8	+ 1.7	+ 1.3
Solution of Pot. Carbonate.	73.8	65.1	− .8	+ 2.9	
	92.3	82.6	− 1.7	+ 2.4	
	105.5	93.9	− 1.2	+ 3.3	
	113.4	99.7	+ .1	+ 4.8	
	126.5	111.6	− .2	+ 5.2	
Mercury.	73.8	43.0	− 1.8	+ 2.8	
	92.3	53.3	− 1.3	+ 4.0	
	105.5	61.9	− 2.2	+ 3.5	
	113.4	65.8	− 1.5	+ 4.7	
	126.5	73.4	− 1.5	+ 5.2	

An attempt was made to see what would come from assuming that there was always attached symmetrically to the rod when in motion a constant mass of liquid of elliptical cross section. It will be seen from the way in which the quantity *R* was introduced that it is the ratio of the volume of this attached liquid to the volume of the rod. Using Table IX. it was found that if the major axis of the ellipse were 2 w and the minor axis $t + \frac{1}{2} w$, the periods calculated, on the assumption (used in this paper) that the lowering of pitch is due only to the added mass of liquid, agreed very well with the observed values.

PHYSICAL LABORATORY, BRYN MAWR COLLEGE,
May, 1901.

POYNTING'S THEOREM AND THE DISTRIBUTION OF ELECTRIC FIELD INSIDE AND OUTSIDE OF A CONDUCTOR CARRYING ELECTRIC CURRENT.

BY WILLIAM S. FRANKLIN.

INTRODUCTION.

PROFESSOR J. H. POYNTING has derived,[1] from Maxwell's equations, an expression for the flow of energy in the electromagnetic field. According to Poynting the energy stream is, at each point, perpendicular both to the electric field and to the magnetic field, and the amount of energy which in one second streams across an area of one square centimeter at right angles to the stream is equal to $\frac{1}{8\pi} EH$, where E is the electric field intensity and H is the magnetic field intensity. J. J. Thomson[2] has pointed out that, although Poynting's expression for the energy stream is perhaps the simplest expression that will satisfy the necessary conditions, it is not the only expression that will do so, inasmuch as any energy flow which has no convergence, or in which the energy streams are reëntrant, may be superposed upon the Poynting stream without changing the distribution of energy which actually takes place in any given case.

In his paper above referred to Poynting gives a number of examples illustrating his theorem. Thus he describes, in a general way, the flow of energy from the dielectric of a charged condenser to the various parts of a wire through which the condenser is discharged; he describes the flow of energy from the seat[3] of the electromotive force in a voltaic cell to the various parts of the circuit to which the cell delivers current; and he describes the flow of energy from the hot junction of a thermoelement, showing how an excess of energy flows in at the cold junction thus producing the heating effect discovered by Peltier.

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[1] Phil. Trans., Vol. 175, Part II., p. 343, 1884.

[2] Recent Researches, p. 313.

[3] The author, for the purpose of his discussion, assumes this seat to be mainly at the surface of contact of acid and zinc.

Poynting's derivation of his theorem is entirely satisfactory and convincing to one who is familiar with the use of differential equations in the discussion of electromagnetic theory, and the only uncertainty which would appear to affect the result is the uncertainty of the fundamental truth of Maxwell's equations.

The examples given by Poynting are not, however, entirely satisfactory, and for a sufficient reason. An example of the theorem is, in the nature of things, a solution of Maxwell's equations for a particular case. Such a solution requires a complete specification of all conditions at the boundary of the region in which the solution applies and a complete knowledge of all singularities occurring within this region. One who appreciates this fact is likely to be satisfied with the examples given by Poynting, notwithstanding their lack of numerical detail, for the solution of Maxwell's equations for any ordinary current generator and its circuit is impracticable.

In a series of short articles appearing at intervals from 1897 to date, Mr. P. S. Wedell-Wedellsborg[1] subjects Maxwell's equations in general and Poynting's theorem in particular to criticisms which have suggested to the writer the desirability of supplementing Poynting's original discussion in three particulars, namely, by stating as clearly as possible the conditions which determine the electric field distribution inside and outside of a conductor carrying an electric current, by working out the numerical details of several simple illustrative examples, and by calling attention to a mechanical model, due originally to Maxwell, which not only gives a good representation of an energy stream in an electro-magnetic field, but gives vivid pictures of the steady electric current and of the action of an electric oscillator with the accompanying energy fluxes. The writer is of the opinion, however, that an understanding of Poynting's theorem and a conviction of its truth can be based only upon Maxwell's differential equations, and that particular solutions are not only generally impracticable, but useless as well. Still the discussion given in the following pages may be of interest to those who are pleased only when a mathematical discussion concludes with particular integrals and concrete results.

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[1] "Notiz ueber Poynting's Theorem." Zeit. f. Phys. Chem., Vol. 22, p. 222.

"Ueber die Gültigkeit der Maxwellschen Gleichungen." Zeit. f. Phys. Chem., Vol. 24, p. 367.

"Antwort an Herrn Scheye." Zeit. f. Phys. Chem., Vol. 29, p. 494.

"Widerlegung eines sehr allgemeinen und wichtigen Safzes der modernen Elektrizitätslehre." Zeit. f. Phys. Chem., Vol. 33, p. 631.

"Ueber Poynting's Theorem." Zeit. f. Phys. Chem., Vol. 35, p. 604.

The earlier criticisms of Wedell-Wedellsborg are met, in a manner that seems to the writer satisfactory, by D. A. Goldhammer,[1] and the need of a numerical solution of Maxwell's equations for a particular case involving the flow of energy into a conductor is well met by G. Mie[2] who gives a solution for the case of two long parallel cylindrical conductors carrying equal and opposite currents.

Wedell-Wedellsborg's criticisms are quite largely based upon the question of the distribution of electric field inside and outside of the current conductor. In regard to this matter, A. Scheye[3] points out that in most ordinary cases the effect of the energy of the electric field is very small and is ignored in the theory of current induction. In a later article Scheye[4] meets two explicit objections raised by Wedell-Wedellsborg, namely, that the electric field distributions required by Maxwell's equations do not agree with experiment, and that, granting the existence of this electric field we are led to inconsistent results. Scheye agrees with the first objection in so far as to admit that further decisive experiments are required. The second objection he combats, pointing out that inasmuch as Maxwell's equations are consistent among themselves they cannot lead to inconsistent results if no foreign assumptions are introduced.

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[1] "Einige Bemerkungen über die von Hernn Wedell-Wedellsborg vermütete Nichtgültigkeit der Maxwellschen Gleichungen für das Innere der Konduktoren." Zeit. f. Phys. Chem., Vol. 23, p. 686.

[2] "Ein Beispiel zum Poyntingschen Theorem." Zeit. f. Phys. Chem., Vol. 34, p. 522.

[3] Zeit. f. Phys. Chem., Vol. 26, p. 159.

[4] Zeit. f. Phys. Chem., Vol. 32, p. 145.

ON ELECTRIC FIELD DISTRIBUTION.

Electric Field in the Conductor.—The electric field intensity at a point in a homogeneous conductor, of which the specific resistance is r, is equal to ri when i is the electric current density at the point. From this expression we can draw two important inferences, namely: (*a*) There is no free electric charge anywhere in the interior of a conductor carrying a steady electric current, for a steady electric current has no convergence,[1] and therefore the field intensity ri has no convergence if r is constant. (*b*) Near the boundary of the conductor the electric field in the conductor is parallel to the surface of the conductor, for i is parallel to the surface of the conductor, and so also is ri.

Electric Field Outside of the Conductor.—The distribution of electric field outside of the conductor is determined by certain conditions which must be satisfied at the surface of the current conductor and at the surfaces of other conductors in the region. At each point of the surface of the current conductor the tangential component of the external electric field must be equal and parallel to ri inside the surface, and at each point of the surfaces of other conductors the tangential component of the external field must be zero.

An immediate result of these conditions is that the electric lines of force are never normal to the surface of a current conductor, and that the electric lines of force may be parallel to the surface of the current conductor. In fact, however, it is only in exceptional cases that the electric lines of force are parallel to current conductors. Thus, in the particular case worked out by G. Mie[2] the electric lines of force are very nearly perpendicular to the surfaces of the current conductors, and in one of the examples given below the lines of force are parallel to the surface of the current conductor. There is always at least one point on any electrical circuit where the electric lines of force are parallel to the current conductor, this is the point where the conductor has no surface charge.

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[1] That is, $\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0$, when u, v, and w are the components of i.

[2] Reference given above.

Reproduction by Stationary Charges of the Electric Field Distribution around a Current Conductor.—The distribution of electric field in the region surrounding an electric current, including the distribution inside of the conductor, can be reproduced by an arrangement of stationary electric charges, the electric current being reduced to zero. Imagine a given current conductor to have its specific resistance increased indefinitely, the size and shape of the conductor, and the generator electromotive force being unchanged. Then ri remains unaltered while i approaches zero. When the limit, $r =$ infinity, is reached the current is zero, the distribution of electric field and of electric charge is the same as at first, the electric charges are evidently stationary, and the magnetic field and energy stream have vanished. This ideal change of specific resistance of the current conductor might be realized by cutting the conductor into a very great number of lamina perpendicular to the lines of flow of the current, all the cross-cuts being made simultaneously.

That the electric charge which is distributed over a current conductor is stationary is further shown by the fact that these charges are surface distributions so that their movement would constitute current sheets in the surface of the conductor and there is no experimental evidence of the existence of such surface currents.

The Conception of Moving Lines of Force.—The idea of the side motion of the lines of force of the electric field surrounding a current conductor has been developed by Poynting,[1] by J. J. Thomson,[2] and by Wien,[3] and this idea has been extensively used by other writers to give concreteness to discussions of electromagnetic action. The writer is of opinion that the fundamental notions of the partial derivatives with respect to time and space, such as form the basis of Maxwell's equations, are quite adequate to the full description of the phenomena of electromagnetic action; and that the only advantage of the idea of the side motion of the lines of force is its concreteness. In the case of a plane electromagnetic wave, for example, it seems to the writer to be quite as satisfactory to consider the rates of growth or decay of the electric and magnetic field intensities at a point in their relation to the space variations of these intensities in the neighborhood of the point, as to imagine an actual side motion of the lines of force although the latter idea is certainly better adapted to descriptive purposes. Either method appears to be applicable to any case inasmuch as Webster[1] has recently derived Maxwell's equations from a consideration of the side motion of lines of force.

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[1] "The Connection between Electric Current and the Electric and Magnetic Induction in the Surrounding Field." Phil. Trans., Vol. 176, p. 277.

[2] Recent Researches, pp. 1–50.

[3] "Ueber den Begriff der Localizirung der Energie." Wied. Ann., Vol. 45, p. 685.

"Ueber die Bewegungen der Kraftlinien im electromagnetischen Felde." Wied. Ann., Vol. 46, p. 352.

In applying the idea of the side motion of the lines of force to the case of a steady electric current a difficulty arises. The charges in which the electric lines of force terminate are stationary and it seems that the lines of force also must be stationary. When the electric lines of force are parallel to the surface of the current conductor their side motion involves no immediate inconsistency, but these lines of force may in every case be traced to termini in stationary charges at some more or less remote part of the current conductor.

In the case of unsteady electric currents, as for example, the transmission of electric waves along wires,[2] the idea of side motion of the lines of force does not seem to lead to inconsistencies, for in this case the charges are not stationary.

Neglect of Electrical Energy in Ordinary Current Problems.—The energy associated with an electric current is in part the energy of the magnetic field and in part the energy of the electric field. In many cases the electrical energy is negligible in comparison with the magnetic energy. Thus, at a point distant one centimeter from a long copper wire one millimeter in diameter carrying a current of ten ampères, the magnetic energy is about 0.16 erg per cc. and the electrical energy is about 2×10^{-12} ergs per cc.[3] This example is somewhat misleading inasmuch as the magnetic energy falls off rapidly as the distance from the wire increases while, in case of a long straight wire, the electrical energy does not fall off. This is equivalent to the statement that for short circuits the electrical energy is negligible while for extended circuits, such as long distance transmission lines, the total electrical energy may be as great as or even greater than the total magnetic energy.

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[1] In a paper read before the American Physical Society, December, 1899.

[2] See paper by A. Sommerfeld, Wied. Ann., Vol. 67, p. 233. A number of diagrams are given in this paper from which a student can get a good idea of the trend of the electric and magnetic lines of force near a wire over which an electromagnetic wave is passing.

[3] This expression for the electrical energy is calculated from the component of the electric field parallel to the wire.

Experimental Evidence of Electrical Field Surrounding an Electric Current.—The writer is not inclined to agree with Scheye[1] that the experimental evidence as to the character of the electric field surrounding an electric current is to any considerable extent deficient. Ohm's law itself shows that the potential gradient parallel to a wire is equal to ri and this law is verified by a great variety of experimental evidence. On the other hand, the so-called capacity effects on long transmission lines and in submarine cables are due immediately to the component of the electric field at right angles to the surface of the conductors; all practical calculations of these capacity effects are based directly or indirectly upon Maxwell's equations, and these calculations agree with experiment as closely as could be expected when we consider first that such calculations are usually abbreviated and that dielectrics and magnetic substances deviate from that beautiful simplicity of behavior which we usually assume as a basis of our calculations.

The submarine cable tests by Crehore and Squier[2] constitute, perhaps, the most complete experimental verification, hitherto published, of the theory of the electric circuit involving both electric and magnetic energy.

EXAMPLES ILLUSTRATING POYNTING'S THEOREM.

The following solutions of Maxwell's equations, illustrative of Poynting's Theorem, apply to two dimensions, so that the respective diagrams represent the solutions completely. The shapes of the conductors have been chosen with a view to simplicity and the electric lines of force and the energy stream lines belong to families of curves with which everyone is thoroughly familiar. A minimum of effort of the imagination is therefore required for a full comprehension of these solutions and their value, as examples of Poynting's theorem is correspondingly great.

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[1] Reference given above.

[2] Presented in a paper read before the annual meeting of the American Institute of Electrical Engineers in May, 1900.

The Slow Discharge of Two Parallel Charged Cylindrical Conductors through a Cylindrical Current Sheet.—The only case, exemplifying Poynting's theorem, for which a complete solution of Maxwell's equations can be set up for all space, is the case of the discharge of a condenser. This problem has been treated in a variety of cases by Hertz[1] and by J. J. Thomson.[2] These authors have treated mainly the case of the oscillatory discharge. The case of very slow discharge is of course much simpler. This case is described in a general way by Poynting[3] as an example illustrating his theorem. The following is an exact solution of the problem of the slow discharge of two parallel charged cylinders through a cylindrical current sheet.

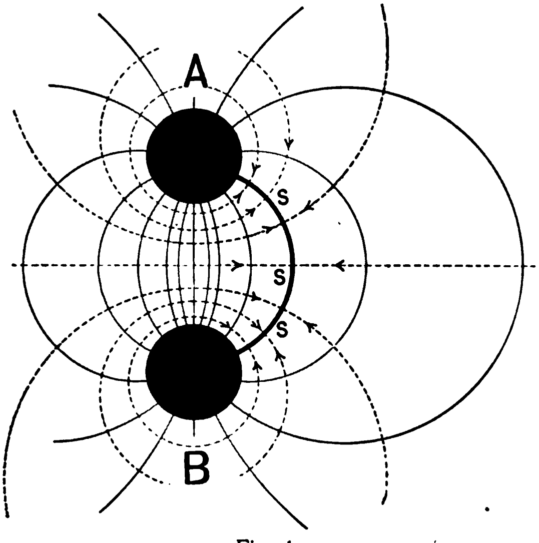

Fig. 1.

A and *B*, Fig. 1, represent two cylinders which are oppositely charged, and connected by a very high resistance sheet of metal *SSS*, through which the cylinders are slowly discharged. The charges, which are uniformly distributed on the cylinders A and B, produce an electric field of which the lines of force are circles, as represented by the full lines in the figure. If this electric field due to the charges on A and B happens to satisfy the necessary boundary conditions along the current sheet SSS, then this field alone will constitute the solution of the problem under consideration. If this field does not satisfy the boundary conditions along SSS, then charge will be distributed over the two faces of the sheet SSS in such a manner as to modify the electric field distribution and satisfy the boundary conditions along SSS. Since we are seeking the simplest possible solution we will assume that the resistivity r, of the sheet SSS per unit width (perpendicular to the plane of the paper) varies from point to point in such a way that ri at each point of SSS is equal and parallel to the electric field at that point due to the charges on A and B, where i is the current per unit width of the sheet SSS. In this case the electric field due to the charges on A and B satisfies all necessary conditions, and the electric lines of force are circles touching A and B orthogonally. The magnetic field[1] is everywhere perpendicular to the plane of the paper and the energy stream lines are circles cutting the electric lines of force orthogonally.

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[1] "Die Kräfte der electrischen Schwingungen, behandelt nach der Maxwell'schen Theorie." Wied. Ann., Vol. 36, p. 1. A paper dealing with an oscillating electrical doublet, the simplest case, perhaps, of a discharging condenser.

[2] Recent Researches, pp. 251–387. The author treats of several types of electrical oscillators of finite dimensions.

[3] Phil. Trans., Vol. 175, Part II., p 351.

Current in a Cylindrical Sheet, Nearly Closed.—The above ideal example may be realized for the case of a finite electric current flowing steadily in a cylindrical sheet as follows: Two plane current sheets AB and CD, Fig. 2, are very near together and they separate

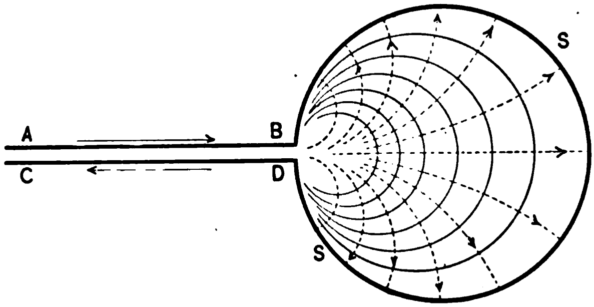

Fig. 2.

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[1] The magnetic field is not everywhere of the same intensity. In fact it grows less and less intense at increasing distances from the current sheet. The energy stream has divergence.

at *BD* forming a cylindrical shell *SS*. The electric current flows as indicated by the arrows and it is required to find the distribution of the electric field and of the energy stream inside of the shell. In this case the region in which the field distribution is to be determined is not a closed region, that is, it is not completely shut off by a conducting screen from the region occupied by the electric generator but since *AB* and *CD* are very near together the boundary conditions across the gap *BD* may be represented by an electric doublet (line doublet). The problem is therefore determinate whatever the shape of *SS* and whatever the manner of variation of the resistivity of the shell from point to point. The simplest case is that in which *SS* is a circular cylinder. The lines of force from a linear doublet are circles and if *ri* at each point of the shell is equal to the field intensity at that point due to the doublet at *BD* then the boundary conditions are completely satisfied by the electric field due to the doublet alone and this, therefore, will be the actual electric field inside of *SS*. In this case there is of course no electric charge on the inner surface of *SS*. The electric lines of force are represented by the full line circles in Fig. 2 and the energy stream lines are represented by the dotted circles. These lines are not extended into the region immediately adjacent to the gap *BD* inasmuch as the solution under consideration does not hold near *BD*. The magnetic field inside of *SS* is uniform and the energy stream has no convergence.

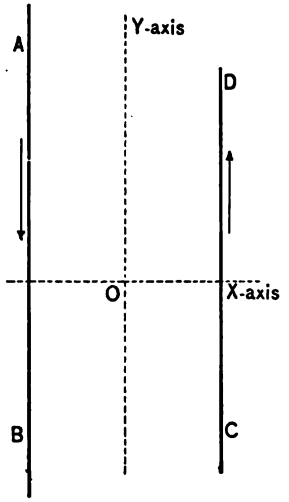

Fig. 3.

Electric Current in Two Parallel Infinite Conducting Sheets.—Electric current flows down an infinite plane conducting sheet *AB* and up an infinite conducting sheet *CD* parallel to *AB*. The current per unit width in *AB* and *CD* is *i*, and the resistivity of each sheet per unit width is *r*. Required the distribution of the electric field and of the energy stream in the region between *AB* and *CD*.

Choose the *Y*-axis midway between the planes and parallel to the electric currents. Choose the *X*-axis so that it cuts the two planes at points which are at the same potential. Let X and Y be the components of the electric field at any point. Then

$$\frac{dX}{dx}+\frac{dY}{dy}=0 \tag{1}$$

expresses the fact that the convergence of the electric field is zero throughout the region between AB and CD.

The magnetic field is everywhere constant so that

$$\frac{dY}{dx}-\frac{dX}{dy}=0. \tag{2}$$

From equations (1) and (2) we have:

$$\frac{d^2X}{dx^2}+\frac{d^2X}{dy^2}=0 \tag{3}$$

and

$$\frac{d^2Y}{dx^2}+\frac{d^2Y}{dy^2}=0. \tag{4}$$

These equations permit separate solutions to be set up for X and for Y.

In order that the boundary conditions for X and Y may be expressed as simply as possible let the distance between the planes AB and CD be taken as two units of length, so that the abscissas of the planes will be -1 and $+1$ respectively, and let the value of ri be represented by the letter a.

The boundary conditions for Y are:

$$\left.\begin{aligned} Y&=-a \text{ when } x=-1,\\ Y&=+a \text{ when } x=+1. \end{aligned}\right\} \tag{5}$$

The solution of equation (4) under these conditions corresponds to the steady flow of heat (Y being temperature) through a homogeneous substance between two infinite planes AB and CD, these planes being at temperatures $-a$ and $+a$ respectively. Therefore, Y, is a linear function of x which has the prescribed values $\pm a$ when $x=\pm 1$. Therefore:

$$Y=ax. \tag{6}$$

The boundary conditions for X are determined with the help of equation (1) as follows: Y being constant along AB and along CD, $\frac{dY}{dy}$ is zero along these planes. Therefore from equation (1) $\frac{dX}{dx}$ is zero along these planes. That is:

$$\frac{dX}{dx} = 0 \text{ when } x = \pm 1 \tag{7}$$

The solution of equation (3) under these conditions corresponds to the steady flow of heat (X being temperature) through a homogeneous substance between two thermally insulating planes AB and CD. . Therefore X is a linear function of y. That is:

$$X = by + c \tag{8}$$

Equations (1) and (2) are not necessarily satisfied by solutions of (3) and (4). Equation (1) however has been used in finding the boundary conditions of X from the given boundary conditions of Y, so that equation (1) is everywhere satisfied.[1]

The condition imposed by equation (2) fixes the value of b, equation (8). Thus, from (6) we have $\frac{dY}{dx} = a$ whence by equation (2) we have $\frac{dX}{dy} = a$; but by equation (8) $\frac{dX}{dy} = b$. Therefore $b = a$. The value of the constant c, equation (8), is determined as follows: The difference of potential between AB and CD at a distance y above the X-axis is equal to $2X$, or by equation (8) it is equal to $2by + 2c$, since the distance between AB and CD is 2 units. Therefore for $y = 0$, this potential difference is $2c$, but the X-axis has been chosen so as to pass through equipotential points on AB and CD, so that $c = 0$, and equation (8) becomes:

$$X = ay. \tag{9}$$

The differential equation of a line of force of the electric field is:

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[1] The two continuous functions X and Y satisfy equation (1) over the entire boundary to the region between AB and CD. Therefore, according to the theory of functions, they must satisfy this equation throughout the region. The small number of disposable constants in the above solutions of equations (3) and (4) is due to the fact that the boundary conditions are fully considered in the thinking out of the solutions by the aid of the heat-flow analogy.

$$\frac{dy}{dx} = \frac{Y}{X}$$

or, using equations (6) and (9),

$$\frac{dy}{dx} = \frac{x}{y}$$

of which the integral is

$$x^2 - y^2 = C. \qquad (10)$$

The lines of force are, therefore, equilateral hyperbolas of which the asymptotes are the two straight lines ($y = x$) and ($y = -x$).

The magnetic field in the region between *AB* and *CD* is uniform and perpendicular to the plane of the paper in Fig. 3. The energy stream lines are, therefore, in planes parallel to the paper in Fig. 3 and since these stream lines are perpendicular to the electric field, the differential equation to a stream line is:

$$\frac{dy}{dx} = -\frac{X}{Y}$$

or, using equations (6) and (9),

$$\frac{dy}{dx} = -\frac{y}{x}$$

of which the integral is

$$xy = C. \qquad (11)$$

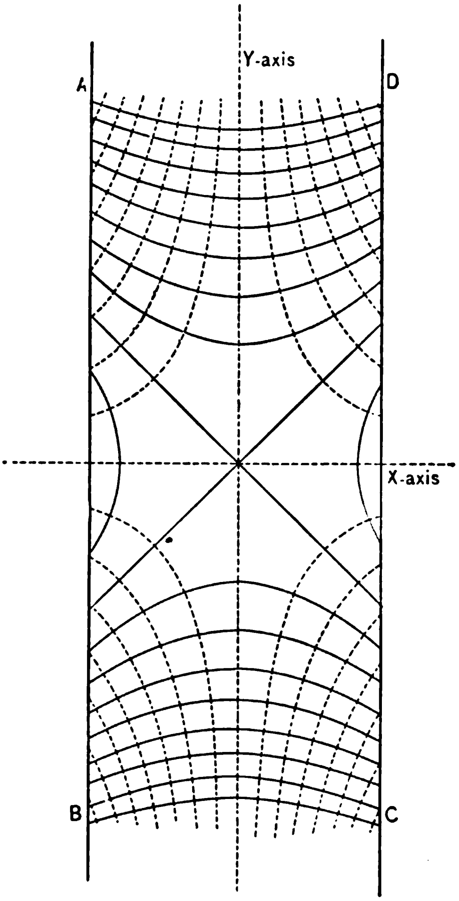

Fig. 4.

The energy stream lines are, therefore, equilateral hyperbolas of which the axes of reference are the asymptotes.

Fig. 4 shows the electric lines of force and the energy stream lines. It is evident that the electric lines of force touch the conducting surfaces obliquely, except in the immediate neighborhood of the two points where the planes are cut by the X-axis. At these points the electric field near the planes is parallel to the planes, and the planes have no surface charge.

The above solution, as shown in Fig. 4, holds for a variety of conditions as follows:

1. Either conducting plane AB or CD may be replaced by another parallel plane at any distance x' from the Y-axis, provided the resistivity of the new plane is: $r = \frac{ax}{i}$; for, the value of Y along the new plane, as given by equation (6), will be the value required by the boundary conditions, namely $Y = ri$.

2. A conducting plane, parallel to the X-axis and perpendicular to the paper in Fig. 3, may be connected across from AB to CD at any distance y' from the X-axis and the flow of current changed so as to come down on AB to the new plane, thence across to CD, and up on CD, provided the resistivity of the new plane is

$r = \frac{ay'}{i}$; for the value of X along the new plane, as given by equation (9), will be the value required by the boundary conditions, namely $X = ri$.

3. A conducting sheet of metal may be placed in the position ofany one of the equipotential surfaces (indicated by the energy stream lines in Fig. 4) without in any way disturbing the electric field, the magnetic field, or the energy stream. In particular a plane sheet of metal may be placed midway between AB and CD without producing any disturbance. This fact is made use of in a discussion, which follows, of the effect of a metal shield near a current conductor.

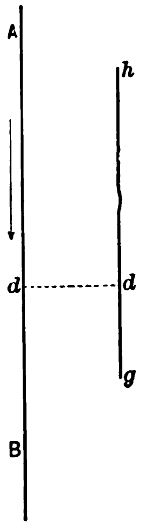

Fig. 5.

Flow of Energy Around a Metal Shield Placed near a Current Sheet.—An insulated metal plate gh, Fig. 5, is placed as a shield near and parallel to a plane current sheet AB. The potential of the shield is equal to the potential at the point d on AB, that is, the shield may be supposed to be connected to AB by a wire dd. Required the distribution of the electric field and of the energy stream between the shield and the current sheet.

The required distribution is represented by the portion of Fig. 4, which is between AB and the Y-axis, the line dd, Fig. 5, being on the X-axis.

If the potential of the shield is equal to the potential of a remote point on AB, then the distribution of electric field and of energy stream between AB and gh will be represented by a portion of Fig. 4 between AB and the Y-axis and entirely above or entirely below the X-axis. The distribution of electric field near the edges of gh will be similar to the distribution of electric field near the edges of a small plate which is near a large plate, forming an ordinary air condenser. The character of the electric field in this case is, in a general way familiar to the reader, and the energy stream lines are of course perpendicular to the electric lines of force, so that the energy stream lines diverge into the surrounding open region near the edges of the plate gh, Fig. 5.

When the potential of gh is equal to the potential of AB at a point between g and h the energy stream enters the shielded region at g and at h, otherwise the energy streams into the shielded region at one end only, a portion of this energy flows into the conducting sheet AB, and the remainder streams out at the other end.

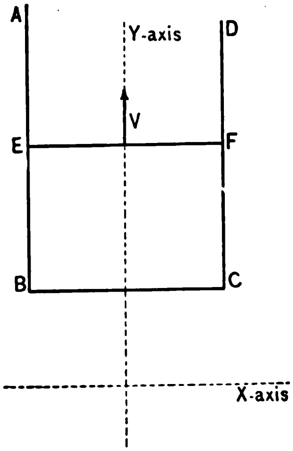

Fig. 6.

Flow of Energy in Case of an Induced Current.—AB, BC and CD, Fig. 6, are conducting planes, EF is a sliding plane in contact with AB and CD, and the entire region is a uniform magnetic field perpendicular to the plane of the paper. The sliding plane EF moves at low velocity V which increases[1] in proportion to its distance from *BC* so that the electromotive force induced in *EF* may suffice to maintain a constant current in the increasing circuit *EBCF*. It is required to find the distribution of the electric field and of the energy stream inside and outside of *ABCD*.

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[1] This velocity is assumed to be small so that the field distribution may everywhere accommodate itself to the changing conditions without unsteadiness. The velocity would have to be at each instant a little in excess of the value required simply to overcome the increasing resistance of the circuit, on account of the fact that the inductance of the circuit is increasing also.

The curved lines in Fig. 7 represent the electric lines of force and the energy stream lines. The regions *AEFD*, *EBCF*, and the external region are treated separately.

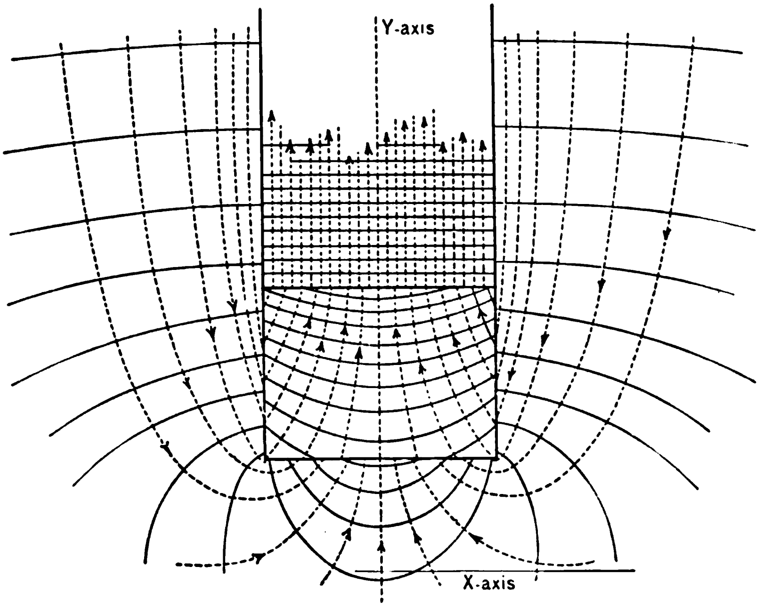

Fig. 7.

Let *e* be the electromotive force induced in *EF*, let *r* be the resistivity of the conducting planes per unit width, and let *i* be the current per unit width of planes. Let *m* be the length of *EF*, and let *l* be the length *EB* at the instant for which the solution is to be found. Let the axes of reference be chosen as shown in Fig. 6.

(*a*) In the region *AEFD* the electric field is uniform and parallel to *EF*. The intensity of this field is $\left(\frac{e}{m} - ri\right)$; less than $\frac{e}{m}$ because of the loss of electromotive force in overcoming the resistance of the slide *EF*. The energy stream in the region *AEFD* is uniform and parallel to *EA*.

(*b*) The distribution of the electric field and of the energy stream in the region outside of *ABCD* is approximately as shown by the lines in Fig. 7. The energy stream passing up between *A* and *D* returns, as indicated by the dotted lines, outside of *AB* and *DC*, and converges upon *EB*, *BC* and *CF*. A portion of this energy stream changes into heat in the conductor and the remainder flows through the conductor into the region *EBCF*.

(*c*) The distribution of the electric field and of the energy stream in the region *EBCF* is exactly the same as that shown by a portion of Fig. 4 as indicated by the hyperbolic lines of force and lines of flow in Fig. 7, except that the energy stream is reversed in direction. The energy which streams up to *EF* in the region *EBCF* exceeds the energy which streams away from *EF* in the region *AEFD* by the amount $(mri^2 + 2lri^2)$ which is the equivalent of the heat appearing in *EB*, *BC* and *CE*, and is equal to $(P - mri^2)$, where P is the total power expended in moving the slides *EF* and mri^2 is the power equivalent of the heat appearing in *EF*.

The total fluxes of energy in the various energy streams above described may be calculated independently of a precise algebraic solution of the electric field distributions and the entire consistency of the above statements is easily shown.

The statements given above concerning the distribution of electric field and of energy stream in the two regions *AEFD* and *EBCF* are rigorously true as is easily seen from foregoing discussions given in this paper. On the other hand, the distribution in the region outside of *ABCD* is only approximately represented in Fig. 7. The building up of algebraic functions for the electric field components X and Y in the external region so as to satisfy the boundary conditions over *ABCD* is easily reduced to well known and soluble cases of heat flow, but the developments are more complicated than would be warranted in the present paper which is intended to be as simple as possible.

South Bethlehem, Pa.,
April, 1901.

POLARIZATION AND INTERNAL RESISTANCE OF ELECTROLYTIC CELLS. II.

By Martin D. Atkins.

Part II. The Area of Electrodes in Relation to the Constant *K*.

HAVING determined, as it seemed, with a fair degree of certainty, that the formula under consideration, while not in all cases applicable to electrolytic action, was yet an approximate working formula for fresh zinc or copper electrodes in the solutions tested, attention was turned to the question of the relation of the constant K to

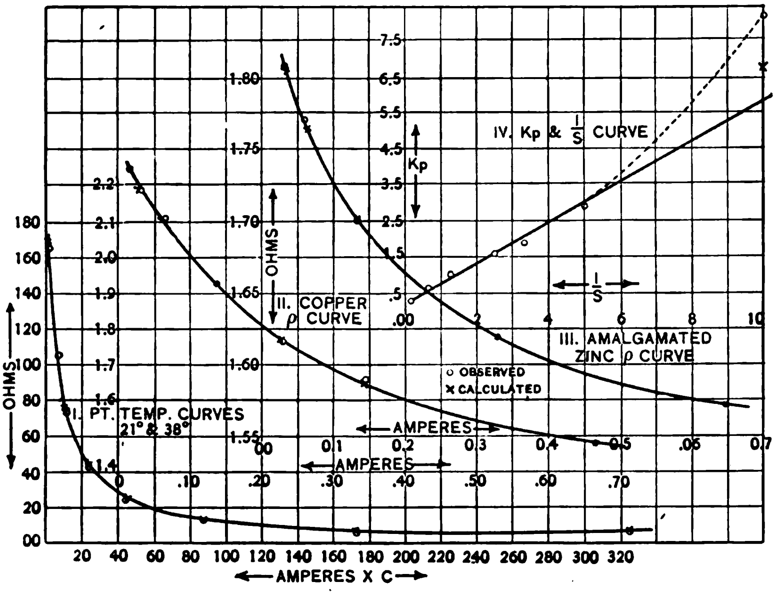

Plate III.

the area of the electrodes. The theory demands, as is seen from the substitution $K = \left(\frac{Bt}{S}\right)$, that K shall be inversely proportional to the area of the electrode. It was felt that, could this point be determined, and should the results affirm such a relation, the conclusions already reached could be very much more confidently asserted. It may be said here, that the results, up to the present obtained, do not establish this relationship.

As has already been said, a large number of observations were made with zinc plates in zinc sulphate. The results proved conclusively that to obtain like conditions by amalgamation was very difficult, if not impossible. Even when handled, as far as could be seen, in a precisely similar manner, widely variant results would be obtained with the same plates; giving no encouragement to hope for concordant results with plates with widely varying areas. While very suggestive results were often obtained, it was found impossible to repeat them with certainty; and hence they can remain only as suggestive. Such a series of results is given below.[1] It should be understood that these are not results selected from a large number; but a series, taken under conditions as nearly like as could be arranged: The objection to basing any argument upon them, lies in the fact that they could not be repeated, even when, as far as could be seen, like conditions were realized.

TABLE XIX.

Areas.	*K*	*P*	*KP*
1	5	.025	.13
2	11	.058	.64
4	20	.052	1.04
8	41.3	.053	2.29

TABLE XX.

Areas.	*K*	*P*	*KP*
1	9.9	.034	.34
2	9.4	.065	.61
4	16.7	.060	1.00

Another series, taken with plates amalgamated and allowed to stand in air four hours, and then used, gave a set of curves which cut off from the y axis values, corresponding to $KP = \rho$ for $I = 0$, as follows:

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[1] Tables XIX.–XX.

TABLE XXI.

Zinc, Amalgamated, Dry. Jan. 11. *T*, 19°-20°.

Dimensions.	Areas (relative).	Range of App. Res., ρ	Differences, *i. e.*, $KP = (\rho - r)_{i=0}$
10 x 13 cm.	8	1.23 - 1.16	.07
10 x 6.5 "	4	1.99 - 1.83	.16
5 x 6.5 "	2	3.00 - 2.68	.32
5 x 3.25 "	1	5.34 - 4.68	.66

While these coincidences, if accidental, are certainly remarkable, the curves are so irregular, and the total changes so small, that the computation of K and P, from values so near each other, and giving differences in themselves so extremely small, gave no satisfactory results.

Most nearly satisfactory conditions having been obtained, for securing like results, by flowing the surface of the plates with pure mercury, the thought suggested itself that, perhaps, liquid electrodes would insure the condition of like surfaces, necessary for our purpose. Small cups, with areas as 1 : 3 : 5, were, therefore, prepared from short sections of glass tubing, and these were filled with an amalgam containing a small amount of C. P. zinc. Using these as electrodes, in the solution before employed, gave us again very suggestive results; which, however, owing perhaps to solution changes, could not be repeated.[1] It is the intention to soon repeat these observations, the solution having been left standing four months over the amalgam, with the hope that the irregularities might adjust themselves.[2] The results follow.

TABLE XXII.

Areas (relative).	Observation					
	1		2		3	
	K	P	K	P	K	P
1	60.6	.290	55.6	.375	42.3	.224
3[1]					18.9	.250
5	12.6	.119	10.15	.220	14.6	.190

---

[1] Used only in last observation.

[2] The repetition of these measurements since made indicates changes still going on, and is equally inconclusive.

There was an advantage in using these electrodes over any others employed, viz., their rate of change with the time alone was very small, as may be seen by reference to their time curve, Plate I, No. III. Comparison with the zinc in the form of plates, even when best amalgamated, shows a much slower rate of increase of the apparent resistance on the part of the Hg — Zn electrodes. Even the copper, later used, was more rapid in its time change, as its time curve shows, Plate I., No. IV. This fact, together with the suggestive relations found for K, is sufficient to encourage to a further investigation of this combination.

Cu in CuSO$_4$.

These observations with the zinc proving far from satisfactory, we returned to the use of copper electrodes in the copper sulphate solution before employed.

Inspection of the Bartoli-Wiedeburg formula (1) in its original form,

$$P = \frac{P_1}{2}\left(1 - e^{-\frac{\beta_1 q}{s}}\right) + \frac{P_2}{2}\left(1 - e^{-\frac{\beta_2 q}{s_2}}\right),$$

shows that, if one electrode be made very large in comparison with the the other, the same form results, as from the equalities of areas, and the assumption that β_1 equals β_2,

$$i.\ e., p = P(1 - e^{-Kt}), \text{ as before.}$$

To determine which electrode could be most advantageously eliminated in this manner, the following tests were made. First, a large kathode was used and the anode varied, and then, *vice versa.* The results indicated that the polarization was practically not affected by varying the sizes of the anode, so long as they were not too small. On the other hand, change in the area of the kathode was attended always with change in the values of K and P. Table XXIII. contains the results of these tests.

With the purpose of rendering the anodic polarization negligible in all cases, an anode of 12 plates was constructed. These plates furnished a surface 24 times that of the largest kathode, and over 1,200 times that of the smallest. A large battery jar, containing ten

TABLE XXIII.

Cu in $CuSO_4$. 100 *G. in* 1 *L.*

Varying Anode.				Varying Kathode			
Relative Areas.		Constants.		Relative Areas.		Constants.	
Kathode.	Anode.	P	K	Kathode.	Anode.	P	K
24	1	.066	18	1	24	.155	16?
	2	.054	14	2		.105	21
	8	.047	11	8		.049	18
	16	.043	10	16		.046	11
	24	.041	9	24		.039	10

liters of solution, was used; and the cell thus constructed, placed in a crock filled with ice water. The temperature thus secured was about 8° C.

The time curve of this cell has already been referred to. The low temperatures retarded the change somewhat, although not so much as it was hoped they might.

As with the zinc, the first experiments aimed at securing surfaces always of the same character. To attain this end was found, as had been the case there, a matter of extreme difficulty. The plates were taken to the cell from acids, from various grades of emery paper, from the polishing wheel, alike with indifferent results. They were placed in the cell from air, alcohol, water, and the solution to be tested, with results that left much, in the way of constant values, to be desired. As a final attempt the plates were electroplated with the same current density for the same times. But, even in this way, it was impossible to obtain like conditions of surface, for the greatly varying areas employed, and the large number of observations necessary.[1]

While the curves show values for KP for $I = 0$ very nearly in inverse proportion to the respective areas, the computed values are far from the desired agreement. While often a single observation in a series falls entirely out, there are, on the other hand, many cases where the values show a very near approach to those demanded by the theory. It is very evident that the disturbing influences are hard to control. Very slight changes in the value for P

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[1] Cf. B. E. Moore, Pol. Copper Voltameter, PHYS. REV., Vol. X., 1900.

make very large differences in those for *K*. The value of *P*, itself, is so sensitive to surface conditions that it is doubtful if more conclusive results should be expected, until these conditions are better understood and controlled. Since varying currents must be used, it is evident that initial conditions of exact similarity could not remain such long. An attempt was made to return to the initial conditions by replating after each observation, but, owing tp the time change doubtless, this resulted usually in an increase in the value of the constants obtained. The following tables contain the results of two series of observations. No attempt has been made usually to completely solve these results for *r*. *K* and *P* have been computed by the approximate method before described. It was felt that too much uncertainty attended the conditions to warrant the large outlay of time and labor necessary to completely solve all these curves; while a sufficient number were solved, and

Table XXIV.

Table XXV.

$CuSO_4$, 100 G. in 1 L. Copper Anode of twelve Plates. Copper Kathodes of Areas from 1 to 64.

Relative Areas.	*K*	*P*	*KP*
2	66.5	.051	3.33
4	39.5	.043	1.69
8	15.9	.053	.84
16	22.2	.031	.67
64	6.3	.039	.25

Relative Areas.	*K*	*P*	*KP*
1	91.5	.109	10.10
2	26.2	.090	2.36
3	40.2	.046	1.85
4	29.4	.049	1.44
8	15.8	.072	1.14
16	19.1	.025	.48
64	11.0	.016	.18

Table XXVI.

Combined Results from Table XXIV and XXV.

Areas.	*K*	*P*	*KP*	$\frac{1}{KP}$	$\frac{1}{KP}$ using .15.
1	91.5	.109	10.10	.10	.15
2	66.5	.051	3.33	.297	.30
3	40.2	.046	1.85	.55	.45
4	39.5	.043	1.69	.594	.60
8	15.9	.053	.84	1.18	1.20
16	19.1	.025	.48	2.12	2.40
64	11.0	.016	.18	5.77	9.60

are given in Tables XXVII. to XXIX., inclusive, to prove that they are in agreement with the formulæ, as was to be expected from Part I. of this paper.

If we divide the observed values for KP in tables XXIV. and XXV. by the inverse areas of the plates we obtain from the first set the mean ratio .154; from the second .121; a mean of .138. Setting up a series upon this value, and taking the mean observed values, we have the results shown in the following table:

Obs. Mean. KP	**Computed KP for mean .138.**	$\frac{1 \times 10}{S}$
.22	.14	.16
.56	.55	.63
.99	1.10	1.25
1.57	2.20	2.50
1.85	2.84	3.30
2.85	4.40	5.00
8.38	8.80	10.00

If these values (observed) be now plotted as ordinates of a curve, which has as abscissæ the values $\frac{1}{S}$, they lie very nearly upon a right line, with the exception of the last. As this value is obtained from a single observation, and the value for P is in that very large, it is quite possible that this point is too high. The curve is given in Plate III., No. IV.

Table XXVII.

Copper $\frac{2}{64}$. *Apr.* 18. $T = 8.0°$; $R = 10.0$ *ohms.* $K = 34.67$; $P = .062$; $KP = 2.15$.

Cap.	dC	dR	I	ρ	$(1 - e^{-KI})$	r	P
.05	135	136.5	.5348	9.89	1.0000	9.77	
.1	150	150.5	.2948	10.02	1.0000	9.81	.06315
.2	207.5	204	.1998	10.14	.9990	9.83	.06046
.3	197.5	189.5	.1237	10.34	.9863	9.84	.05904
.5	173.5	161	.06307	10.66	.8877	9.78	.06708
3.2	249	206	.01261	11.50	.3555	9.74	

Solved results. Copper electrodes in $CuSO_4$, from part II investigation.

TABLE XXVIII.

Copper $\frac{1}{32}$. *Apr.* 16. $T = 12.0°$; $R = 10.0$; $K = 52$; $P = .0405$; $KP = 2.08$.

Cap.	dC	dR	I	ρ	$(1 - e^{-Kt})$	r	P
.05	153	132.5	.5191	11.55	1.0000	11.473	
.1	167.4	144	.2821	11.62	1.0000	.479	.04641
.2	238	204	.1998	11.67	1.0000	.471	.03038
.3	225	190	.1241	11.80	.9984	.480	.03979
.5	195	161.5	.0633	12.08	.9624	.474	.04165
1.0	209	164.5	.0322	12.52	.8128	.516	.03486
3.2	282	207.5	.0127	12.96	.4833	.445	.04565
3.2	182	129	.0079	13.18	.3368	.482	

TABLE XXIX.

Copper $\frac{1}{8}$. *Mar.* 28. $T = 25.4°$; $R = 2.0$ *ohms.* $K = 7.10$; $P = .1480$; $KP = 1.05$.

Cap.	dC	dR	I	ρ	$(1 - e^{-Kt})$	r	P
.5	252	340	.6650	1.474	.9911	1.249	
1.0	294	351.8	.3445	1.654	.9134	1.244	.1466
1.0	207	231.8	.2270	1.765	.8005	1.260	.1523
1.0	136	138	.1351	1.927	.6168	1.251	.1474
3.2	233.8	213	.0652	2.101	.3705	1.243	.1470
3.2	126	106	.0324	2.182	.2057	1.262	
3.2	68.3	52	.0159	2.242	.1068	1.253	

SUPPLEMENTARY EXPERIMENTS.

Upon scrutinizing the whole field investigated, two results, worth especial consideration, were apparent. *First,* the marked increase in apparent resistance all the cells showed upon standing. *Second,* the current effect, or cutting down of the resistance on sending large currents through the cells.

Absorption of the ions of metal by the plates having been a suggested cause for the first of these phenomena, the following test was made. Plates prepared by silvering glass were used as electrodes; the thought being that the absorption within the material of the electrodes would, because of their microscopic thickness, be soon completed. So far as tested this was found to be the case. The apparent resistence increased rapidly upon first placing such electrodes in the solutions, becoming constant within the first quarter hour, and remaining thus for several hours—in some instances 24. Unfortunately the films soon disintegrated, or scaled from the glass, and so conclusive data could not be gathered. These tests will be repeated.

An investigation was made as to the cause of the second, the current, change. With a highly polished, small copper electrode, the reading on the most sensitive point (smallest current), was taken. Then the plate was plated with a large current for ten minutes, readings being taken every 2 minutes. The curve for drop in ρ thus obtained is shown on Plate I., No. V. It will be noticed that the time change was entirely overcome in this experiment. It is probable that with the smallest plates used, and the largest currents, a single swing of the pendulum would cut the deflection down even while the current was passing. This influence would give us on the large currents and small plates, always relatively small values for P. This they are found to be, with those metals which are most susceptible to the current effect. By plating a short time with a small current, the deflections could be forced up to their original value. It is difficult to see how the error thus introduced can be eliminated.

PART III. TIME TESTS.

Although it was evident that the relationship of areas demanded by the Wiedeburg theory had failed of establishment, the further relationship which would demand that K be proportional to t was tested. By varying the positions of the keys, the time during which the current was allowed to flow through the cell was widely varied. In all cases such variation had no effect whatever upon the values of the deflections produced.[1]

PART IV. INITIAL POLARIZATION CAPACITY FROM OBSERVED DATA.

The applicability of this method to measuring the polarization capacity of an electrolytic cell has already been described.[2] Below are given several such values, for the cells employed in this research, computed from the formula[1]

---

[1] Since this paper was written an article has been published by Mr. B. E. Moore, describing investigations upon the copper voltameter. In this report Mr. Moore has found that P increases as the time of current flow is increased. The time intervals used were however, very much shorter than any employed by this investigation. Vide, PHYSICAL REVIEW, January, 1900, pp. 47–49. "Polarization and Internal Resistance of the Copper Voltameter," by B. E. Moore.

[2] This article, p. 103, also 3.

$$C = \frac{t}{Kp}.$$

In all cases $t = 0.2$ second.

I. Zinc plates in $ZnSO_4$.

Area sq. cm.	Init. P. Cap. pr. 1 cm. sq.	
130	.00462	Mean .00529
65	.00482	
32.25	.00596	
16.125	.00577	

II. Copper plates in $CuSO_4$.

Area sq. cm.	Init. P. Cap. pr. 1 cm. sq.	
12.18	.0076	Mean .0062
12.18	.0078	
24.36	.0055	
48.72	.0040 (?)	

III. Silver plates in $AgNO_3$. *One trial* .000567 *per* 1 *cm. sq.*

IV. Platinum in H_2SO_4. *One trial* .0106 *per* 1 *sq. cm.*

A complete computation for all the data has not been attempted.

CONCLUSIONS.

A review of the results of this investigation justifies the following special conclusions:

1. Electrodes of copper, in $CuSO_4$, and zinc (amalgamated, both plates and liquid amalgam), in $ZnSO_4$, when freshly placed in their respective solutions, give, with varying currents, curves completely satisfied by a formula of the Bartoli-Wiedeburg form, and its derived equations.

2. With such electrodes, therefore, the internal resistance r is, according to our assumption, constant.

3. Electrodes of silver in $AgNO_3$, and zinc in $ZnSO_4$, in the main, give curves satisfied by such a formula.

4. Electrodes of platinum (polished), in H_2SO_4, do not conform, as far as investigated, to the formulæ. The maximum value for

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[1] Guthe and Atkins, Proceed. A.A.A.S., 1899, p. 109.

the polarization, however, is in agreement with that found by other methods.

5. Electrodes of all these metals in their respective solutions show upon standing a marked increase in apparent resistance. All results obtained concur in their testimony that this is not an increase in the real resistance of the cell.

6. Neither the inverse proportionality of the constant K to the area of the electrodes, nor its direct proportionality to the time of flow of current, is conclusively established by the results obtained.

While the theory of Wiedeburg is, therefore, not sustained as to its constants by the results of this research, an equation of the same general form would seem to be suggested as satisfying the curves for internal resistance and polarization of electrolytic cells.

GENERAL CONCLUSIONS.

(*a*) The method of this research furnishes a means of determining, to a degree of accuracy of 1 per cent., the internal resistance of electrolytic cells. Even those which do not admit of determination by the Kohlrausch method yield readily to this.

(*b*) The maximum value of the polarization, for a given combination of metal and solution, can be determined by this method.

(*c*) The method can be used for determining the polarization capacity of electrolytic cells even when this is very large.

In conclusion, I wish to express my sense of obligation to Professor Henry L. Carhart, who has placed at my disposal the facilities of his laboratories, for the purpose of this research. I wish also to avail myself of this opportunity to thank, most heartily, Dr. K. E. Guthe, with whom the investigation originated, for giving so generously of his personal direction and assistance to the details of the work at every stage.

UNIVERSITY OF MICHIGAN,
PHYSICAL LABORATORY, June, 1899.

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THE PHYSICAL REVIEW

A Journal of Experimental and Theoretical Physics

CONDUCTED BY

EDWARD L. NICHOLS, ERNEST MERRITT and FREDERICK BEDELL

LXV. July, 1901

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New Books: *Crew:* The Elements of Physics; *Snyder and Palmer:* One Thousand Problems in Physics; *Caldwell and Fish:* Notes and Questions for the Dynamo Laboratory

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THE
PHYSICAL REVIEW

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THE
PHYSICAL REVIEW.

THE METALLIC REFLECTION OF ULTRA-VIOLET RADIATION.

By Perley Gilman Nutting.

SPECTRA reflected from such substances as glass, silver and benzene, that strongly absorb but part of the ultra-violet radiation, exhibit some very interesting peculiarities that have been but little studied. Langley[1] measured the reflecting power of silver out as far as wave-length 350 ($\mu\mu$), where it was 61 per cent., having fallen off very rapidly from nearly 80 per cent. at 400. Beyond the limit of Langley's observations we have found that the reflecting power falls off still more rapidly to a rather sharp minimum of less than 10 per cent. between 320 and 324. Glatzel[2] has observed this minimum. Corresponding with this reflection minimum is a very pronounced transmission band, the spectrum between wave-lengths 310 and 340 being quite freely transmitted by a coating of silver on quartz so thick as to completely absorb all other radiation between 800 and 185. If then all the radiation in this region were visible, silver would possess a very strong surface color, transmitting only those waves which it does not reflect. Strong reflection minima of this form are difficult of theoretical explanation. Nor do any of the other ten metals examined exhibit this peculiarity. Beyond this minimum the reflecting power of silver increases again to about 30 per cent. at about wave-length 250 and then falls off again to less than 10 per cent. at 185. Silver then, at normal incidence, reflects but little better than many crystalline salts in at least two spectral regions, and in these regions its reflecting power could hardly be termed metallic. On the other hand, the reflecting power of other metals, notably mercury, remains high and nearly constant, falling off but slightly in the neighborhood of wavelength 185.

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[1] S. P. Langley, Phil. Mag., 27, 10, 1889.

[2] B. Glatzel, Phys. Zeitschr., 2, 173, 1900; Beibl., 25, 35, 1901.

The reflecting power of solid cyanin is interesting in view of its well-known strong and narrow absorption band in the optical region, and its suspected second band in the ultra-violet. Wood and Magnusson[1] have recently determined its refractive index through the first band and predicted the existence of the second from the form of the index curve in its neighborhood. In the region of the first band the reflecting power of cyanin is well up among those of the metals, in excellent agreement with its value calculated from the refractive and absorptive indices; but in the region of the ultra-violet band, the reflecting power appears to follow quite a different law.

Such substances as glass and many of the crystalline salts and hydrocarbons, which strongly absorb the shorter waves, have been examined with a view to testing Drude's and Beer's formulas for the reflecting power of absorbing substances. According to each of these formulas, the reflecting power should be large when the absorptive indices are large. Their disagreement with observed phenomena indicates an absorption not due to molecular resonance or to the electrical conductivity of particles imbedded in a dielectric. On the other hand, the great reflecting power of the benzines in the ultra-violet, where they have so many strong absorption lines, [2] is in close agreement with the above reflection formulas. Carbon bisulphide, with its narrow, well-defined transmission band in the extreme ultra-violet, is a particularly good subject for study.

Again, the volatile petroleum ethers and napthas, absorbing all waves beyond 280, offer a striking contrast to dense, viscous glycerine which absorbs little or nothing out as far as wave-length 185, yet the reflection spectra of all these substances do not materially differ beyond 300. The compound distillation products of petroleum suggest, by their behavior, another possible field for the existence of metallic reflection. The series of products with increasing boiling points possesses the remarkable property of alternately absorbing and transmitting that part of the spectrum extending from near the visible out to 185 $\mu\mu$. Napthas of increasing boiling points from 80° to 130° show a slightly increasing absorption. Kerosene, with a slightly higher boiling point, does not absorb at all. Paraffin oil, next in the series, absorbs moderately and uniformly from near the visible to 185 mm. Vaseline, fluid at about 50°, can scarcely be distinguished from paraffin oil, yet it strongly absorbs all the spectrum from near the visible. Finally pure paraffin, fluid at about 80°, is again a non-absorber. Not only was there no trace of metallic reflection from the partially absorbing members of this series, but neighboring members had reflecting powers as nearly alike at 200 as in the optical region.

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[1] Wood and Magnusson, Phil. Mag. (6), 136, Jan., 1901.

[2] See Pauer, Wied. Ann., 61, 367, 1897.

Metallic reflecting power might be looked for in the ultra-violet in the case of fluorescing substances like uranium glass, common glass and the heavier members of the paraffin series. Nearly all glasses, particularly the harder varieties, fluoresce strongly in the region of the four zinc lines between 200 and 210, much more strongly than in regions of greater or less wave-length. The reflecting power of these substances has been examined with especial care in this region.

Apparatus and Method.

The methods employed have been purely photographic. The intensity of the radiation was taken to be inversely proportional to the time necessary to bring two exposures to the same density, provided certain precautions were observed. A number of brands of plates were first tested for this proportionality. Each was exposed, in adjacent squares, to a very short spark having a large capacity in parallel so as to be nearly non-oscillatory, a single spark at 25 cm., four sparks at 50 cm., sixteen sparks at 100 cm. The photometric sensitiveness of the plates was tested by making exposures of 15, 16 and 17 sparks side by side. Cramer's isochromatic plates were found to be thoroughly satisfactory for photometric work and very sensitive to the extreme ultra-violet. Seed and Lovell plates, particularly the smaller commercial grades, could not be used with success. The use of photography in photometry depends primarily upon the bringing out of slight differences in the photographic impressions. Plates must be considerably under-exposed and completely developed with a rather slow developer. Much depends upon the kind of developer used. What are known as "contrasty" developers are best. We find the ordinary metol-hydrochinone to be entirely satisfactory when diluted to one-fourth strength and used fresh each time.

As a source of radiation, an induction coil spark was used between terminals of an alloy of about two parts aluminum to one of zinc. This alloy gives an excellent spectrum to work with, the whole region from the visible to the extreme aluminum line at 185 being filled with lines of very nearly the same intensity very evenly distributed. This alloy possesses the further advantage of being very durable, much more so than either metal alone. The spark was completely enclosed, but for the slit, within a short brass tube of about five cm. diameter having one cap of wood for holding the inner electrode. The spectrum lines themselves being entirely unsuitable for photographic photometry, a capacity of seven large Leyden jars was placed in parallel with the spark so that the spectrum was quite continuous between the lines. The interrupter on the coil was refitted and adjusted until it would run continuously for hours without perceptible variation in the intensity or frequency of the spark.

For accurate photometry, the two spectra to be compared must be brought to exactly the same focus. Faint lines, slightly out of focus, do not appear at all on the negative. A corresponding weakening appears through the whole spectrum so that great care is necessary in adjusting the reflecting surfaces.

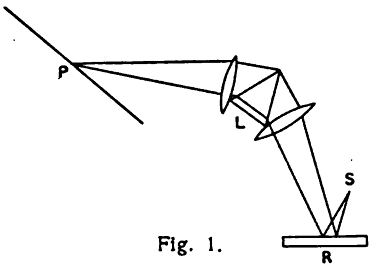

Fig. 1.

Fig. 1 shows the arrangement of the apparatus used in the greater part of the work. Radiation downward from the horizontal spark S is reflected upward at nearly normal incidence from the reflecting surface R through the quartz spectrometer train L. The reflecting surface rested upon a plane-faced, massive block of metal. An absorbing plate or quartz cell of an absorbing liquid could be inserted between the reflecting surface and the first lens. A plate holder of special design at P was inclined at an angle of about 45° so that all parts of the spectrum were in focus upon a fluorescent screen within it. After the first exposure, the plate holder was displaced laterally in its own plane by a mechanical device and another exposure made close beside the first. For a known standard reflector for comparison a quartz plate was used, set in a brass block and backed with a mixture of balsam, boneblack and turpentine to prevent reflection from its lower surface. Photographic plates about three by twelve centimeters were used and on each plate three exposures were made, one of reflection from quartz and two from the substance under examination. The time of exposure was determined by means of a metronome beating seconds. Exposures varied from ten to fifty seconds.

Discussion of Results.

Silver.—Two silver reflectors were used, one freshly deposited from Rochelle salt silvering solution and the other a piece of mirror plate from which the covering had been removed with carbon bisulphide. The surfaces used in the determination of the reflection curves were lightly polished with a clean, dry, bristle buffing wheel. Unpolished coatings of silver and quartz backed with silver gave the same minimum in their reflection spectra. About sixty exposures were made in the determination of the curve. The accompanying photograph (see Plate I.) shows the transmission band corresponding with the reflection minimum. The faint transmission in other regions is probably due to a few pin holes remaining in the silver coating. The (lower) transmission spectrum was exposed for two minutes, mercury being used for the lower reflector. The middle spectrum is the direct reflection from the mercury—exposure two seconds. The upper spectrum is reflected from silver—exposure twelve seconds. Beyond wave-length 200, silver is by far the poorest reflector of the metals studied and its reflection can hardly be called metallic.

Gold.—Gold foil was pressed between two glass plates, on one of which was a thin coating of warm sealing wax. The surface was rather dull with a reflecting power nearly constant throughout the whole spectrum.

Lead and Cadmium.—Small blocks of these metals were shaved clean of impurities and pressed against plate glass in a vise. Excellent optical surfaces were obtained in this manner. The reflecting power of these substances is nearly constant, falling off slightly in the region of extremely short waves.

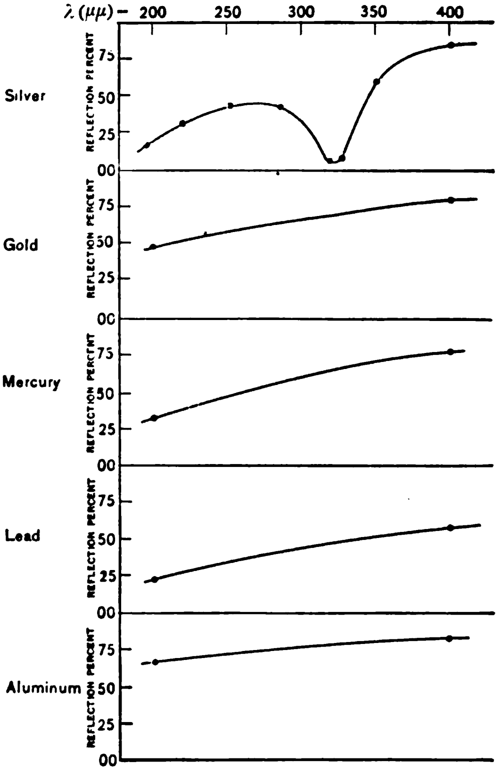

Fig. 2.

Mercury, distilled and free from grease, showed a reflecting power high and constant.

Aluminum, *zinc*, *copper* and *brass* were polished dry on a clean oil stone of very fine grain and gave fairly good optical surfaces with very constant reflecting power.

Grating Metal.—The spectrum was reflected from the polished surface bordering the rulings of a small plane grating. The reflecting power falls off considerably in the region of the shorter waves.

The reflecting power of these metals is shown graphically in Figs. 2 and 3.

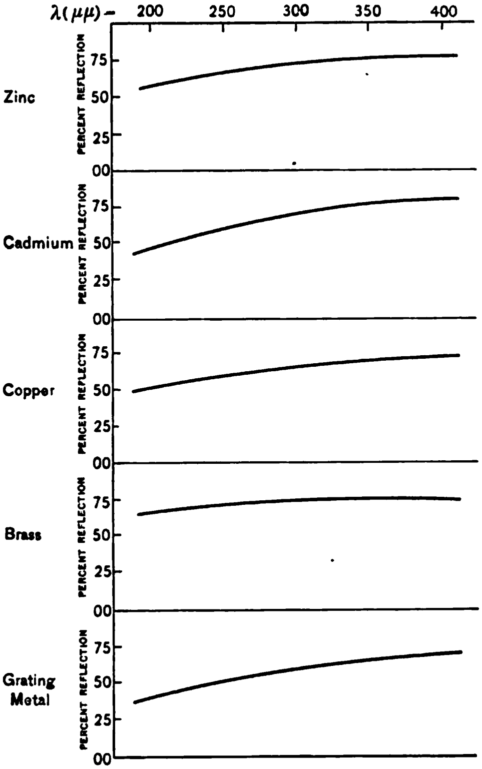

Fig. 3.

Glass.—Ordinary French plate and mirror plate, from which the silver had been removed, were used. Both were backed with Canada balsam and boneblack to avoid reflection from the lower surface. Glass shows strong absorption beginning abruptly, even when layers but half a millimeter thick are used. The reflecting power, however, remains constant in this region in which the absorption begins so abruptly and has a constant ratio to that of quartz from the visible to the extreme end of the spectrum at 185. According to Drude's formula for reflection[1] we should have for the reflecting powers at wave-lengths 350 and 400, the anomalous relation

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[1] Drude, Lehrbuch der Optik, Leipzig, 1900, p. 336.

$$\left[\frac{n^2(1+x^2)+1-2n}{n^2(1+x^2)+1+2n}\right]_{\substack{\lambda=350\\ \chi=20}}=\left[\frac{n^2(1+x^2)+1-2n}{n^2(1+x^2)+1+2n}\right]_{\substack{\lambda=400\\ \chi=0}}.$$

Glass certainly does not show metallic reflecting power in the region in which its absorption is very great. Its properties in this respect are like those of solutions. An alcohol solution of cyanin for example, strongly absorbing all the ultra-violet, shows a low and

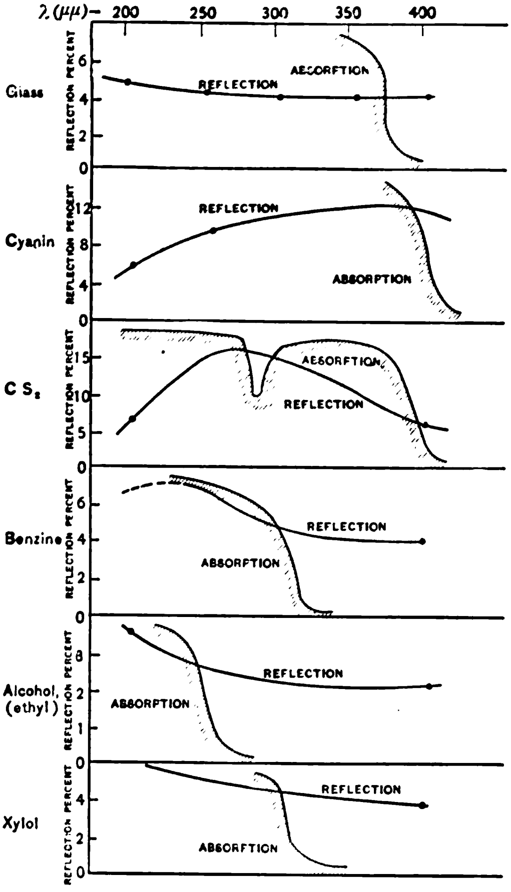

Fig. 4.

nearly constant reflecting power, practically the same as that of pure alcohol. The lack of metallic reflecting power in the case of glass may be explained if we consider it a solution of absorbing salts in a non-absorbing silicate solvent. No variation in reflecting power could be detected in the regions of strong fluorescence either of common glass or of uranium glass.

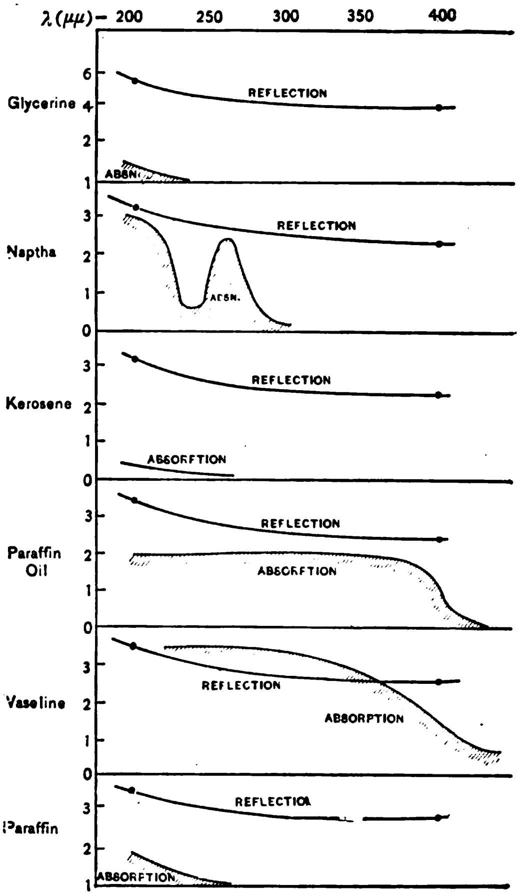

Fig. 5.

Cyanin.—Excellent plane mirrors were obtained by fusion of cyanin between plates of glass after the method used by Wood for making thin prisms. When one plate is quartz, the results are better, as the glass usually comes off clean, leaving the cyanin unbroken on the quartz. The accompanying photograph, in Plate II., shows the second absorption band covering the whole ultra-violet beyond

380. The reflecting power is considerably increased in the region of the absorption, but could hardly be called metallic.

Carbon Bisulphide.—The absorption region covers the whole ultra-violet beyond 380, with the exception of a narrow transmission band between 280 and 290. The reflecting power is somewhat increased in this region.

Benzine.—Pauer[1] obtained numerous narrow bands with very thin layers of benzine in the region in which the photograph shows the absorption of a layer 1.5 mm. thick to be complete. The ultra-violet lines of benzine vapor show every characteristic of molecular resonance, and the very considerable reflecting power in this region is additional evidence of such a mechanism of absorption.

Alcohol, xylol, glycerine and the petroleum products show more or less absorption but give no indication of metallic reflection. The curves for these and the preceding substances are given in Figs. 4 and 5.

BERKELEY, CAL., April, 1901.

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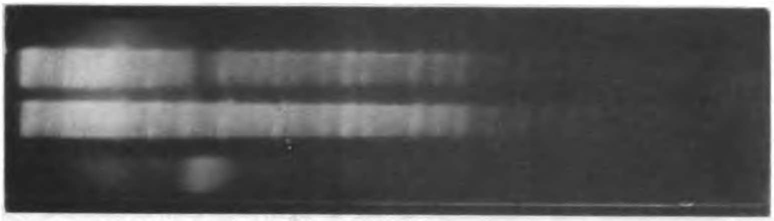
[1] J. Pauer, Wied. Ann., 61, 368, 1897.

SILVER I.

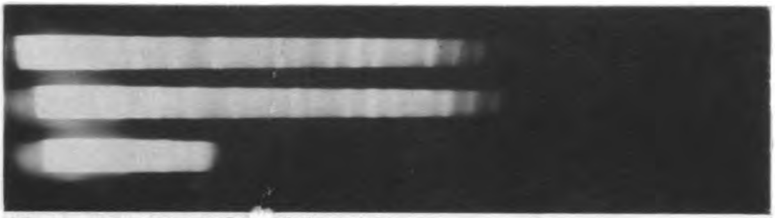


Upper spectrum reflected from silver.
Middle spectrum reflected from mercury.
Lower spectrum transmitted through a thin sheet of silver on quartz.

SILVER II.

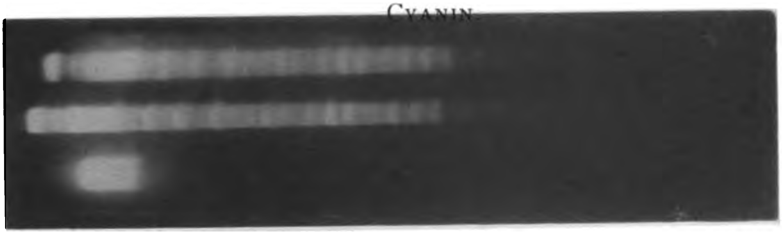
Upper spectrum reflected from quartz.
Middle and lower spectra reflected from silver.

GLASS.



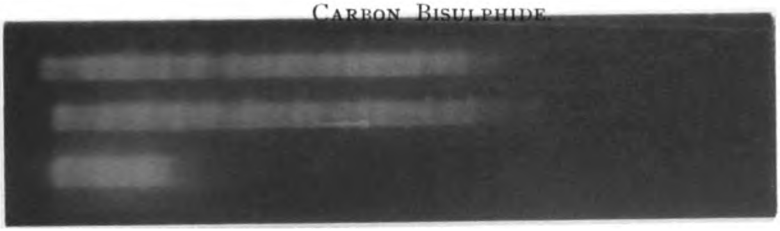
Upper spectrum reflected from glass.
Middle spectrum reflected from quartz.
Lower spectrum transmitted through plate glass.

CYANIN.



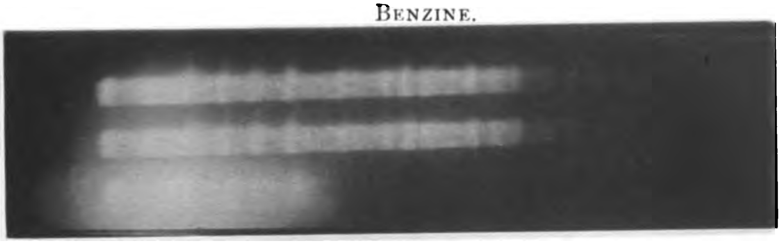
Upper spectrum reflected from cyanin mirror.
Middle spectrum reflected from quartz.
Lower spectrum transmitted through ½ mm. of alcoholic solution of cyanin.

CARBON BISULPHIDE.



Upper spectrum reflected from Carbon Bisulphide.
Middle spectrum reflected from quartz.
Lower spectrum transmitted by thin layer of Carbon Bisulphide.

BENZINE.



Upper spectrum reflected from benzine.
Middle spectrum reflected from quartz.
Lower spectrum transmitted by a thin layer of benzine.

ON THE MAGNETIC EFFECT OF ELECTRICAL CONVECTION.

By Harold Pender.

Historical Review.

FROM a series of experiments extending over the last four years, M. V. Cremieu[1] has come to the conclusion that a moving electrified body produces no magnetic effect. The conclusion is in direct opposition to the classical experiment performed by Professor Rowland[2] in Berlin in 1876, which was subsequently repeated in this laboratory by Professor Rowland and C. T. Hutchinson[3] in 1889, and confirmed also by experiments performed by Röntgen[4] and Himstedt[5] in Germany. A further investigation of this very important question was therefore considered desirable. Accordingly in the fall of 1900, under the supervision of Professor Rowland, the following research was undertaken. As a result it is has been shown conclusively that electrical convection does produce magnetic action.

The idea that a moving electrified body might produce a magnetic effect similar to that produced by an electric current first occurred to Faraday[6] in 1837. The first to perform an actual experiment on the subject, however, was Professor Rowland. The experiment was carried out in Germany in 1876, though the idea of it had occurred to him as early as 1868, and is recorded in a notebook of that date. His method was similar to that described by

---

[1] C. R., T. 130, p. 1544, 1900; T. 131, p. 578 and p. 797, 1900; T. 132, p. 327 and p. 1108.

[2] Pogg. Ann., 158, p. 487. Am. Jour. Sci., 1878, p. 30.

[3] Phil. Mag., 27, p. 445, 1889.

[4] Ber. d. Ber. Akad., 1885, p. 195.

[5] Wied. Ann., 38, p. 560, 1889.

[6] Exp. Res., Vol. I., art. 1644.

Maxwell in his Treatise on Electricity and Magnetism (§ 770), written presumably in 1872 or 1873.

In the Berlin experiment, Professor Rowland used a horizontal hard rubber disc (diameter 21 cm.) coated on both sides with gold, revolving between two glass plates with their inner surfaces also gilded. Each side of the disc formed a condenser with the gilded surface opposite it. The condensing plates were earthed; and the disc charged by means of a point brought up within ⅓ mm. from the periphery. An astatic needle was suspended above the upper condensing plate, so that its lower magnet was nearly over the edge of the disc, and perpendicular to a radius of the same. The needle was enclosed in a metal case, so as to screen it from electrostatic action. The speed of the disc was 61 revolutions per second. On reversing the sign of electrification of the disc, a deflection of from 5 to 7.5 mm. was obtained, depending on the conditions. The deflection was the same whether the gilded surfaces of the disc and condensing plates were divided into sectors, or left continuous. The deflection as observed and calculated agreed quite well, but as the needle was always very unsteady, it was impossible to make the readings with great accuracy. In 1883 Lecher[1] repeated Rowland's experiment, but obtained negative results. Only a short account of this investigation was published, no details of the apparatus being given. It is impossible, therefore, to say why he obtained no effect. It may have been because his apparatus was not sufficiently sensitive.

In 1885, in an attempt to detect the existence of displacement currents, Röntgen[2] had occasion to use an apparatus similar to that employed by Professor Rowland in his Berlin experiment. The ebonite disc in this case was not covered with gold. Incidentally, the effect on the suspended needle of charging the disc, when rapidly rotating, by a brush discharge from a series of points, was tried. The Rowland effect was readily observed. On reversing the sign of electrification a deflection of from 8 to 10 mm. was noted. In 1888[3] this apparatus was again used to investigate the action of a dielectric moving in a homogeneous electrostatic field. If the upper condenser plate is charged negatively (say) and the lower positively, then there will be an apparent positive charge on the upper surface of the disc, and an apparent negative charge on the lower surface. If then the motion of an apparent charge due to the polarization of the dielectric produces magnetic action, the needle, which was nearer the upper than the lower surface, ought to show a deflection on rotating the disc and charging the condenser formed by the two stationary plates. After all precautions had been taken to prevent the disc from assuming a real charge, a deflection of from 2 to 3 mm. was observed on reversing the sign of electrification of the condenser plates.[1] On increasing the difference of potential between the condenser plates until a brush discharge took place between the upper plate and the disc (the upper plate was .14, the lower .25 cm. from the disc) the Rowland effect was again readily observed.

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[1] Rep. d. Phys., 20, p. 151, 1884.

[2] Sitzb. d. Ber. Akad., 1885, p. 95.

[3] Sitzb. d. Ber. Akad., 1888, p. 23. Wied. Ann., 40, p. 93.

In 1889 Professor Rowland and C. T. Hutchinson[2] repeated in this laboratory the original Berlin experiment, employing, however, a different form of apparatus. Instead of a single disc rotating in a horizontal plane, they used two vertical discs rotating about horizontal axes in the same line; the needle system, enclosed in a brass tube, was placed between the discs, opposite their centers. Each disc was surrounded by a guard ring, both discs and guard rings being gilded on the side facing the needle. Between the discs were placed two condensing plates—glass plates gilded on the surfaces facing the discs. Between the condensing plates was suspended the needle. This arrangement permitted of accurate calculation of the effect which should be expected. The condensing plates were charged from a Holtz machine and battery of Leyden jars, to a potential of about 5,000 volts; the discs were earthed by means of metal brushes bearing on studs fixed in the periphery of the discs. The speed of the discs was about 125 revolutions per second.

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[1] Röntgen gives no calculation of the effect which should be expected. On the assumption that the magnetic force due to any element of surface is proportional to the quantity of apparent charge passing a given point in unit time, I have calculated from the data given in Röntgen's paper that the deflection should be 2.1 mm. I employed the formulæ used by Rowland in his Berlin experiment (loc. cit.).

[2] Phil. Mag. (5), 27, 445.

The deflection of the needle on reversing the electrification was from 5 to 8 mm. This experiment offers a means of determining the ratio of the two systems of electrical units. The mean of all their determinations gave for this 3.19×10^{10}; the determinations which differed from this the most being 3.74×10^{10} and 2.26×10^{10}.

During the same winter of 1889 Himstedt[1] carried out a series of experiments on this subject in Giessen. In his first experiments he employed the same apparatus as had been previously used by Röntgen, with some modification in details. Later on, apparatus was constructed similar to that used by Rowland and Hutchinson, but considerably more sensitive. Discs of ground glass 20 cm. in diameter were mounted so as to revolve about horizontal axes in the same line. A strip on the edge of both sides of the discs was made conducting by rubbing into the ground surface a thin coat of graphite. On both sides of each disc were placed condensing plates, and between the two inside plates was suspended an astatic needle, properly shielded, the upper needle being just above, the lower just below the conducting strip. The discs were charged by an induction machine and a battery of Leyden jars by means of a sliding contact. With a speed of 117 revolutions per second and discs charged to 5,000 volts (condenser plates earthed), a deflection of 100 mm. on a scale 3 m. distant was obtained on reversing the sign of electrification of the discs. This apparatus did not permit of ready calculation of the deflection to be expected, so no absolute measurements were made. Himstedt, however, showed that the deflection was in the direction to be expected, reversed with the direction of rotation of the discs, was proportional to the speed of rotation, and for the disc at potentials of from 400 to 4,000 volts was proportional to the surface density of the charge on the disc. Above 4,000 volts this latter proportionality ceased to exist, the deflection remaining approximately constant up to the highest potential tried 14,000 volts. Himstedt explains this by assuming that there is a limit to the amount of charge which a moving body can carry with it. For these high potentials, however, the leakage was considerable ; in one minute the potential dropped from 14,000 to 4,000 volts. Any theoretical conclusion of the above nature must therefore be accepted with reserve.

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[1] Wied. Ann., 38, p. 560, 1889.

So stood the question as to the existence of a magnetic action due to electric convection in 1897, when Cremieu began his experiments. The first problem[1] attacked was the reverse of the above experiments, viz., the detection of the mechanical force exerted on a charged body by a rapidly changing magnetic field. This quantity is extremely small, varying as the reciprocal of the square of the ratio of the two systems of electrical units. Cremieu attempted to detect this force by the rotation of a charged aluminium disc suspended between the poles of a strong electromagnet. No deflection of the disc was observed, though the calculated deflection as read by means of a telescope and mirror on a scale 1.1 m. distant was 100 mm. This is in direct opposition to Lodge's experiment[2] on the same subject performed in 1889, though Lodge's experiment is not altogether satisfactory, as the observed deflection was only from 2 to 3 mm. Cremieu has published only a short note on this experiment, with no details of the apparatus, so that it is impossible to discuss the work further.

The negative results obtained from the reverse experiment led Cremieu to investigate once more the magnetic action of a moving electrified body.[3] A very ingenious method, distinctly different from any of those previously employed, was used. A solid aluminium disc, 37 cm. in diameter, was mounted on an insulating hub of hard rubber turning on a horizontal axis. Concentric with this disc was placed a coil of 13,000 turns of copper wire .15 mm. in diameter, connected in series with a sensitive galvanometer. The diameter of the coil was 44 cm. The coil was enclosed in a brass case. The disc revolved between two thick iron condensing plates. These plates also served to decrease greatly the reluctance of the magnetic circuit through the coil. The whole distance from plate to plate was 8 mm., the disc revolving in the center of this opening. A brush rubbing on a brass ring mounted on the insulating hub and connected to the metal disc, served to connect the disc with one terminal of a high potential battery (5,000 volts), the other terminal of which was earthed. On charging or discharging the disc, the condensing plates being earthed, on the assumption that a moving charge produces a magnetic action, a current should be induced in the coil, and the needle of the galvanometer deflected. The calculated effect of a single charging or discharging was too small to be detected, so that a combined interrupter and commutator was employed. This instrument served to charge and discharge the disc a number of times a second, and at the same time to commutate any alternating current (if there was any) which might have been induced in the coil. The galvanometer should therefore have shown a permanent deflection if the commutator was kept running at a constant speed. No such deflection was observed, though the calculated deflection was from 20 to 50 mm.

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[1] C. R., 131, p. 578, 1900.
[2] Phil. Mag. (5), 27, p. 469, 1889.
[3] C. R., 130, p. 1544, 1900.

Four series of experiments were performed. In the first was used a solid aluminium disc 0.5 mm. thick. This was charged to 5,000 volts, but it immediately discharged itself by sparks across the small gap between the disc and condensing plates. To prevent this, the latter were covered with glass plates, lacquered, 2 mm. thick. This, however, left a space of only 1.75 mm. between the disc and glass surface. It was suggested by Pellat that perhaps the charge brushed across from the disc to the glass surface, thus leaving the disc uncharged. This was tested by Cremieu for *the disc at rest*, and found not to be the case. But it is impossible to make a disc of aluminium only 0.5 mm. thick and 37 cm. in diameter, run perfectly true at a speed of 100 revolutions per second. Consequently, when the *disc was rotating*, the interval between it and the glass surface must at certain points have been considerably less than 1.75 mm., and therefore the above objection still holds. This is the explanation of the negative results of Cremieu's experiments which was given by Professor Rowland.

In his second series ef experiments Cremieu used an aluminium disc 1 mm. thick, covered with a thin coating of caoutchouc, the condensing plates being left bare. Again negative results were obtained. When it is remembered that the distance between the disc and the condensing plates was only 3.5 mm., and that an aluminium disc of the diameter used, even though 1 mm. will not run perfectly true, it seems by no means improbable that there may not have been a brushing over of the charge from the condensing plates to the caoutchouc surface. The magnetic action of this charge would then just neutralize that of the charge on the disc. This same objection applies to the third series of experiments, in which was used an ebonite disc 2 mm. thick, gilded on both sides in three sectors and covered with caoutchouc; and to the fourth series, in which the iron condensing plates were covered with sheets of mica, over which was pasted tin foil in six sectors, covered with a thin layer of caoutchouc. (The object of the third and fourth series was to see if the negative results were due (1) to the slipping of the charge with respect to the disc, or (2) to the movement of the charge induced on the condensing plates with the inducing charge on the disc.)

During the past winter Cremieu has continued his investigations,[1] replacing the coil and galvanometer by an astatic needle, and again with negative results. The rotating apparatus consisted of the same ebonite disc, mounted in a bronze frame of the same dimensions as the iron frame previously used. To this frame were screwed brass plates, covered with mica, and over the latter was pasted tin foil in six sectors, connected to the earth. The astatic needle was placed outside of these, with the lower needle opposite the upper edge of the disc. To this experiment applies the same objection as to those previously performed, viz., the rotating disc probably did not keep its charge. It is interesting to note that in a second series of experiments with this apparatus, in which the tin foil was mounted on ebonite plates and these screwed directly to the bronze frame, so that between the needle and disc there was only one metal plate, a deflection was obtained, in the proper direction and reversible with the sign of the electrification of the disc and the direction of rotation. This Cremieu states was suppressed by the introduction of a brass plate between the needle and the condensing plates. Without a more detailed account of the exact conditions under which this effect was obtained than is given by Cremieu, it is difficult to say exactly what this means.

Believing that the deflection observed by previous investigators might have been due in some way to the condensing plates, a different arrangement was then employed.[2] The same ebonite disc, with the three sectors completely insulated from one another, was again used, but the condensing plates formed by tin foil on ebonite were each divided in half. The lower half of each was put up close to the disc (how close Cremieu does not state); the upper halves were 10 mm. distant. The foil covering a sector of 60° on each of the upper halves was removed. The needle, enclosed in a copper tube with walls .4 mm. thick, connected to the earth, was placed directly over the edge of the disc opposite this break in the condensing plates. Each sector of the disc was charged when immediately between the lower halves of the condensing plates. When it came under the needle it was completely insulated from the other two and from the charging apparatus. With this apparatus Cremieu again found that no deflection of the needle was produced on charging or discharging the rotating discs, so long as the needle was properly screened against direct electrostatic action. If, however, the copper tube above mentioned was replaced by a tube of some less perfect conductor, *e. g.*, graphite paper, a deflection could be obtained, which had all the characteristics expected of an effect due to electric convection. But this effect could be obtained equally well when the magnets forming the astatic needle were removed and only the mica vanes on which they were mounted were left. Here are Cremieu's final conclusions:

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[1] C. R., 131, p. 797, 1900.

[2] C. R., 132, p 327, 1901.

" Enfin, toutes les fois que le système suspendu, quel qu'il soit, est protégé par un écran électrique très conducteur, on ne peut obtenir aucune déviation.

" De plus, il arrive souvent qu'au moment où l'on change le signe de la charge du disque, une étincelle, ou même une sorte de pinceau de décharge à peine lumineux, se produit entre le disque et les plateux fixes.

" Ceci cause, sur l'aiguille aimantée, des impulsions réversible a la fois avec le signe de la charge et le sens de la rotation, le peu de stabilité du zéro de systèmes aussi sensibles fait qu'il peut résulter de ces impulsions des déviations permenentes faibles."

According to Cremieu's explanations of these slight deflections there is no reason why they should reverse with the direction of rotation of the disc. In all probability, however, these deflections were really caused by electric convection, and should have reversed as observed. Though in the short note which Cremieu has published concerning this experiment there is no mention made of a caoutchouc coating over the disc, the disc was undoubtedly thus covered as in all the previous experiments. The sparks observed were then due to the passage of the charge from the condensing plates to the caoutchouc surface. A slight charge might still have remained on the condensing plates, so that the charge taken by the caoutchouc surface was less than that on the surface of the disc. The slight deflection observed was, probably, then really due to the difference in magnetic action of these two surfaces. It is also seen why a deflection of the magnitude expected was not obtained.

If Cremieu's conclusions are true, then "open currents" can exist. Cremieu has tested this deduction by the following experiment.[1] An ebonite disc, 37 cm. in diameter and 2.5 mm. thick, was gilded (presumably on one side, if one may judge from the diagram given) in 25 sectors, the distance between sectors being 1 cm. This disc turned about a horizontal axis between two fixed ebonite plates. One of these plates (presumably the one facing the side not gilded) carried a sector of tin foil twice as wide as the sectors on the disc. On the second plate, opposite this sector was fixed a brush which made contact with the sectors on the disc when immediately under the fixed sector. This brush was also connected to a second brush on the same plate, about 60° ahead of the first brush. Then, according to Cremieu, if the fixed sector is connected to a source of electricity and the disc set rotating, each sector, as it comes under the fixed sector, will receive a charge by induction; when this sector makes contact with the second brush it will be discharged. There will then be a conduction current flowing in the wire connecting the discs, which circuit is completed by the charge carried convectively between the two brushes. An astatic system was suspended with its lower needle near the edge of the disc, between the two brushes. The conduction circuit between the two brushes contained a galvanometer. The fixed sector was charged to a potential of 100 to 130 C.G.S. electrostatic units, its distance from the rotating disc was .5 cm. When the disc was set rotating the galvanometer indicated a current of from 10^{-4} to 2×10^{-4} ampères. The astatic system, however, remained at rest; though a conduction current of 10^{-4} ampères in a circuit occupying the mean effective position of the charged disc caused a deflection of 15 mm. on a scale 4 meters distant. Cremieu therefore concludes that the electricity carried convectively between the two brushes is without magnetic effect, *i. e.*, that the current in the wire connecting the two brushes is an "open current."

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[1] C. R., 132, p. 1108.

There is, however, a possible explanation of this experiment which requires no such conclusion. As was stated above, the potential of the fixed sector was from 100 to 130 C.G.S. electrostatic units or 30,000 to 39,000 volts, while its distance from the disc was only .5 cm. According to J. J. Thomson (Rec. Res., p. 72) the sparking distance between planes at a difference of potential of 100 C.G.S. electrostatic units is .95 cm. Ebonite is not a perfect insulator, especially for such high potentials. Hence the charge from the fixed sector may have brushed across to the surface of the ebonite facing the disc. This charge would be of opposite sign to that induced on the metallic sectors as they passed under the first brush, and the magnetic effect of that part of the disc between the brushes and nearest the needle would therefore be approximately null.

PRESENT RESEARCH.

Method and Apparatus.

The method which has been employed in the present research to put in evidence the existence of the magnetic action of electrical convection is identical in principle with that used by Cremieu in his first experiments, viz., the measurement of the current induced in a coil, on reversing the convection current due to a rotating charged disc.

All experimenters who have worked on this problem have been met by three difficulties, which have required no little patience and considerable experimenting to overcome successfully. These difficulties are: (1) the mechanical jarring caused by the rapidly rotating discs, (2) direct electrostatic effects due to the charged disc, and (3) external magnetic disturbances. The present instance has been no exception to this rule. In the first place, the galvanometer which it was found necessary to employ was so sensitive, that it was impossible to make any observations whatever except between 1:30 and 5 A. M., when the electric cars in the vicinity of the laboratory had ceased making their regular trips.

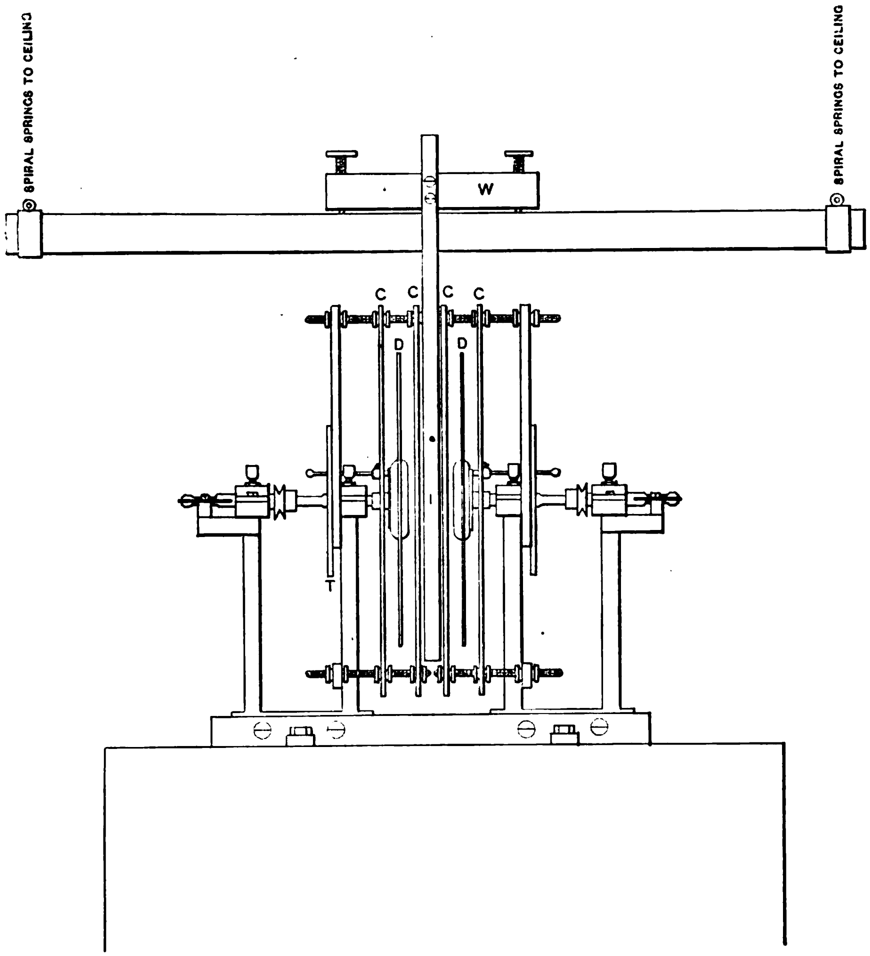
It is needless to describe the succession of trials which were made to eliminate first one disturbing factor and then another, and the various modifications made from time to time in different parts of the apparatus. I shall describe the apparatus only as it was used in the final quantitative measurements.

The Disc Apparatus.—This part of the apparatus was constructed several years ago under Professor Rowland's direction, shortly after his experiment in 1889, but had not been previously used. A drawing of it is given in Fig. 1. Two micanite discs *DD*, .336 cm. thick and 30.4 cm. in diameter, gilded on each side, and mounted on ebonite cores, rotate about horizontal axes in the same line. On each side of the discs are condensing plates, formed by rings of tin foil on ebonite plates *cccc* 40 cm. square. The external diameter of these rings is 40 cm., internal diameter 10 cm. By means of adjusting screws it is possible to vary the distance between the discs and condensing plates, and to adjust the latter accurately parallel to the discs. Fixed to the frame of the apparatus is a test coil *T*, through which a conduction current can be sent at will. The frame carrying each disc is fixed separately to base plate of the apparatus, so that the two discs can be placed at any desired distance apart. To each disc is attached a speed counter formed by a cog wheel with 199 teeth gearing directly with a worm on the shaft of the disc.

On the side of the ebonite cores next the frame are fixed brass rings, diameter 6 cm., width 5 cm., making contact with the gilded surfaces by strips of tin foil. Contact is made with these rings by brushes of thin brass mounted on the ebonite plates *cccc*. The brushes nowhere come within 1 ½ cm. of the frame or the tin-foil condensing plates.

At first ebonite discs were employed, but even though they were 3.5 mm. thick, they soon became so badly warped that their use was extremely unsatisfactory. Besides the excessive jarring produced when they were in rapid rotation, it was impossible to make accurate measurements of the distance between discs and condensing plates. The micanite discs have served admirably. They are as plane and run as true now as when first put up two months ago.

In the first experiments the discs were charged by points brought up close to their edge. The effect sought could be observed with



$\frac{1}{8}$ ACTUAL SIZE.

Fig. 1.

this arrangement, but the needle of the galvanometer, when the latter was connected in series with the coil, was exceedingly unstable when the discs were being charged and discharged, and often even became demagnetized, so that it was frequently necessary to take the galvanometer apart and re-astatize the needle. After weeks of experimenting it was found that this was due to the sparking at the points, unavoidable in this method of charging. On charging by means of the brushes and rings above described, so that there was no sparking at the discs, the spot of light, though it never remained perfectly steady, rarely oscillated over 10 mm.

The Coil.—The constants of the coil, the induced current in which was to be measured, are as follows:

Mean diameter	30 cm.
Width	3.8 cm.
Thickness	1.27 cm.
Number of turns	1295
Resistance	140 ohms
Size of wire	No. 21 B. & S.

This coil I, (Fig. 1) was suspended between the two inside condensing plates, the distance between which was just great enough to allow the coil to hang perfectly free. With regard to this coil, two things were found absolutely necessary; first, to have it completely shielded from all direct electrostatic action, and secondly, to have it so suspended as to be perfectly free from mechanical jarring. The following arrangement, finally adopted after considerable experimenting, proved entirely satisfactory.

Two brass hoops 1.5 cm. thick were placed around the coil, one hoop outside, the other inside. The coil had been previously wrapped with insulating tape. The space between the two hoops was then carefully covered with tin foil. In this way was formed a closed metal case only slightly wider than the thickness of the coil. The terminals of the coil were brought out from this case, and were carefully wrapped in tin foil. The metal case and foil were connected to earth. To the outside hoop were soldered two heavy brass strips fixed to the wooden block *W*, provided with leveling screws. This frame containing the coil was stiffened by a brass rod connecting the strips just above the coil. The block *W* rested on a heavy marble slab suspended from the ceiling by four spiral springs. To prevent the air currents generated by the rapid motion of the disc from setting this system in vibration, cardboard strips were screwed down over the openings between the condensing plates, so that each disc was completely boxed in.

The Galvanometer.—As has already been stated, the galvanometer employed was an extremely sensitive one. It is an astatic instrument of the ordinary four-coil form, with a low resistance, 58.4 ohms. The needle, which together with its mirror weighs only 2.5 mg., is suspended by an extremely fine quartz fiber, invisible almost in ordinary light. The period of the needle when vibrating in the earth's field was from 3 to 5 seconds, depending on how perfectly the needle was rendered astatic. The period was usually increased to about 25 seconds by cutting down the earth's field by means of control magnets. With this period a current of 1×10^{-10} ampères gives a deflection of 1 mm. on a scale 1.1 meters distant. The deflection of the needle was read by the deflection of the image of an incandescent light filament reflected from the needle mirror, on a ground-glass scale 1.1 meters distant. The needle was absolutely dead beat.

A heavy marble block, resting on a bed of excelsior on a marbel slab bracketed to the wall, formed a support for the galvanometer which rendered it entirely free from mechanical jarring. The galvanometer was also shielded magnetically by three coaxial cylinders of soft iron, closed at the bottom and top by soft iron plates. With this shield on, the needle was fairly steady during the period above mentioned. The accidental deflections of the spot of light with the disc uncharged rarely amounted to over 5 mm. during the time required for a single determination of a deflection—*i. e.*, about one minute, though there was at times a considerable shift of the zero.

The expression "sensibility of the galvanometer" will be frequently used in what follows. By it is meant the deflection in millimeters on the scale 1.1 meters distant from needle, reduced to an infinitesimal arc, produced by a current of 2×10^{-8} ampères flowing through the galvanometer. This value of the current was used simply because it gave a convenient deflection. The scale remained fixed with reference to the galvanometer throughout the experiment. To obtain a constant testing current a Clark cell was connected in series with 101,648 ohms. Around 1,648 ohms of this was a shunt circuit with a resistance of 10,000 ohms. Around 101 ohms of this a second shunt circuit containing the galvanometer was formed, the total resistance of this circuit being 10,000 ohms. This arrangement gives a current of 2×10^{-8} ampères through the galvanometer. A simple mercury key served to throw the galvanometer in or out of this circuit.

The Reverser and Commutator.—The reverser was similar in construction to an ordinary rotary interrupter. Connected to the shaft of the reverser was an eccentric, 1 mm. off center, which operated a commutating device in the circuit between the coil and galvanometer. To the eccentric was attached a lever carrying at one end a small silver rod 1.2 cm. long. When the shaft of the interrupter was turned this rod was given a slight up and down motion between two flat silver springs, between which also was fixed a second silver rod. When the moving rod was at its highest point it made contact with the upper spring, the fixed rod making contact with the lower spring. When the moving rod was at its lowest points, the contacts were reversed. This special form of commutator was found necessary in order to avoid thermal currents. The two rods were connected to the terminals of the coil, the two springs to the galvanometer terminals. The commutator was entirely enclosed in a metal shield connected to earth.

The Charging Apparatus.—The discs were charged by means of a large Voss machine—diameter of revolving plate 50 cm.—and a battery of six gallon Leyden jars. Each pole of the Voss machine was connected to the inside coating of three of the jars. The outside coatings were earthed. It was found convenient to drive the reverser at a speed of from 3 to 5 revolutions per second, which reversed the charge on the discs 12 to 20 times a second. As the discs and condensing plates formed a condenser of considerable capacity, it was therefore found necessary, in order to keep up the potential of the battery, to keep the Voss machine running at a considerable speed, in fact, as fast as the construction of the machine permitted. This was accomplished by means of a small electric motor, the speed of which remained admirably constant. When the discs were being charged, the potential seldom varied over 3 per cent.

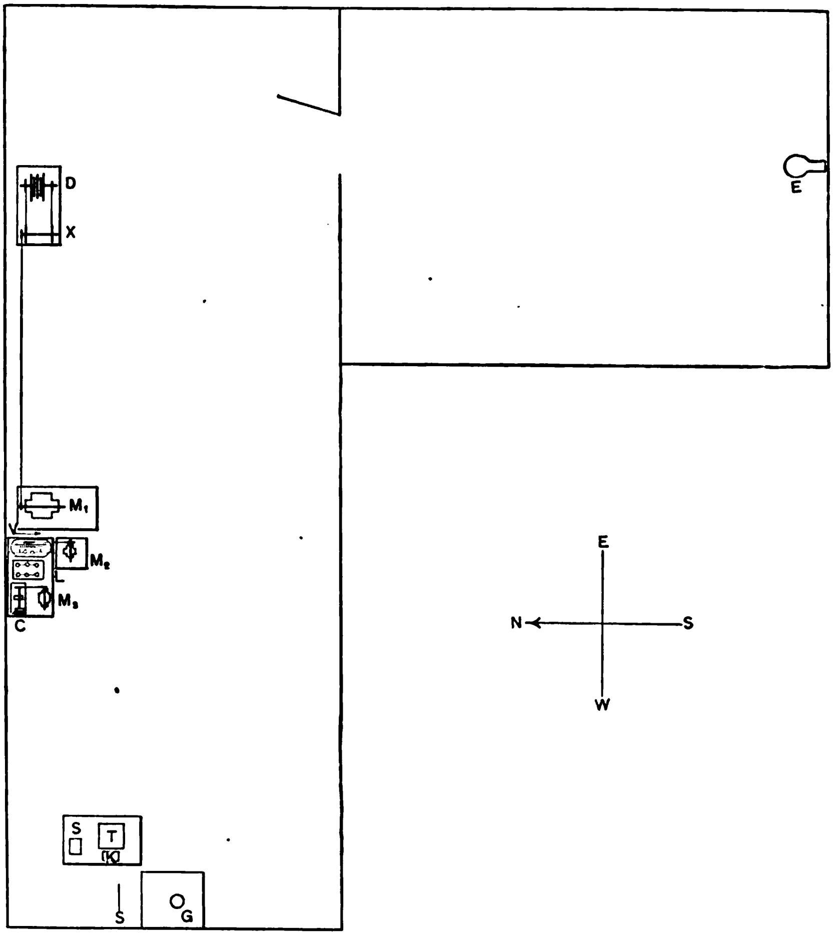
General Arrangement of the Apparatus.—This is shown in Fig. 2. The letters indicate the following:

E an absolute electrometer.
D disc apparatus and coil.
M_1 motor driving discs.
M_2 motor driving Voss machine.
M_3 motor driving commutator.
V Voss machine.
L Leyden jars.
C commutator.
S a high potential reversing switch.
K key in galvanometer circuit.
T resistance boxes and cell forming test circuit.
G galvanometer.
s scale of same.
X counter shaft.

The advantage of this galvanometer method of detecting the existence of the magnetic action of electrical convection over the magnetometer methods is clearly seen in this possible arrangement. Instead of having the delicate magnetic detector which must be employed, up close to the rotating discs, where it is subject to all kinds of disturbing elements, as is necessary when a magnetometer is used, it can in this method be placed as far as one may desire from all the rest of the apparatus. The three motors M_1, M_2, M_3 were so placed as to reduce their action on the coil and galvanometer to a minimum. When placed as shown there was no appreciable action on the coil, but on turning the current on the large motor M_1, the zero of the galvanometer was changed about 15 mm. As the speed of the motor was kept constant, however, during a series of observations, this was a perfectly steady deflection, and consequently of no particular disadvantage. The discs were set up at right angles to the meridian, for convenience in placing the rest of the apparatus. There was no appreciable effect due to their thus cutting the lines of force of the earth's field, *i. e.*, there was no effect on running the apparatus with the discs uncharged.

In the figure all connections are omitted for the sake of clearness. These, however, can be readily described.

Galvanometer Circuit.—This circuit was completely enclosed in a continuous metal shield throughout its entire extent. This shield was connected to earth. The lead wires from the galvanometer lead first to a specially constructed mercury switch *K*, enclosed in a wooden box, lined with tin foil—the switch was thus protected both from thermal and from electrostatic effects. One of the leads was connected to the earth. From the switch ran two lead wires to the springs of the commutator. The rods of the commutator were connected to the terminals of the coil.



12′ TO THE FOOT.

Fig. 2.

Charging Connections.—All high potential wires were suspended by silk threads. From the two poles of the Voss machine ran two wires to two terminals of the reversing switch S. From the other two terminals ran two wires to the reverser. The brushes of the reverser could be arranged so that at any instant the two discs could be charged either alike or oppositely, as desired.

If the discs are set rotating in the same direction, the commutator and Voss machine also set rotating so as to charge the discs at any instant with the same sign, then on throwing the galvanometer in series with the coil, the spot of light on the galvanometer scale should move off a distance D from its position on open circuit. If the order of charging with respect to the commutation is reversed, as can be done by reversing the high potential switch S, then the spot of light should move off a distance D on the other side of its position on open circuit. Hence, on reversing the high potential switch S, keeping everything else unchanged, the spot of light should change its position by an amount $2D$. This was the deflection always observed. This deflection varied from 30 to 80 mm., depending upon the conditions. Its direction always reversed with the direction of rotation of the discs, and was always such as should be expected from Ampère's rule. This latter was determined by comparison with the deflection produced by a conduction current, reversed in the same manner as the charge on the discs, flowing in the test coil T on the disc apparatus.

At times a deflection was noticed on reversing the switch S with the discs at rest, but the other parts of the apparatus running. This, however, was invariably traced to the temporary failure of the insulation either of the ebonite cores of the disc or of the ebonite between the brushes and the tin foil condensing plates, due to the surfaces becoming greasy. By cleaning the surfaces with benzine and then with alcohol, this deflection was always completely suppressed. The insulation was usually thus cleaned at once every night, and frequently oftener. *No readings were ever taken when there was any perceptible deflection with the discs at rest.*

CALCULATION OF THE EFFECT.

The strength of the convection current produced by the rotation of an elementary annular surface of radius r and width dr, carrying a charge of surface density σ is

$$dI = 2\pi r dr \frac{N\sigma}{v}$$

where N is the number of revolutions per second and v the ratio of the two systems of electrical units. The quantity induced in the coil I (Fig. 1) connected with the galvanometer by reversing this, *i. e.*, by reversing the sign of σ is

$$dq = 4\pi r dr \frac{N\sigma}{v} \times \frac{M'}{R}$$

where M' is the coefficient of mutual induction of the ring and the coil I, and R the total resistance of the circuit consisting of coil and galvanometer. Let the charge σ be reversed C times a second and the induced current be commutated as above described. The value of the rectified induced current will be

$$4\pi r dr \cdot \frac{N\sigma}{v} \frac{M'}{R} C$$

and the steady deflection of the spot of light from its equilibrium position will be

$$dD = 4\pi r dr \frac{N\sigma}{v} M'K$$

where K is a factor depending on the resistance of the circuit, the number of reversals per second, and the sensibility of the galvanometer, or $2M'K$ is the deflection given by a unit current reversed in the ring C times a second under the same conditions. Hence the total deflection due to one surface of the disc is

$$D_1 = \frac{4\pi NK}{v} \int_{R_1}^{R_2} \sigma r M' dr.$$

Since there are four surfaces the entire effect due to them all can be written

$$D = 4D_1 = \frac{16\pi NR}{v} \int_{R_1}^{R_2} \sigma r M dr$$

where M is the mean value of M' for the four surfaces.

Considering D the observed deflection, we have

$$v = \frac{16\pi NK}{D}\int_{R_1}^{R_2}\sigma r M dr.$$

The value of v thus determined is a test for the accuracy of the assumption that a convection current is magnetically equivalent to a conduction current.

The surface density of the discs is uniform except on the edge and at the center opposite the opening in the tin-foil condensing plates. The uniform surface density over the main body of the disc is

$$\sigma = \frac{V}{2\pi(B-\beta)}$$

where V is the potential of the disc, the condensing plates being earthed, B the distance between the condensing plates, and β the thickness of the disc.

The excess on the edge of each side is (Max. Elec. and Mag. § 196)

$$q = 2\pi R_1\sigma\frac{B}{\pi}\log_e\left(2\cos\frac{\pi\beta}{2B}\right)$$

$$= \frac{VR_1B}{\pi(B-\beta)}\log_e\left(2\cos\right)\frac{\pi\beta}{2B}$$

The effect of this excess can be calculated by considering it as concentrated in a circle of slightly smaller radius R' than that of the disc. The lack of uniformity opposite the internal edge of the condensing plates may be allowed for by taking for R_1 a value somewhat less than the radius of the opening in the tin foil, and considering the surface density uniform. This last correction is less than one per cent., since the velocity of this portion of the disc is very small and its effect on the coil also very slight. We then have

$$v = \frac{4vNK}{(B-\beta)D}\left[\int_{R_1}^{R_2}rMdr + \frac{R_1B}{\pi}\log_e\left(2\cos\frac{\pi\beta}{2B}\right)M_R\right].$$

M and $\int Mrdr$ can be calculated from the dimensions of the apparatus by means of elliptic integrals, provided the coil I is perfectly uniform. However, there were considerable variations in the dimensions of the coil from point to point, so that this method was not deemed sufficiently accurate. Hence the following scheme, which amounts practically to an experimental determination of M and a graphical calculation of $\int Mrdr$ was adopted. Twelve concentric coils of one turn each were mounted on a plane board. This board could be clamped in turn up against each surface of each disc. The radii of the coils varied from 4.3 to 16.0 cm. All leads were carefully twisted. Connections with a storage battery, 110 volts, were so arranged that a current of ¾ of an ampère could be sent at will through the reverser and any one of the coils on the disc, and the current induced in the coil I on running the reverser measured. Instead of employing for this purpose the sensitive galvanometer above described, an ordinary D'Arsonval about 1,000 times less sensitive was used. Hence the large value of the inducing current. The D'Arsonval could be used during the day; also its readings were much more reliable than those of the astatic instrument. The "calibration of the disc apparatus" then consisted in sending a given current through the test coil T with the reverser running, and noting the deflection of the D'Arsonval connected in series with the coil I. The same current was then sent through coil No. 1 on the disc, the deflection noted; then again through coil T, the deflection noted, and so on for each of the twelve coils when clamped up in turn against each face of each disc. The deflection due to the coil T was determined before and after every determination for a coil on the disc, in order to eliminate any error due to the variation in speed of the commutator. The ratio ρ of the deflection due to the given current in each coil to the deflection due to the same current in coil T_1 was determined from these readings. To find the value of the factor $2MK$ it is then necessary to know only the deflection of the astatic galvanometer produced by a unit current in coil T with the reverser running. Call this Δ. Then $2MK = \Delta\rho$. (This of course depends on the sensibility of the galvanometer and the number of reversals per second—see below.) It is hardly necessary to state that the relative positions of the discs, coil I, and coil T, remained fixed throughout the experiment.

It was found that the values of ρ for any one coil in corresponding positions on the two discs were very nearly the same, within about one per cent., thus showing that the two sides of the apparatus were very nearly symmetrical with respect to the coil. The values for the two sides of the same disc differ about 5 per cent. The mean of the ratio for the four positions of each coil was then plotted as ordinates to the radii of the coils as abscissæ. The curve thus found represents $\frac{2KM}{J}$. Multiplying the ordinate of each point by its abscissæ, we get a curve representing $\frac{2KMr}{J}$. The area of this from R_1 to R_2 is

$$\frac{2K}{J}\int_{R_1}^{R_2} Mrdr.$$

Call this μ. The value of $\frac{2KR_1}{J} M_{R'}$ is also given from the first curve. Call this ν.

Then

$$v = \frac{4\pi NJ}{(B-\beta)D}\left[\mu + \frac{\nu B}{\pi}\log_e\left(2\cos\frac{\pi\beta}{2B}\right)\right]$$

Two independent "calibrations" of the disc apparatus were thus made, the first after set 7, the second after set 17. Table I. gives the radii of the coils, the corresponding values of ρ for each calibration, and the mean of the two sets.

Table I.

Radii.	ρ_1	ρ_2	Mean ρ	Radii.	ρ_1	ρ_2	Mean ρ
4.39	.146	.143	.145	12.00	1.293	1.294	1.294
5.98	.284		.284	12.82	1.522	1.501	1.512
7.87	.505	.504	.505	13.57	1.692	1.697	1.695
9.04	.678	.682	.680	14.34	1.880	1.885	1.883
10.02	.845	.848	.847	15.21	2.036	2.065	2.051
11.05	1.071	1.070	1.071	15.96	2.159	2.162	2.161

The factor J is equal to the deflection of the galvanometer due to the current, commutated, induced in the coil I on reversing a unit current in the coil T, as above described. This quantity therefore depends on the number of reversals of the current per second, and the sensibility of the galvanometer. The first idea which suggested itself was to arrange a test circuit so that the deflection produced by a given current thus reversed in the test coil T could be determined alternately with the deflection produced by reversing the charge on the rotating discs, keeping the speed of the reverser and the sensibility of the galvanometer unchanged. This, however, was not practicable. In order to insure a steady contact between the brushes and the brass sectors of the reverser, it was necessary that the brushes bear with some little pressure on these sectors. With the brushes thus adjusted, however, the particles of brass gradually rubbed off soon formed a thin metallic bridge across the intervening ebonite sectors, thus destroying the insulation for high potentials. When the reverser was therefore being used to reverse the charge on the discs, it was necessary to have the brushes so adjusted that they barely touched the brass, in which case the contact for the test current was destroyed. Moreover, when the brushes were adjusted first one way and then another, the speed of course changed. Hence another method of procedure was adopted.

By a series of preliminary experiments it was found that, on reversing a given current in the coil T, as above described, the factor Δ was directly proportional to the sensibility of the galvanometer and to the number of reversals per second, at least within the limits used—8 to 20 reversals per second. Consequently, instead of determining Δ directly for each set of readings, the sensibility of the galvanometer S, and the number of reversals per second C, were determined instead. Writing

$$\Delta = ASC$$

the above formula for v becomes

$$v = \frac{4\pi NASC}{(B-\beta)D}\left[\mu + \frac{\nu B}{\pi}\log_e\left(2\cos\frac{\pi\beta}{2B}\right)\right].$$

The factor A was determined from time to time throughout the experiment as follows. The sensibility of the galvanometer S was first determined. The brushes were then properly adjusted, a known current i sent through the coil and reverser, and the reverser and commutator set running. The deflection due to the current induced in I by this current i in T was then determined; also the number of reversals per second C. The double deflection $2D$ was the quantity measured directly. We have the formula

$$A = \frac{D}{SCi}.$$

The current i was obtained from an ordinary commercial cell in series with several thousand ohms. The e.m.f. of this cell was determined by comparison with a standard Clark cell. The total resistance of the circuit was measured by a standard bridge. The internal resistance of the battery was less than an ohm. In some determinations a Daniell cell was used in place of the commercial cell. Table II. gives a series on determinations of A, for various values of the quantities involved. i is given in C.G.S. electromagnetic units. The value of A is seen to be constant within the limits of error of observation. After using the commutator for these determinations it was always necessary to thoroughly clean the reverser with emery cloth and to reset the brushes, before it could again be used to reverse the charge on the discs.

TABLE II.

S	$2D$	C	i	A
153	158	7.46	$.628 \times 10^{-4}$	$.1101 \times 10^{4}$
205	107	7.34	.317	.1122
197	200	14.7	.317	.1089
172	176	14.2	.317	.1136
177	158	12.8	.317	.1100
171	318	13.4	.632	.1098
214	245	16.8	.314	.1085
86.7	88.2	16.5	.276	.1117
160	198	20.3	.276	.1105
215	217	16.8	.276	.1089
			Mean:	.1104

The quantities to be measured in conjunction with the deflection due to the rapid reversal of the charge on the rotating discs are the following:

1. The number of reversals per second. This was determined by a velocimeter attached to the shaft of the commutator. The

time required for 200 revolutions was determined by a stop-watch. One turn of the shaft corresponds to four reversals.

2. The speeds of the two discs. The time required for 20 complete turns of each of the speed counters was determined by a stop-watch. Each counter has 199 teeth, so that this corresponds to 39,800 turns of the disc. The speeds of the two discs agreed very well. The average speed of the two is the quantity recorded below under N.

3. The sensibility of the galvanometer. This was determined as already described.

4. The difference of potential between the discs and the condensing plates; the condensing plates were always earthed. This was determined by means of an absolute guard ring electrometer, the same instrument as that used by Rowland and Hutchinson in 1889. The disc attached to the balance arm and its guard ring were earthed. The lower plate was connected to the discs through the reverser. By a simple device it was so arranged that this connection was made only when the discs were charged in one way, positively, say, so that the disc of the electrometer did not fluctuate up and down as it would have done had the lower plate been connected permanently to the rotating disc. Had the lower plate connected directly to a pole of the Voss machine, the potential measured would have been higher than that of the discs, since in the interval during which the discs were disconnected from this pole, its potential would rise, due to the storing of the charge in the Leyden jars, for the Voss machine was kept rapidly rotating all the time. The potential then measured by the electromoter was that to which the discs were brought when charged positively, say. By reversing the switch S, the potential to which they were brought when charged negatively was similarly determined. The mean of the positive and negative values of the potential thus determined is the quantity recorded below under V.

The difference of potential between the two plates of the electrometer is

$$V = \delta \sqrt{\frac{8\pi g M}{A}}.$$

In this laboratory $g = 980.1$. A, the area of the disc, corrected for the annular space between it and its guard ring, is 80.11. Hence $V = 17.56\,\delta\sqrt{M}$. δ is the distance apart of the plates, read from a scale attached to the lower plate. M is the mass on the scale pan; δ and M are the quantities determined for each measurement.

A series of measurements was then made as follows. First one observer determined the sensibility of the galvanometer. The zero was noted, the test current 2×10^{-8} ampères sent through, a reading made, the connection broken, and a zero again noted. This was usually done three times, and the mean deflection taken as the sensibility. The commutator and Voss machine were then started, and a test made to see that there was no effect when the discs were at rest. The discs were then set in rotation, and when they had reached a constant speed, a second observer noted the speed of each, and also that of the commutator, while the first observer measured the potential of the discs, with the switch S first thrown north, and then south. The deflection produced by the convection current was then determined by the first observer as follows: With the switch S thrown north, the galvanometer was thrown in circuit, and a reading taken. The switch was then thrown south, a second reading taken, then north again, and a third reading taken. The difference between the second and the mean of the first and third readings was taken as the value of $2D$. The switch was then thrown south, then north, then south, a reading taken each time. Again the difference between the second and the mean of the first and third was taken. In this way variations in the zero were largely eliminated. The above operation was repeated twice. The speeds and potentials were then determined, a similar set of readings taken, and the speeds and potentials once more determined. This usually required the greater part of one of the forty-minute periods during which the galvanometer could be used. During the next period the direction of rotation of the discs was reversed, and a similar set of readings made. It is hardly necessary to state that with apparatus as complicated as that used, accidents were frequent, and often a whole night was spent without obtaining a satisfactory set of readings.

There are two ways of employing the disc apparatus to put in evidence the effect sought. The two discs may be rotated in the same direction, and, at any instant, charged alike; or, they may be rotated in opposite directions, and, at any instant, charged oppositely. Both methods have been employed, but the majority of readings were taken with the discs charged oppositely and rotating in opposite directions, as the needle of the galvanometer was much more steady under these conditions.

Table III. gives an example of a complete set of readings, taken under these latter conditions. The first four columns give the readings for the sensibility, the next three the readings for the potentials, the next eight the readings for the deflection, and the last three the speeds. S is the sensibility of the galvanometer; M the mass on the scale pan of the electrometer; δ and δ' the distance apart of the electrometer plates, less .26 cm. the zero correction of the scale of the electrometer, when the discs are charged positively and negatively respectively; N and S indicate the position of the handle of the reversing switch; C' is the time required for 200 turns of the commutator shaft, N_1 and N_2 the time required for 20 complete turns of the speed counters on the two discs.

This table is a fair example of the general run of readings. The large variations in the separate determinations of $2D$ are due to the fact that the spot of light was always unsteady when the charge on the discs was being rapidly reversed, oscillating over from 10 to 15 millimeters. This unsteadiness was probably due to the fact that the metal case enclosing the coil is not a complete shield against rapidly varying charges. It will also be noticed that at times there was a change of the zero. This was due to variations in the field at the galvanometer due to outside causes.

In Table IV. are given the mean values of the various factors of a number of sets of determinations, taken between the middle of March and the first of May; also the value of v calculated for each set in the manner above described. Observations were attempted only on those nights when the atmosphere was comparatively dry. A bracket in the following table indicates that the sets included under it were taken immediately following one another on the same night.

In connection with sets 14 and 15 a further series of readings was made, which may be looked upon as an additional proof of the ex-

TABLE III.

March 27, 1901. *Rotation* : *I*—, *II*+.

Zero.	Read.	Zero.	S	M	δ	δ'	N	S	N	S	N	S	$2D$	$2D$	C	N_1	N_2
69	283	66	215	.25	.65	.98	177	93	170	105	142	70	81	54	65.1	38.3	38.6
66	282	60	219				115	67	117	66	126	64	49	61			
60	279	63	217		.66	1.01	118	73	109	55	117	62	41	58			
							97	23	93	75	110	42	72	51	64.8	39.0	40.2
							125	62	120	58	120	70	61	56			
					.68	1.00	105	65	118	68	119	75	47	47	62.8	38.0	39.8
			217		.66	1.00							Mean	56.5	64.2	38.5	39.5
			Cor. 214		.83								Cor.	56.5		39.0	
					.26												
					1.09									Hence			
					Hence $V = 9.55$									$C = 12.5$			
					$v = 3.16 \times 10^{10}$									$N = 102.1$			
31	241	29	211	.25	.94	1.15	55	145	65	150	68	147	85	81	61.2	38.2	39.0
29	243	31	213				55	155	65	149	68	138	95	76			
30	245	28	216		.86	1.09	67	150	68	144	58	148	82	88			
							50	115	40	132	65	115	70	59	7.7	38.6	39.6
							45	132	48	130	52	118	85	72			
					.92	1.12	55	120	52	124	55	121	66	77	59.0	38.0	39.8
			213		.91	1.12							Mean	78.0	59.3	38.3	39.5
			Cor. 210		1.015								Cor.	77.8		38.9	
					.26												
					1.275												
					Hence $V = 11.20$									Hence			
					$v = 2.84 \times 10^{10}$									$C = 13.5$			
														$N = 102.3$			

TABLE IV.

No.	*B*	*I*	*II*	*S*	*V*	*C*	*N*	*2D*	*v*
1	2.08	+	—	240	10.52	10.8	99.0	65.3	2.84×10^{10}
2	"	—	+	240	10.10	10.9	99.4	58.7	3.05
3	"	—	+	192	8.51	12.1	90.6	42.2	2.90
4	"	—	+	214	9.55	12.5	102.1	56.5	3.16
5	"	+	—	210	11.20	13.5	102.3	77.8	2.84
6	"	+	—	202	9.81	13.2	99.1	63.3	2.82
7	"	—	—	174	9.30	11.7	83.5	36.3	2.97
8	2.047	—	+	189	10.35	15.4	97.6	63.7	3.19
9	"	+	—	189	10.83	15.0	77.6	53.2	3.09
10	"	—	—	171	9.30	13.4	89.9	41.0	3.24
11	"	+	+	165	11.40	19.8	86.5	69.1	3.23
12	2.394	—	+	88	10.41	19.1	83.2	29.2	2.93
13	"	+	—	99	10.12	17.6	89.7	33.8	2.75
14	"	+	—	195	9.56	22.0	75.6	63.7	2.83
15	"	—	+	193	9.90	21.2	82.3	68.3	2.87
16	"	—	—	190	9.96	20.6	87.9	66.2	3.00
17	"	+	+	201	10.31	20.8	78.3	62.0	3.17
								Mean :	3.05×10^{10}

istence of the effect sought. The discs were charged oppositely throughout. First was taken a complete set of readings on reversing the switch *S* while the discs were at rest, the remainder of the apparatus running as usual. The discs were then set rotating, both positively, and a second series of readings taken; then one of the belts from the counter shaft to the disc was crossed so that I. rotated +, and II. –, and a third series taken; then the main belt from the motor was crossed so that I. rotated — and II. +, a fourth series taken; finally, the belt from counter shaft to disc was put straight again, so that both discs rotated negatively, and a fifth series taken. In connection with sets 16 and 17 a similar series of readings was made, except that here both discs were charged alike. Here are the observed and calculated values for the deflection for the various cases.

In all of the preceding the surfaces both of the discs and of the condensing plates were uniform. A series of experiments early in the winter had shown that the qualitative effect was the same under these conditions as when the surfaces of the discs were divided into a number of separate sectors, and charged at the periphery by a

TABLE V.

Speeds.		Charged.		Deflection.	
I	*II*	*I*	*II*	Obs.	Cal.
0	0	±	∓	+ .9	0.0
+83.2	+88.9	±	∓	+ 2.0	+ 2.1
+78.4	−73.4	±	∓	−63.7	−60.1
−79.3	−74.2	±	∓	+ 2.3	+ 2.1
−83.0	+81.9	±	∓	+68.2	+65.0
0	0	±	±	+ .1	0.0
−86.9	−88.8	±	±	−66.2	−68.2
+88.0	−89.6	±	±	− 1.0	− .4
−88.0	+89.6	±	±	+ 2.0	+ .4
−78.3	+78.3	±	±	+62.0	+66.0

point. To see if the quantitative effect was also the same, the following experiment was performed. The conducting surfaces of both the discs and the condensing plates were each divided into six sectors by radial scratches .75 mm. wide. The internal edges of the condensing plates were connected by a ring of tin foil 1 mm. wide. The sectors of the discs were connected only by the charging rings on the ebonite cores. If then the effect observed was in any way due to conduction currents in the metal surfaces, this arrangement should certainly greatly alter its magnitude. Observation, however, showed that this was by no means the case. In Table VI. are given the results of four series of measurements. The values of v calculated from these lie well within the range of accuracy of the experiment.

TABLE VI.

No.	*B*	*I*	*II*	*S*	*V*	*C*	*N*	*2D*	*v*
18	23.94	+	—	159	11.99	16.9	90.0	58.7	2.89
19	"	—	+	218	13.60	17.1	81.4	82.0	2.96
20	"	+	+	218	12.45	17.2	76.9	76.7	2.75
21	"	—	—	195	14.00	15.3	106.3	80.3	3.23
								Mean :	2.96

The above results, therefore, show beyond any doubt that *electric convection does produce magnetic action*, or, more exactly, that, within the errors of observation, the current induced in a coil by reversing a convection current in its vicinity, is equal to the current induced in this coil by reversing a conduction current of the same strength as the convection current in a circuit coinciding with the path of the convection current.

The conditions of the experiment above described are still capable of considerable improvement. A number of alterations are to be made next winter, and it is hoped that much more consistent results will then be obtained. Cremieu's recent experiments will also be repeated, with the view of seeing if the explanations above offered will bear experimental test. There also remain a number of questions in regard to the effect of moving electrified bodies still unanswered, at least satisfactorily; for example, the motion of a dielectric in a uniform field; also the reverse of the above experiment. These problems are also to be attacked.

It is a source of great satisfaction to me that the above investigation was practically completed before the death of Professor Rowland. His advice and criticism throughout the work were invaluable.

To Professor Ames also I am indebted for much valuable advice. I also take this opportunity of thanking Mr. N. E. Gilbert for his valuable assistance in the early part of the work, and Messrs. J. T. Barrett and Jas. Barnes for their help in taking the final readings.

EFFECT OF DRAWING ON THE ELASTICITY OF COPPER WIRE.

By J. R. Benton.

WHEN different specimens of the same substance are used for determining elastic moduli, the results vary considerably, so that the values obtained for any one specimen describe the behavior of that specimen only, and do not hold universally for the substance in question. This discordance is explained by the assumption of differences in the physical condition of the various specimens. Such differences are produced in the case of metals by rolling, hammering or drawing, or by varying the manner of cooling from high temeratures.

Determinations of the elastic moduli of metals are most often made on wires. It is therefore especially important to know how the elasticity of wires is affected by the processes used in their manufacture; of these, drawing is the most important.

This article describes some experiments on the changes in elasticity which the process of drawing produces in copper wires. I shall first give a statement of the results obtained from the experiments, and leave till afterwards the discussion of the methods of observation, as they will be of less interest to the general reader.

Statement of Results.

The following are the principal facts that have come to light in the experiments:

1. If a copper wire is first annealed by heating electrically[1] to redness and is then drawn, its modulus of rigidity decreases, and Young's modulus increases.[2]

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[1] It has been shown that the passage of an electric current through copper wire does not influence Young's modulus, excepting in so far as it heats the wire. It is probably safe to assume further that the current has no specific effect on the modulus of rigidity. See M. C. Noyes, Phys. Rev., 3, 432–447, 1896.

[2] The increase of Young's modulus in drawing copper wire was observed by Wertheim, Compt. rendus, T. XV., p. 110; Pogg. Ann. Ergänzungsband, II., pp. 1–70.

2. Each successive drawing causes a further diminution of the modulus of rigidity, and a further increase of Young's modulus; for both moduli the amount of change becomes less with each drawing.

3. If finally the wire is annealed after having been drawn several times, the modulus of rigidity returns to a value considerably greater than its original value, and Young's modulus to one considerably less than its original value.

The numerical results for two pieces of wire taken from the same coil are given in Table I.

TABLE I.

Treatment of Wire.	Modulus of Rigidity in Dynes per sq. cm.	Young's Modulus in Dynes per sq. cm.	Diameter in cms.
	$10^{11}\times$ (Wire A.)	$10^{11}\times$	
Annealed and drawn once.	4.017 ± 0.010		0.15042
Drawn 2 times. . . .	3.946 ± 0.013		0.13914
" 3 " . . .	3.919 ± 0.011	13.87 ± 0.14	0.13060
" 6 " . . .	3.876 ± 0.011	13.90 ± 0.13	0.11220
" 9 " . . .	3.863 ± 0.008	14.20 ± 0.12	0.09410
Annealed again . . .	4.322 ± 0.008	11.90 ± 0.12	0.09318
	$10''\times$ (Wire B.)	$10''\times$	
Annealed.	4.177 ± 0.012	12.82 ± 0.14	0.16174
Drawn once. . . .	4.015 ± 0.011	13.21 ± 0.14	0.15082
Drawn 3 times .	3.961 ± 0.010	12.90 ± 0.13	0.13662
" 5 " . .	3.948 ± 0.008	13.26 ± 0.13	0.12298
" 7 " . . .	3.945 ± 0.008	13.40 ± 0.12	0.11238
" 9 " . .	3.897 ± 0.008	13.32 ± 0.12	0.10012
Annealed again . . .	4.305 ± 0.008	11.09 ± 0.12	0.09856

The values given for Young's modulus hold for isothermal extension, while those given for the modulus of rigidity hold for adiabatic torsion.

The results are shown graphically in Figs. 1, 2, 3 and 4. The vertical line through each plotted point in the diagrams gives the limits of probable error for the corresponding determination.

It is probable that the difference in elastic properties between the wires annealed before drawing and those annealed after drawing is due to the fact that the latter cooled from red heat more rapidly on account of their smaller diameter, and were therefore more completely annealed.

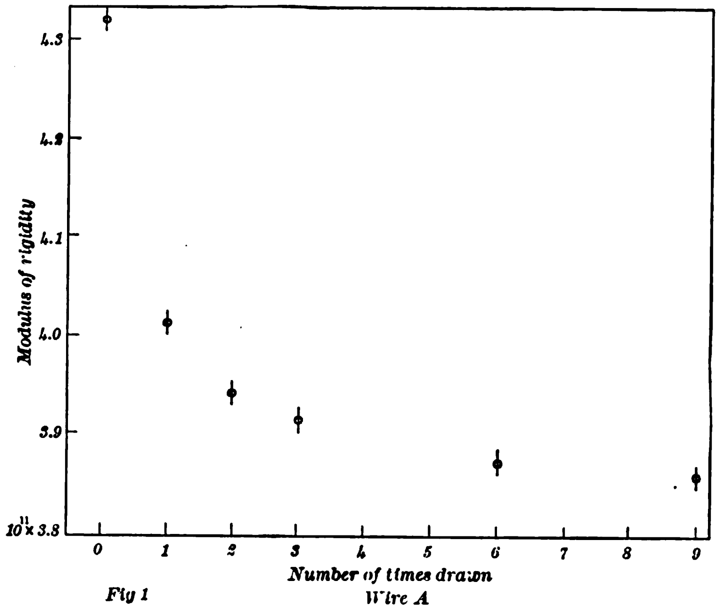
Modulus of rigidity
4.3
4.2
4.1
4.0
3.9
10¹¹ × 3.8
0
1
2
3
4
5
6
7
8
9
Number of times drawn
Wire A
Fig 1

Fig. 1.

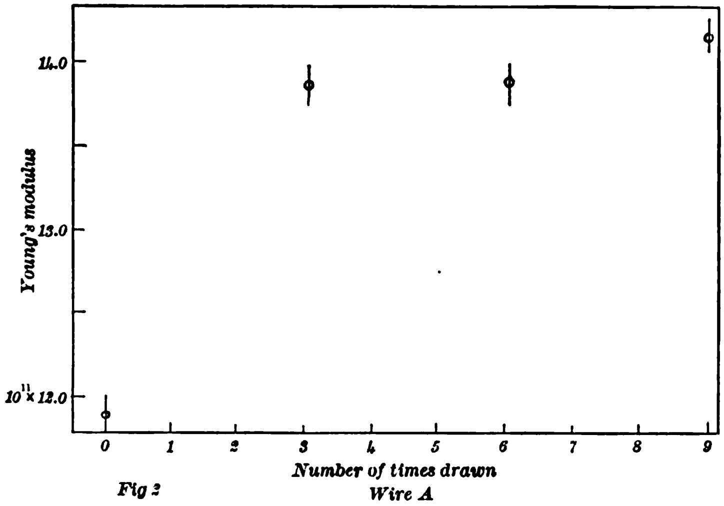
Young's modulus
14.0
13.0
10¹¹ × 12.0
0
1
2
3
4
5
6
7
8
9
Number of times drawn
Wire A
Fig 2

Fig. 2.

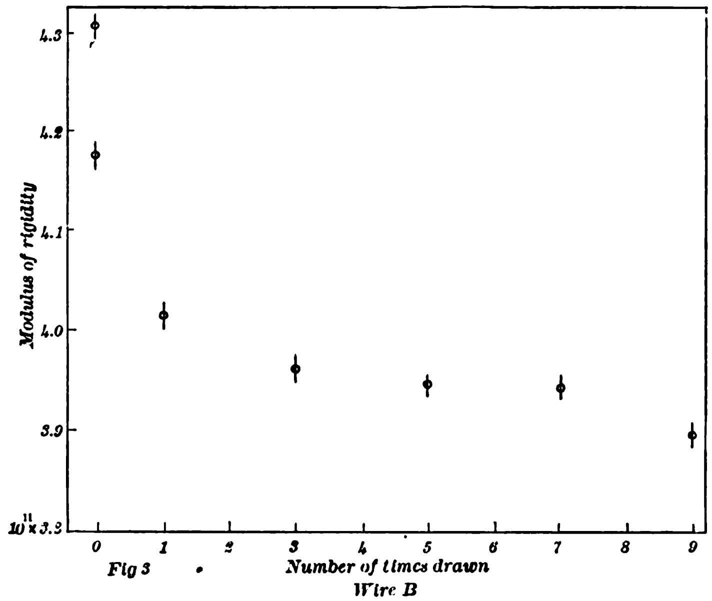
Modulus of rigidity
4.3
4.2
4.1
4.0
3.9
$10^{11}\times 3.8$
0
1
2
3
4
5
6
7
8
9
Fig 3
Number of times drawn
Wire B

Fig. 3.

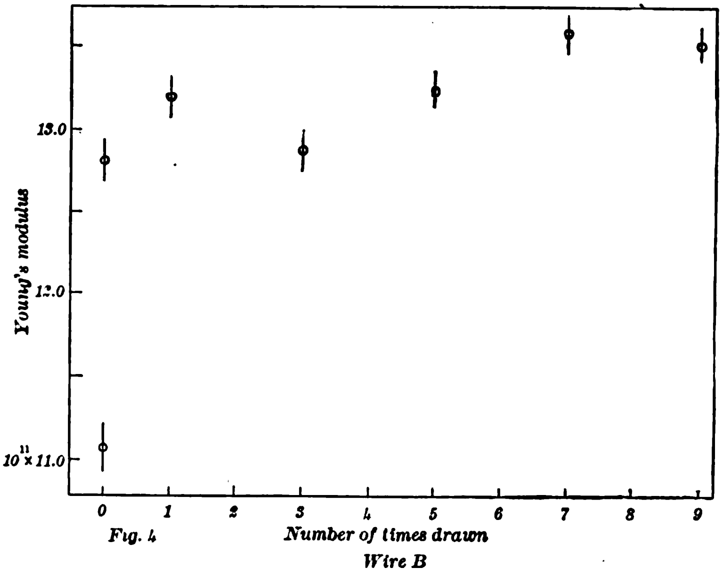
Young's modulus
13.0
12.0
$10^{11}\times 11.0$
0
1
2
3
4
5
6
7
8
9
Fig. 4
Number of times drawn
Wire B

Fig. 4.

It is a matter of common observation that drawn copper wires offer much greater resistance to bending than annealed ones. This would be explained by an increase in Young's modulus. But the change in Young's modulus, as seen in Table I., is not sufficiently great to account for the very considerable increase in resistance to bending. We have no right, however, to assume that the change in Young's modulus is uniformly distributed over the cross-section of the wire. If, for example, it should happen that the modulus increased for the outer part of the cross-section, but decreased for the inner part, we might have no difference in the elasticity of extension, but would nevertheless have an increase in the elasticity of bending.

These considerations seemed to make it worth while to determine Young's modulus for the same wires by the method of bending. The results are given in Table II. Contrary to expectation the values obtained by flexure were considerably less than those obtained by extension. This would seem to ndicate (though it would by no means prove) that Young's modulus increases more in the interior of the wires than near the surface. It is to be noticed that the values obtained by the two methods very nearly agree for the annealed wire. The greater stiffness of drawn wires as compared with annealed wires must then be due simply to the increase in the *limit* of elasticity. The flexural elasticity itself increases only slightly in drawing.

TABLE II.

Young's modulus determined by bending. (*Wire B.*)

Treatment of Wire.	Young's Modulus in Dynes per sq. cm.
Annealed and drawn once.	$10^{11} \times 11.23 \pm 0.12$
Drawn 3 times.	11.79 ± 0.12
" 5 "	11.88 ± 0.12
" 7 "	11.83 ± 0.12
" 9 "	12.08 ± 0.12
Annealed again.	10.82 ± 0.12

DETERMINATION OF THE MODULUS OF RIGIDITY.

In determining the modulus of rigidity the following method was used: A wire (*a*) about 235 cm. long was fastened at the top to a firm bracket (*b*), the end of the wire being tightly clamped between steel plates. On the lower end of the wire was clamped a rod (*c*), the ends of which were fastened so as to prevent torsion of the lower end of the wire. At the middle of the rod was a hook, from which weights (*d*) could be suspended so as to make the wire hang straight. Exactly half way between *b* and *c* a brass disk (*e*), 15 cm. in diameter, and weighing about 1 kg., was fastened. On the brass disk two lead cylinders (*f*, *f*) (4.7 cm. in diameter; 2.5 cm. in height) could be laid. These cylinders were always laid in exactly the same position; this was attained by means of small pegs (*g*) which projected from the disk *e*, each cylinder being brought into contact with the proper pegs.

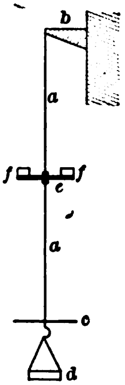

Fig. 5.

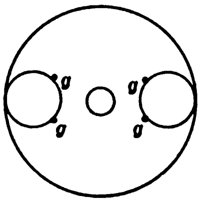

Fig. 6.

The method of observation was as follows: the period of torsional vibration was determined when the lead cylinders were in place; then the lead cylinders were replaced by hollow brass cylinders of the same dimensions externally, and the period was again determined. The purpose of the brass cylinders was simply to insure that the resistance of the air to the motion of the disk should be the same for both determinations. The period of swing in the two cases, and the difference in the moment of inertia, together with the dimensions of the wire, give the necessary data for determining the modulus of rigidity.

The modulus of rigidity *F* can be calculated from the formula

$$F = \frac{4\pi l' K}{r^4(t_2^2 - t_1^2)}, \qquad (1)$$

where l' is the length between the disk and either end of the wire, K is the difference in moment of inertia due to substituting the lead cylinders for the brass ones, r is the radius of the wire, and t_2, t_1, are the periods with the lead cylinders and the brass cylinders, respectively. If we put $\sqrt{\frac{m}{\pi l \sigma}}$ in place of r, where m is the mass of wire, l is its total length, and σ its density, the formula becomes

$$F = \frac{4\pi^3 l^2 \sigma^2 K l'}{m^2(t_2^{\,2} - t_1^{\,2})}. \tag{2}$$

Of the quantities in this formula, l and l' were determined by direct measurement with a meter scale; their errors were taken as not greater than 0.2 cm., and 0.1 cm. respectively, or about 0.1 % in each case. σ was determined by weighing the wire in air and in water. One determination was made for each wire experimented on, and the mean was used, being 8.849 for the annealed wires, and 8.840 for the drawn wires, the probable error being in each case ± 0.005, or about 0.06 %. If any change in density took place after the first drawing of the wire, it was within the limit of the experimental errors.[1] K was calculated from the dimensions and masses of the two lead cylinders and the two brass cylinders. Its value in C.G.S. units was 21210 with a probable error not greater than 17, or about 0.08 %. m was determined by weighing the wire; its probable error was taken as not greater than 0.0003 gr., or about 0.001 %. t_1 and t_2 were determined by observing the time when a mark on the disk swang past its position of rest at every fifth swing for about 250 swings; by properly combining these observations the period of the disk could be determined with a probable error of 0.0006 sec., or 0.01 % to 0.03 %. This gives for $(t_2^{\,2} - t_1^{\,2})$ a probable error not greater than 0.1 %. In determining the period an ordinary watch was used, with which it was possible, after practice, to read to the nearest fifth of a second. The combination of these errors gives the probable error of F between 0.2 % and 0.4 %.

The formula used assumes (1) constancy of external conditions, (2) homogeneity of the wire, (3) perfect firmness of the supports, and (4) that the wire has the shape of a circular cylinder. The possible imperfect fulfillment of these assumptions leads to the consideration of the following possibilities of systematic error:

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[1] Marchand and Scheerer state that the density of copper wires is increased by drawing. The wire for which they found this result had density between 8.895 and 8.896, and must therefore have been of different copper from the wires I used. Moreover, the changes due to drawing appear only in the third decimal place, and are so irregular that their conclusions appear hardly justifiable. Marchand and Scheerer, Journal für praktische Chemie, 27, p. 193. 1842.

1. As regards external conditions: (*a*) The temperature at which the different determinations were made varied between 19° and 23° C. The variation of the rigidity modulus between these temperatures is within the limits of error of my determinations, as shown by the experiments of Kohlrausch and Loomis,[1] of Pisati,[2] and of Katzenelsohn.[3] (*b*) The weights hung on the wire were not always the same, so that the condition as regards longitudinal stress varied between 4.8×10^8 and 11.2×10^8 dynes per square cm. The variation in modulus of rigidity due to this cause is also within the limits of experimental error, as shown by my work on the dependence of modulus of torsion on tension.[4] The condition of the wire as regards torsion of course varied during each swing, but this had no appreciable effect, as shown by the fact that the swings were sensibly isochronous. The amplitude was always kept less than 10°.

2. Homogeneity of the wires can not be assumed, so that the values obtained for F must be taken as integrated values for the cross section of each wire. Probably the copper itself was in fact not homogeneous; besides that there was always a film of oxide on the surface of the wire, which could not be entirely gotten rid of. After annealing, the surface was cleaned with HCl, which was then washed off with water, leaving the surface perfectly clean; but it soon became tarnished by exposure to the air.

3. Suppose the body to which the wire is clamped at the bottom not to be perfectly firm, the top being perfectly firm; then the motion of the body at the bottom will approximate to that of a body of very great moment of inertia, hung on to the bottom of the wire, and free to twist with the wire. The motion of such a system will be given by the equations

$$\left.\begin{aligned} \frac{d^2\theta}{dt^2} &= -\frac{M}{K_1}\left[\theta + (\theta - \varphi)\right] = \frac{M}{K_1}(\varphi - 2\theta) \\ \frac{d^2\varphi}{dt^2} &= +\frac{M}{K_2}(\theta - \varphi) \end{aligned}\right\} . \qquad (3)$$

where M is the direction-force of the wire, K_1 and K_2 are the moments of inertia of the disk and the body at the bottom respectively, and θ and φ are the respective angular displacements. The elimination of φ gives

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[1] Kohlrausch and Loomis, Pogg. Ann., 141, 1870.

[2] Pisati, Nuovo Cimento, V., pp. 145–150.

[3] Katzenelsohn, Beiblätter der Physik, 12, p. 307.

[4] PHYS. REV., Vol. XII., pp. 100–114, 1901.

$$\frac{d^4\theta}{dt^2} + \left(\frac{M}{K_2} + 2\right)\frac{d^2\theta}{dt^2} + \frac{M}{K_1K_2}\theta = 0. \tag{4}$$

If initial conditions $\theta = 0$, $\varphi = 0$, $\frac{d\theta}{dt} = \omega$, and $\frac{d\varphi}{dt} = 0$, are assumed, the solution of this equation takes the form

$$\theta = c_1 \sin c_2 t + c_3 \sin c_4 t \tag{5}$$

where c_1, c_2, c_3 and c_4 are constants depending on M, K_1, K_2 and ω, and c_3 is very much smaller than c_1 if K_2 is very much larger than K_1. This shows that the average period taken for a large number of swings will not be affected by a slight lack of rigidity of the support; for $c_3 \sin c_4 t$ will be negative as often as it is positive, and the disk will therefore swing through its position of rest too soon as often as it will do so too late.

4. As regards the shape of the wire: (*a*) It is safe to assume that its cross section is the same at all points of its length, as the die-plate through which the wire is drawn is of very much harder material than the wire, and would not wear away appreciably during the drawing of two meters of wire. (*b*) The wire is always more or less bent, so that its axis not a straight line, but a curve of double curvature. That such deviation from straightness had no appreciable effect in my experiments, I proved empirically by purposely bending and kinking a wire and then determining the period of torsional swing, which was found not to differ from that of the straight wire. (*c*) It is possible that the cross section may not be perfectly circular. If this is the case, then as a first approximation it can be considered elliptical. Then $\frac{1}{r^4}$ of formula (1) must be replaced by $\frac{1}{2}\frac{1}{ab}\left(\frac{1}{a^2} + \frac{1}{b^2}\right)$, where a and b are the semi-axes of the ellipse. If we set $a = r + \delta$ and $b = r - \delta$, we find that the difference between $\frac{1}{r^4}$ and $\frac{1}{2}\frac{1}{ab}\left(\frac{1}{a^2} + \frac{1}{b^2}\right)$ is of the order of δ^2. The error in assuming that the

cross section is circular will be less than 0.1 per cent. if δ is less than $\frac{1}{32}r$; and δ was less than $\frac{1}{32}r$ for all the wires I used, as shown by micrometer calipers; so that this source of error need not be feared.

Determination of Young's Modulus.

To determine Young's modulus the sinking of the lower end of the wire (see Fig. 5) under a known increase of load was read with a micrometer. The lower end of the wire was left free, except that it was prevented from swinging to the side by means of three strings at right angles to the wire, so arranged that they did not interfere with its vertical motion. Young's modulus was given by the formula

$$Y = \frac{sgl}{\pi r^2 e}, \tag{5}$$

where s is the mass of the weights added, g the acceleration of gravity, l the length of the wire, r its radius, and e the increase in length due to the weights added. If we put $r = \sqrt{\frac{m}{\pi l \sigma}}$, where m is the mass of wire, and σ its density, we have

$$Y = \frac{sgl^2\sigma}{me}. \tag{6}$$

The method of determination and the errors of l, σ, and m have already been discussed (see p. 240). s was determined within an error of 0.01 per cent. by direct weighing. To determine e, a succession of readings with the micrometer was taken as the wire was successively loaded and unloaded. The mean values were used; a correction had to be made for the sinking of the upper support, which was measured by the micrometer. The probable error of the final value of e was ± 0.0007 cm., or from 0.4 per cent. to 1 per cent. The combination of these errors gives the probable error of Y between 0.5 per cent. and 1 per cent.

The formula (6) assumes (1) constant external conditions, (2) homogeneity of the wire, (3) cylindrical shape of the wire and (4) the applicability of Hooke's law.

1. The temperature varied between 19° and 23° C. The varaition of Young's modulus within these temperatures is negligible, according to the experiments of Katzenelsohn.[1] The fact that the values obtained are those for isothermal extension has already been noticed.

2. Homogeneity of the wire can not be assumed; the values of Y are to be regarded as average values for any cross section of the wire.

3. The shape of the cross section or the wire is immaterial, as that is eliminated in formula (6). The cross section must, however, be the same at all points of the length of the wire; this can be assumed to be the case with a drawn wire. It is possible that the wire may deviate from straightness; if so, it is to be expected that the wire will be partially straightened by increasing the load, so that the lower end will sink more than if only extension took place. Suppose the half length of the wire to be denoted by λ and suppose the middle of the wire to be at a distance δ from the straight line joining its extremities. If a vertical force W be applied to the end of the wire, the horizontal force at the middle tending to straighten the wire will be $W\cdot\frac{\delta}{\lambda}\cdot$ If the wire is made perfectly straight, its lower end will sink through a distance $2\delta\cdot\cos\cos^{-1}\frac{\delta}{\lambda} = 2\lambda\left(\frac{\delta}{\lambda}\right)^2\cdot$ As the wire will not be perfectly straightened by any vertical force, however great, the sinking of the lower end will be less than $2\lambda\left(\frac{\delta}{\lambda}\right)^2$ and the force acting to produce it will be $W\frac{\delta}{\lambda}$, so that the sinking produced will be of the third (or higher) order of small quantities if $\frac{\delta}{\lambda}$ is of the first order.

4. With the stresses used some slight deviation from Hooke's law is to be expected, according to the experiments of J. O. Thomson.[2] The limits of stress between which my determinations of Young's modulus were made, were approximately the same for all the wires, so that the results will serve for purposes of comparison.

[1] Katzenelsohn, Beiblätter der Physik., 12, 307.

[2] J. O. Thomson, Wied. Ann., 44, 555, 1891.

DETERMINATIONS OF YOUNG'S MODULUS BY FLEXURE.

The determinations of Young's modulus by bending were made by laying a piece of the wire across two knife-edges which were rigidly braced at a distance of 7.22 ± 0.01 cm. from each other. Exactly midway between the knife-edges a light scale-pan was suspended, on which weights were laid, and the sinking of the middle of the wire was measured with a micrometer. The weights were so arranged that the sinking always amounted to about 0.08 cm. The sinking of the supports was subtracted; and any possible rolling of the wire on the knife-edges was prevented by bending down one end of the wire and letting it rest against a nail, friction being minimized by lubrication. For these experiments the wires were made as straight as possible; but perfect straightness is extremely difficult to attain without subjecting the wire to processes (such as drawing or hammering) which would change its elastic properties. It is obvious that slight deviation from straightness will cause but slight error. No rigorous solution of the problem of the bending of a rod of any initial shape has yet been made.

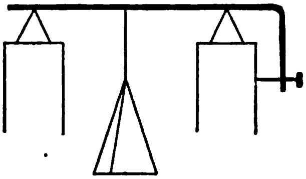

Fig. 7.

The value of Young's modulus (Y_b) is calculated from the formula

$$Y_b = \frac{b^3 g P}{12 \pi r^4 h}, \qquad (7)$$

where b is the distance between the knife-edges, g is the acceleration of gravity, P the mass of the weights hung on, r the radius of the wire, and h the depression of the middle.

The probable errors of these quantities are as follows:

b,	± 0.01 cm.	or	± 0.2 per cent.
P,	± 0.1 mg.	or	± 0.0001 per cent.
r,	± 0.00005 cm.	or	± 0.1 per cent.
h,	± 0.0006 cm.	or	± 0.6 per cent.

giving for the probable error of the result ± 1 per cent.

PHYSICAL LABORATORY, PRINCETON UNIVERSITY.

THE ABSORPTION SPECTRUM OF COLLOID FERRIC HYDRATE.

By B. E. Moore.

IN a recent article upon "A Spectrophotometric Study of the Hydrolysis of Ferric Chloride,"[1] I have pointed out that the probable final product of hydrolysis, $x(FeO_3H_3)$, could be advantageously compared with Graham's colloid ferric hydrate. For, if two solutions of the same base should have identical absorption spectra, it would be natural to assume the same structure of the molecule.

A solution of colloid ferric hydrate was prepared according to Graham's direction. After dialyzing for five weeks, the solution was kindly tested by Mr. R. W. Thatcher who found it to be .1116 normal with respect to iron and that 90.4 per cent. of the chlorine had been removed. From a sample of this solution I prepared a dilute solution, .002462 normal, and another solution of the same concentration from the stock of ferric chloride made twelve months previous. These solutions were placed in two absorption cells of identical absorption coefficients. One cell was placed before each slit,[2] and ten readings of the slit were taken at a definite point in the spectrum. The cells were then interchanged, and ten new readings taken. Similar readings were taken at other points in the spectrum. A comparison of these sets of readings gives double the relative difference of absorption. This method was particularly good at points in the spectrum where absorption differences were small. When the absorption differences were large, in one position the slit must be opened very wide, which necessitated its abandonment. However, the absorption spectra of each could be determined separately as in the previous contribution. This method was also adopted at all points in the spectrum as a check. The ferric chloride solution had been hydrolizing for eight days at room temperature and a comparison of its absorption spectrum with the curve for the same concentration in the previous article shows that the hydrolysis would proceed a little farther. However, the general character of the two spectra is the same. But there is a marked difference between the absorption spectra of these solutions and the absorption spectrum of colloid ferric hydrate, as is shown by the following table of data and curves. The character of the curves is not the same. The ferric chloride solution absorbs more throughout the spectrum, but

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[1] Physical Review, Vol. 12, p. 151.

[2] Physical Review, Vol. 12, p. 160.

TABLE.

Transmission in per cent.

	Colloid Ferric Hydrate.			Hydrated Ferric Chloride.
	$n = .002462$.	$n = .001231$.		
	Observed.	Observed.	Calcu.[1]	
6500	89.3	93.6	94.5	82.1
6000	87.6	92.5	93.6	75.2
5750	82.5	89.3	90.8	61.5
5500	71.5	82.	84.5	40.5
5250	41.3	66.8	64.3	19.8
5000	13.25	34.1	36.4	9.4

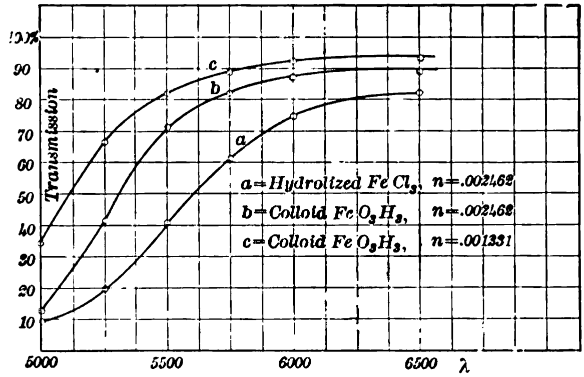

Fig. 8.

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[1] Calculated from formula colog $i_2 = -\frac{\text{colog } i_1}{2}$, where the intensities, i_1 for $n =$ .002462 are assumed correct.

relatively more in the red. The curve for the colloid absorption is much steeper as it approaches the region of strong absorption so that it would intersect several of the previous curves[1] for solutions of the hydrolized ferric chloride. This fact excludes the assumption of the same product but diluter solution. It leaves us two other assumptions. We may assume with Antoni and Giglio, that in the process of transformation there is a combination of iron, chlorine and hydroxyl; or we may assume there is present an appreciable amount of crystalloid ferric hydrate in the colloid hydrate prepared by dialysis. Small quantities of the crystalloid are expected to be present in the colloid solution but the difference in the absorption spectra would necessitate the assumption of larger quantities of the crystalloid than I had anticipated. However, for reasons previously given, I think the latter view more tenable than the first assumption. The nine per cent. chlorine is not responsible for the difference, since in the hydrolized ferric chloride there may be as much as thirteen per cent. of the ferric chloride not transformed.

I now prepared a dilute solution of colloid ferric hydrate and replaced 5 cu. cm. of 90 cu. cm. of water with 5 cu. cm. of strong chlorine water, and compared this solution at different points in the spectrum with the previously prepared solution of colloid ferric hydrate. Tests made at different times during two weeks' interval showed no difference whatsoever. A solution of hydrochloric acid whose density was 1.163 at 25° C. was diluted 100 fold and 5 cu. cm. of this diluted acid replaced 5 cu. cm. of water as before. At different times for two weeks, this solution was compared with the two previous ones and at no time was there a difference observed which could not be attributed to working conditions. That is, neither the chlorine caused the formation of any chlorine products nor the hydrogen effected any transformation of the possible crystalloids. The introduction of 5 cu. cm. of Graham's colloid ferric hydrate into a fresh prepared solution of ferric chloride accelerated the hydrolysis[2] so that in two hours the solution was as dark as the older solution of the same concentration had become upon standing for four hours. Next, 5 cu. cm. of an old solution of hydrolized ferric chloride was added to a new solution and at the end of two hours, the mixed solution was as dark as the old solution had become at the end of 5.5 hours. All the solutions in this test were of the same concentration, viz.: .002462 normal. The action indicates that the accelerators are the same in the two cases but that there is a larger quantity of it present in the hydrolized ferric chloride than there is in the dialyzed ferric hydrate. The two were of the same concentration as to iron and the slower acceleration of the Graham's colloid ferric hydrate can be readily explained upon the assumption previously made of appreciable quantities of inactive crystalloid. hydrate. The absorption curves then suggest that the colloid absorption spectrum lies between the hydrolized ferric hydrate and the ferric chloride spectra.

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[1] Previous contribution, Fig. 5, p. 175.

[2] Goodwin, Phys. Rev., Vol. 11, p. 193.

PHYSICAL LABORATORY,
UNIVERSITY OF NEBRASKA, July, 1901.

THE ABSOLUTE MEASUREMENT OF SELF-INDUCTANCE.

By William Duane.

THE author has found the following to be a relatively easy method of measuring the absolute value of the self-inductance of a coil without using a standard of self-inductance or of capacity.

Suppose the coil is wound in two parts. Let the mutual inductance of the two parts be M; the self-inductance of one be L_1; and, of the other L_2. Then, if the current flows in the same direction through both coils in series the self-inductance of the whole is

$$L' = L_1 + 2M + L_2. \tag{1}$$

If the current flows in opposite directions through both coils in series the self-inductance of the whole is

$$L'' = L_1 - 2M + L_2. \tag{2}$$

In the first part of the method the ratio of L' to L'' is measured by comparing each with a third self-inductance L, which should be of the order of magnitude of L' and L''. Any of the methods of comparing self inductances can be used. The measurement gives us

$$L' = k_1 L$$

$$L'' = k_2 L,$$

and therefore

$$\frac{L'}{L''} = \frac{k_1}{k_2} = A, \tag{3}$$

where A is a known quantity.

In the second part of the method the value of the mutual inductance M is measured. This can be done by any of the usual methods (for example the earth inductor and ballistic galvanometer, or the doubly wound solinoid and ballistic galvanometer method) and is a comparatively easy measurement. From equations (1) and (2) we have

$$L' - L'' = 4M. \tag{4}$$

From (3) and (4)

$$L' = \frac{4MA}{A - 1} \tag{5}$$

and

$$L'' = \frac{4M}{A - 1} \tag{6}$$

which gives us L' and L'' in terms of measured quantities.

In order to use the above method it is necessary to be able to break the circuit of the coil at some point inside of it. This can be done best during the winding, and the author suggests that all coils that are to be used as standards of self-inductance be wound in two sections, so that their values can be easily checked from time to time.

It is not desirable to break the coil in such a way that the mutual inductance is small, for in that case L' and L'' would be very nearly equal, A would not differ much from unity, and any errors in the measurement of A would be greatly magnified when it is substituted in the formulas (5) and (6).

On the other hand it is not desirable (at least when the coils are circular, such as are often made and sold as standards) to break the coil in the middle, for then $M = L_1 = L_2$ approximately and L'' would be very small. The best method of winding lies between these two, the exact division of the coil depending on the relative accuracy of the methods used to measure A and M.

If the coil to be measured is not wound so as to be broken conveniently, its self-inductance can be measured by means of the above method slightly modified. Let L_1 be its self-inductance. Join in series with it another coil of self-inductance L_2, which should be of the same order of magnitude as L_1 and place the two near together so that their mutual inductance is M. Then, considering the two coils as one circuit, measure its self-inductance,

$$L' = L_1 + L_2 + 2M$$

as above.

We have from this equation

$$L_1 + L_2 = L' - 2M = B \tag{7}$$

a determined quantity.

It is now necessary only to measure the ratio of L_2 to L_1, in order to determine their values.

We have

$$\frac{L_2}{L_1} = C \tag{8}$$

a measured quantity. Hence from (7) and (8)

$$L_1 = \frac{B}{1 + C}$$

and

$$L_2 = \frac{BC}{1 + C}.$$

The above remarks as to the best relative magnitudes of L_1 and L for accurate work apply to this case, also.

It is necessary of course that the two coils should retain their relative position during the entire measurement of L_1, otherwise the mutual inductance M would vary.

The members of my class in electrical measurements have tested the above method, and obtained very consistent results. It gives me great pleasure to thank them for the care with which they have performed the tests.

HALE PHYSICAL LABORATORY,
UNIVERSITY OF COLORADO.

NEW BOOKS.

The Fifth and Sixth Books of Euclid. By M. J. M. HILL, F.R.S., Professor at University College, London. Cambridge University Press.

Many of the "Euclids" retain a little of the form of the real Euclid, but replace its rigorous logic by reasoning which often fails to be conclusive. The present work is a refreshing contrast to these in spirit and methods; the proofs are in general not open to question and where an assumption is necessary there is no attempt to get around it by specious reasoning, but the statement that it is an assumption is plainly made. Especially in the first pages, where a foundation is laid for the work which follows, is the desire of the author for rigorous methods apparent.

The book is intended as an elementary text-book, covering the ground of the fifth and sixth books of Euclid; its object, as the author expresses it, is "to remove the chief difficulties felt by those who desire to understand the sixth book." The fifth book is made subservient to this object and there is no attempt in it to follow Euclid; the method of treatment employed dispenses with ratio except as a secondary consideration, and is new in the sense of now being published for the first time in text-book form. The attempt of the author to lead up to the sixth book by an easier path than that of ratio is undoubtedly successful, yet the work is hardly of a grade for beginners, the reason being that a satisfactory treatment of either ratio or subjects nearly related to it involves reasoning too close to be comprehended without preparation.

One of the chief difficulties of ratio to the student is that the definition does not tell him what the thing is, and, although he may have an idea of it, the lack of definiteness makes him feel that he is dealing with something elusive. Professor Hill ingeniously avoids this difficulty by using for the foundation of his work a substitute for ratio called the "relative multiple scale"; this is defined with exactness and is easy of comprehension, yet serves in the sixth book as well as ratio; it is indeed closely related to ratio, the sameness of two "scales" leading to the same results as the sameness of two ratios. The "multiple scale" may be described briefly as a device for showing graphically whether the multiple r of the magnitude A is greater than, equal to, or less than the multiple s of the magnitude B.

The first pages of the book are devoted to an explanation of the "multiple scale" and to proving some of its properties. Then certain applications are taken up and the word ratio is introduced cautiously by means of dual statements of propositions, "ratio" being used in the secondary form. A little later a chapter is devoted to ratio in which it is pointed out that the earlier propositions regarding the multiple scale correspond to ideas of ratio already held by the student. Thus he is "led to the idea that two magnitudes of the same kind determine not only a relative multiple scale but also a ratio." That is, the student gains a considerable empirical knowledge of ratio although the proofs rest strictly upon the multiple scale.

It is unfortunate that the author has made a few exceptions to the rule that the multiple scale should be fundamental and ratio subsidiary; in some of these the difference consists chiefly in the form of statement and but little harm is done beyond the marring of the consistency of the book; in one or two, however, as in dealing with duplicate ratio, propositions are made to depend on ratio alone—to this the objections on the ground of mathematical rigor are more serious. In judging of them, however, the limitations imposed by the fact that the book is a text-book and the utter lack of rigor in most text-books should be remembered.

As in all elementary mathematical works assumptions have been necessary, and the author has frankly pointed out the more important ones to the student; his sound position in this is not so common that it should be passed over without comment.

WENDELL M. STRONG.

NEW YORK, August, 1901.

Leitfaden der Wetterkunde. By Dr. R. BÖRNSTEIN. 8vo, pp. vi + 181. Braunschweig, Vieweg und Sohn, 1901.

This is an elementary text book-on meteorology but it contains many matters of interest to the physicist. The author defines meteorology in his opening chapter as the physics of the atmosphere. He takes up first of all the composition of the air, and then proceeds to consider in turn the six meteorological elements, *temperature*, *moisture*, *cloudiness*, *precipitation*, *barometric temperature* and *wind.* The daily and yearly temperature range in various localities is briefly discussed and the influence upon the same of radiation from the soil, of neighboring bodies of water and of ocean currents such as the gulf stream are shown by means of well-selected curves, the data for which are chiefly taken from German sources.

The chapter on clouds is particularly interesting. It is illustrated by means of a set of colored photographic plates of great beauty which serve better than any description to enable the layman to identify and recognize the various types of clouds. Here as elsewhere throughout the volume the treatment of even important topics is of extreme brevity; to the methods of studying the movements and the heights of clouds, for example, a single paragraph is given. Frequent citations of original sources are made, by means of consecutively-numbered references, to a bibliography which is gathered in a separate section at the close of the volume. This list contains 206 titles and includes many of the most important memoirs of the science.

The chapter on precipitation deals with dew, various forms of frost, and with rain, snow and hail. Here as elsewhere the statistical portion of the subject is reduced to a minimum. The author makes use of it, indeed, chiefly to illustrate such general principles as may be regarded as well established concerning the influence of the sea, of mountain chains and of forests.

The discussion of these six elements occupies the first 94 pages of Professor Börnstein's book. It is followed by a somewhat fuller treatise upon the weather, which is considered as the result of the mutual action of these meteorological elements. In this chapter movements of the atmosphere, with the attendant changes of pressure which accompany storms, are discussed at considerable length. The data are chiefly from European sources. Under the head of weather-bureau service the systems employed in various countries are briefly summarized.

In so short a treatise, and especially in one written merely as an outline of the elements of a science, many things must of necessity be left out. The most striking omission in this book is perhaps the failure to describe the numerous ingenious recording devices that have come into use in modern weather-bureaus. The omission is doubtless entirely justifiable, but one is disappointed nevertheless. In whatever he undertakes to discuss, however, the author is admirably simple, concise, clear and accurate.

E. L. N.

Induction Coils, How to Make, Use and Repair Them. By H. S. NORRIE. (NORMAN H. SCHNEIDER.) Second Edition. Pp. xvi + 269. New York, Spon & Chamberlain, 1901.

This new edition of Mr. Schneider's little handbook has in addition to the contents of the original edition, brief descriptions of coils for gas and automobile engines, medical coils, new forms of contact breakers, electric gas-lighting apparatus and batteries. There is a short chapter on wireless telegraphy with diagrams of the arrangement of the transmitting and receiving apparatus, in which several forms of coherer are briefly described. The treatment in this chapter as throughout the book is of the sort termed *practical*, that is to say explicit directions are given by means of which any handy individual can construct for himself the simpler parts of the apparatus described. The chief value of the book consists in the fact that it gives actual specifications for the construction of the various pieces of apparatus described; indicating the number of turns and the size of wire, nature of the insulation and indeed going into the numerous details which are necessary to successful operation and which are rarely to be found in works in which the attention is directed chiefly to the theory and principles of the subject.

E. L. N.

PORTRAITS

To satisfy a demand which has arisen, the publishers have arranged to issue a limited number of copies of the portraits of scientific men, that have appeared from time to time as frontispieces in the Physical Review.

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THE PHYSICAL REVIEW

A Journal of Experimental and Theoretical Physics

CONDUCTED BY

EDWARD L. NICHOLS, ERNEST MERRITT and FREDERICK BEDELL

LXVI. August, 1901

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THE

PHYSICAL REVIEW.

THE DISTRIBUTION OF ENERGY IN THE SPECTRUM OF THE ACETYLENE FLAME.

By George Walter Stewart.

THE published work on the distribution of energy in flame spectra is not very extensive, and some of it is difficult to find, occurring merely as a minor part of a problem considered. As might be expected, the methods and apparatus of the different investigators are quite similar. Lens and mirror spectrometers and also gratings have been used for the production of spectra, and linear thermopiles and bolometers for the measurement of the radiant heat.

Tyndall[1] explored the spectra of illuminating gas and hydrogen flames. He moved a thermopile backward and forward in the spectrum of each flame, and determined the point of maximum intensity, which was found to be farther toward the long wave-lengths with the hydrogen than with the gas flame. Magnus,[2] while working upon the radiation of platinum, found the distribution of energy in the spectrum of a bunsen flame, but his results are not in accord with those of subsequent observers. Langley[3] found the distribution of energy in the flame of illuminating gas, and, from the curve, calculated the radiant efficiency to be 0.024. In a later article[4] he mentions the fact of having also investigated the candle flame, but gives neither curves nor data. Julius[5] obtained the spec-

[1] Tyndall, Annal. der Phys., 124, 1865, p. 36.

[2] Magnus, Annal. der Phys., 124, 1865, p. 476.

[3] Langley, Science, June 1, 1883.

[4] Langley, Phil. Mag., 30, 1890, p. 260.

[5] Julius, Licht- und Wärmestrahlung verbr. Gase; Berlin, Leonh. Simion, 1890.

tral energy curves for flames of hydrogen, carbonic oxide, illuminating gas in bunsen and the ordinary lava tip burners, carbon bisulphide, sulphur, sulphuretted hydrogen, cyanogen, carbonic oxide in oxygen, mixtures of hydrogen and chlorine and hydrogen and bromine, and phosphuretted hydrogen. He also took observations on several parts of the illuminating gas flame. In work upon the emission of gases, Paschen[1] obtained the distribution of energy in the flames of the bunsen and oxyhydrogen flames. He made most accurate determinations of the positions of the bands of CO_2 and H_2O, and these results will be discussed in connection with the work herein described. The results of Rubens and Aschkinass[2] upon the bunsen flame are in agreement with those of Paschen. Lummer and Pringsheim[3] determined the wave-length of the maximum intensity of radiation of several sources of light, and among them the candle and argand burner flames. As it was necessary for them to take observations over merely a small part of the spectra, the only data given are these values of wave-length.

The great value of acetylene, not only for illuminating purposes, but also in experimental research, makes a knowledge of the distribution of energy in its spectrum of considerable importance. In fact, the problem was suggested by a demand which has already arisen in experimental work conducted in this laboratory. In the work here described, a mirror spectrometer and the radiometer of Nichols[4] were used.

I. APPARATUS AND ADJUSTMENT.

1. The Spectrometer.

The spectrometer available was manufactured by Franz, Schmidt & Haensch, Berlin. The mirrors, made by Brashear, had a focal length of 55.7 cm. and an aperature of 9 cm. As it was necessary to keep the radiometer stationary, the mirrow-prism device of Wadsworth was used.[5] Another advantage of this arrangement is that any wave-length measured is always at the angle of minimum deviation.

---

[1] Paschen, Annal. der Phys., 51, 1894, p. 1.

[2] Rubens and Aschkinass, Annal. der Phys., 64, 1894, p. 584.

[3] Lummer and Pringsheim, Verh. der Deutsch. Phys. Gesell., 1899, p. 215.

[4] Ernest Nichols, PHYS. REV., IV., 1897, p. 297.

[5] Wadsworth, Phil. Mag., 38, p. 337.

The fluorite prism used throughout the work was a very clear specimen of the purple variety. It was 2.5 × 3 cm. and had an angle of 38° 59′ 55″.

The arrangement of the spectrometer and radiometer is shown in Fig. 1. *R* is the radiometer; M_1 and M_2 the spectrometer mirrors; *P* the mirror-prism system; *S* the source; and S_1 and S_2 two slits of the same width, 0.477 mm. S_2 is placed directly in front of the radiometer and in the focus of M_2, thus avoiding the use of any lens whatever. In order to prevent stray radiation, numerous shields of blackened tin were constructed, but only those about the spectrometer are shown in the figure. As will be shown later, the arrangement was found quite satisfactory. The shutter operated to give deflections and zero readings was placed at *A*. The radiometer could not be conveniently mounted on the arm of the spectrometer, and although they stood on the same support, there were changes in their relative positions which caused considerable annoyance.

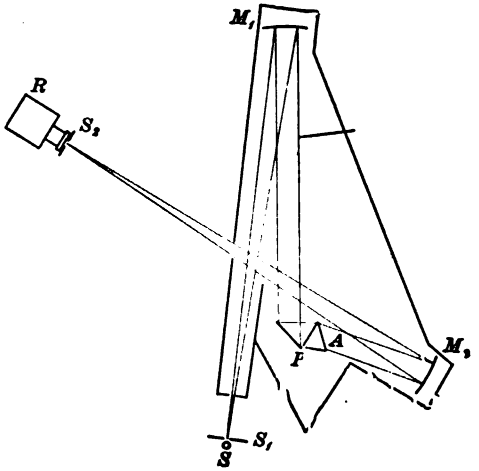

Fig. 1.

The refractive indices used in the computations are due to Paschen.[1]

---

[1] Paschen, Annal. der Phys., 56, 1895, p. 765.

Adjustment on the Sodium Line.—It was necessary to have an accurate method of causing the sodium line, or any other known part of the spectrum, to fall upon the slit S_2. This method had to be not only accurate, but also simple, for, owing to slight changes in the relative positions of the radiometer and spectrometer, it was necessary that the adjustment be repeated quite frequently.

The radiometer was adjusted so that its fluorite window was not quite perpendicular to the incident beam, the reflected light returning in the neighborhood of M_2. The apparatus C in Fig. 2, consisting of a long focus lens and two mirrors, could be lowered into the spectrometer in the region of the reflected beam. The light from the fluorite window would then pass through the lens and be reflected from the mirrors into the microscopic eyepiece D, Fig. 2. Whenever an adjustment on the sodium line was desired, sodium light was produced in front of the spectrometer slit, the prism table was turned until in the proper position for this wave-length, and C was lowered into the spectrometer. The position of the spectrometer was now shifted until the eye at D saw that the image of S_1 coincided with S_2. The radiometer slit was then upon the sodium-line. The error introduced by this method was about 3 seconds of arc as read on the spectrometer, or less than 0.002μ at this point in the spectrum.

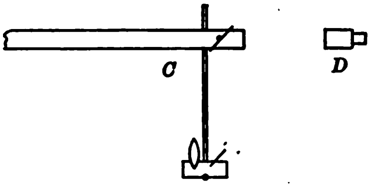

Fig. 2.

2. *The Radiometer.*

(*a*) *Construction.*—There was at my disposal a radiometer which had been constructed according to the description of Nichols.[1]

This instrument, modified to suit the work undertaken, is shown in Fig. 3. The bronze block was 5 × 5 × 10 cm., and the vertical boring 3 cm. in diameter. There were two lateral borings; the one in front of the vanes was 2 cm. in diameter, and the one in front of the mirror, not shown in the figure, was elliptical and longer on the outside. From the crossbar resting on a shelf on the interior just below the top, was hung the vane and mirror suspension. The crossbar fitted loosely in the boring so that the suspension could be shifted horizontally. In front of the vanes was a thin sheet of mica supported by a brass cylinder which fitted in the lateral boring. The window which had been at *a* was discarded, and in place was soldered a brass tube closed at one end, but with an aperture as shown in the figure. A mixture of beeswax and tallow was rubbed upon the ground surface of this partially closed end. A fluorite plate, 4.5 mm. in thickness, was then pressed firmly into place. A piece of mirror glass was sealed over the other window in a similar manner. If the metal surfaces were first slightly warmed, then, when the fluorite and glass were put into place, there resulted joints which were, so far as the writer could observe, perfectly air-tight. For a top, the neck of a round bottle was utilized. Into this neck was ground a glass tube which led off to a stopcock, and thence through two rubber-mercury seals to the mercury pump. The same mixture of beeswax and tallow was used at all four joints at the radiometer. Melted beeswax was poured into the bottle-neck to prevent that joint being disturbed when the top was removed.

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[1] Ernest Nichols, PHYS. REV., IV., 1897, p. 297.

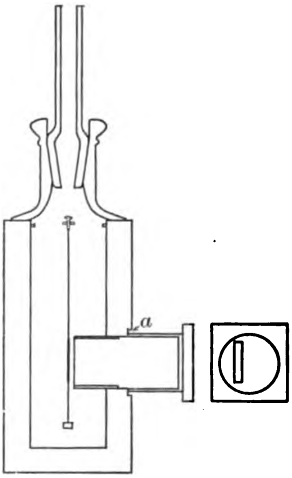

Fig. 3.

The deflections of the mirror were read by the telescope and scale method.

Two different suspensions were used. In both the vanes were approximately 2 × 15 mm., and were fastened together by glass fibers, the distance between them being about 1.5 mm. The mirrors were about 2 × 3 mm. The vanes, mirror and fibers were fastened by shellac.

In the first suspension, which was used in exploring the visible spectrum, the vanes were of thin mica coated with lampblack. A firm black coating was obtained by first dipping the mica into a very dilute alcoholic solution of shellac, and then holding it above a piece of burning camphor. The loose particles were then wiped off gently, and the operation repeated until a coating sufficiently thick was obtained. The weight of this suspension was 4.2 mg. The quartz fiber, by which it was suspended, was so fine that even at the point of maximum sensitiveness the system was aperiodic, and one had to wait five minutes in order to take one reading.

In the second suspension platinum-black was used. While lamp-black and platinum-black are far from being perfect absorbers of long rays, yet the absorption of the latter covers a much wider range than that of the former.[1] According to Kurlbaum,[2] platinum-black is preferable for all experiments, not only because of its better absorption, but also because of its greater conductivity, and above all because a uniform layer is deposited and the degree of absorption can be defined by the data of electrolysis. By determining the ratio of emission of different thicknesses of these two absorbers to that of an ideally black body and assuming Kirchoff's Law, he has found the percentage of radiation which will be absorbed by layers of different thickness. He shows that a layer of platinum-black $190 \frac{mg}{dm^2}$ absorbs almost 97 per cent. and fully as much as a layer of twice that thickness.

The composition of the electrolyte as given by Kurlbaum and as used in this case, is 1 part platinum chloride, 30 parts water, and 0.008 part lead acetate. An attempt was made to deposit the platinum-black upon aluminium foil; but it failed because the aluminium was attacked by the solution. This would not have been wholly unexpected had the writer been aware that what is commonly called platinum chloride is really chlorplatinic acid. Platinum foil then seemed to be the most practicable material for the vanes. Pieces were rolled to the thickness of about 0.01 mm., a thin coating of shellac placed on one side to prevent electrolysis, and the platinum-black deposited. The layer obtained was about $250 \frac{mg}{dm^2}$. The weight of this suspension was about 15 mg. The fiber was not so fine as in suspension number one, and at the point of maximum sensitiveness the time required for a deflection was about 40 seconds.

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[1] Rubens and Nichols, PHYS. REV., IV., 1894, p. 314.

[2] Kurlbaum, Annal. der Phys., 67, 1899, p. 846.

(*b*) *Sensitiveness.*—As is well known, the sensitiveness of the radiometer does not continue to increase with higher vacua, but reaches a maximum. The following curve was taken to show this point, the pressures being read with a McLeod gauge.

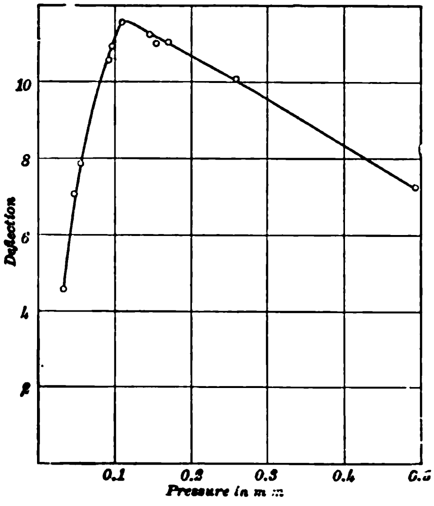

Fig. 4.

Ernest Nichols has found this maximum point to be about 0.05 mm., while our determination is about 0.1 mm. It is difficult to see how this difference can be accounted for in the construction of the radiometers. On the other hand, such a discrepancy might easily be due to the gauges themselves.

During the greater part of the experiment, I had no gauge at my disposal. This was of very little inconvenience as the period of vibration, and also the action of the valves in the pump served fairly well as methods of ascertaining the vacuum.

With suspension number two, an ordinary candle at a distance of 300 cm. gave a deflection of 103 mm. on a scale 63 cm. from the radiometer vanes. The instrument used by Nichols[1] in his recent work on star radiation, gave a deflection of 11 mm. on a scale 183 cm. from the mirror, the candle being 811 cm. distant. The radiomicrometer used by Boys in attempting to measure the radiation from the stars, gave a deflection of 1.7 cm. under similar conditions. Computing our deflections under these same conditions, we have for number two 42 mm., and for number one, which was about four times as sensitive, 170 mm. But comparison should be made considering like areas of the surface exposed to radiation. When this is done, the sensitiveness of each instrument and suspension is as follows: Boys 0.9, Nichols 11, suspension number two 4.2, and suspension number one 17.

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[1] Ernest Nichols, Astrophys. Jl., Mch., 1901, p. 101.

It must be remembered that it is not a very difficult matter to make a radiometer suspension more sensitive than any of these, but the difficulty lies in its use.

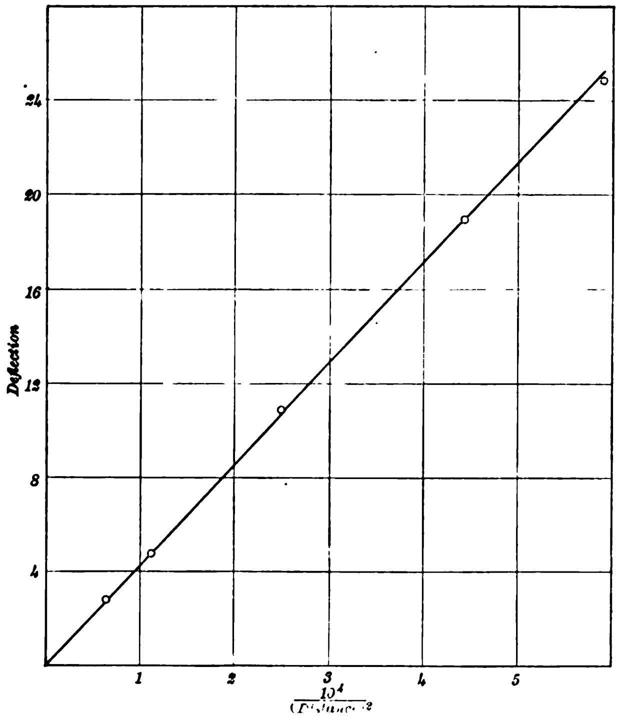

Fig. 5.

(*c*) *Accuracy.*—That the deflections of the radiometer are proportional to the energy when the vanes are 2 or 3 mm. from the window, has already been proven by Nichols. However, it seemed desirable to make some sort of a test. I used the convenient method of observing the deflections produced by a constant source, placed at different distances. At first, results obtained in this way did not follow the inverse square law, varying as much as 15 or 20 per cent. The source, an acetylene flame, was constant to within much less than 1 per cent., and the large errors were due to the reflections from the walls of the room. The accompanying curve shows a series of observations made after various precautions were taken.

It will be noticed that, with one exception, the deviations from the mean are less than 2 per cent., and the character of these deviations show that with sufficient care a curve could be obtained which would agree with the inverse square law to within less than 1 per cent.

(*d*) *Difficulties.*—The advantage which the radiometer possesses over other devices for measuring radiant energy are well known, but as the instrument is somewhat new, the difficulties encountered with these sensitive adjustments will be mentioned in considerable detail.

1. The zero reading drifted almost constantly. The changes of temperature in the room, the lack of proper protection against the radiations from different parts of the room, the air draughts, and the lack of perfect symmetry in the construction of the vanes, tended to produce this effect.

The first of these causes could not be completely removed as the apparatus could not be conveniently set up in a constant temperature room.

The glass and fluorite windows were carefully shielded from all sources of radiation. It was found necessary to place a shutter in front of the glass window, and, with the more sensitive suspension, this shutter was opened only when taking a reading. Radiation from the room entering the top of the instrument caused no disturbances. In fact, a gas light several feet away and shining obliquely into the top seemed to have very little or no effect.

In the construction of the vanes care was taken to make them alike, but how nearly this was accomplished could not be learned as the construction of the instrument prevented such a test being made.

This drift of the zero of course affected the accuracy of the readings, but by taking the reading of the zero before and after each deflection, the error was less than one small scale division, and quite often less than one-fifth of this.

2. The annoyance which could be the least easily controlled, was the vibration of the suspension due to the tremor in its support. The entire apparatus was set upon a stone pier and this effect minimized as far as practicable. The tremor did not prevent the observer from reading the scale, but caused the suspension to vibrate over several small scale divisions. This effect seemed to be due to the large area and small inertia of the vanes, and the viscosity of the residual gas.

3. Throughout the experiment the radiometer was kept connected to the pump so that the vacuum could be changed at any time. Only twice did any trouble from leakage occur, and in both cases the trouble was located in the mercury-rubber seals. The radiometer itself would stand for weeks with practically no leakage. Even when in connection with the pump, at the end of a week the change in the vacuum would be less than 0.01 mm.

4. If the air were allowed to rush in or out of the radiometer too rapidly, the vanes would strike the mica window and become electrified. To prevent this, the rush of air was controlled by the pump stopcock, while the movement of the vanes was watched through the telescope.

Several methods of discharging the vanes were tried. The simplest and most successful was to permit the instrument to stand several hours in the open air, after first freeing the clinging vanes from the window. This electrification could be prevented by a little care, but, when it did occur, was a source of considerable annoyance.

5. In constructing a radiometer, the difficulties in using a suspension of long period should be borne in mind. In such a vibrating system, the moment of inertia is of small importance as compared with the viscosity of the gas, and so an effort must be made to make the area of the vanes as small as the work undertaken will permit. An improvement could have been made in our instrument by placing the slit closer to the vanes and making the vanes smaller. As in

many instruments, the accuracy is not proportional to its sensitiveness. For example, the sensitiveness of suspension number one was four times as great as suspension number two, yet its accuracy was not nearly so great in proportion, due to the fact that changes of temperature, air draughts, etc., produced a greater effect.

II. Observations in the Visible Spectrum.

The visible spectrum was explored previously to the employment of the device for the accurate setting of the sodium line upon the radiometer slit. The method here used consisted merely of adjusting until the reflection from the fluorite surface appeared the color of sodium light.

It was not supposed that this would give an accurate, but merely a consistent setting. Indeed, the average deviation from the mean of several adjustments was not more than 5″ on the spectrometer, or about 0.003μ. The later method showed that the adjustments made in this part of the experiment were in error by about 45″ toward the violet, or about 0.025μ. It was then a simple matter to correct the wave-lengths, no account being taken of the fact that the light did not pass through the prism at minimum deviation.

The source of light was a cylindrical acetylene flame from a single tip burner. This type was used because the intensity per unit area was greater than in the flat flame.

On account of the drift of the vanes, the zero was read before and after each deflection and the average taken in the computations. The time required for one deflection being five minutes, ten minutes was consumed in taking one reading.

The table below gives the results of the observations in the visible spectrum. The deflections recorded are averages of a large number of observations, taken with different degrees of sensitiveness, but reduced to similar conditions. They are about the actual size of the deflections which would be obtained with the most sensitive adjustment of suspension number one. The first column indicates the rotation of the prism table, the positive sign meaning a rotation toward longer waves. The points 0.800μ and 0.665μ were obtained from observations given under III.

TABLE I.

Angular Rotation.	Wave-Length.	Def.	Angular Rotation.	Wave-Length.	Def.
——	0.800μ	64.80	− 5′	0.460μ	.60
+4′	.710	28.60	− 8′	.415	.44
——	.665	16.25	−12′	.374	.44
+3′	.662	16.35	−15′	.352	.25
+2′	.620	8.90	−18′	——	.25
0′	.552	3.00	−21′	——	.25
−1′	.527	1.92	−25′	——	.17
−3′	.489	.97			

These data are plotted to a different scale in Fig. 6, curve 1. The deflections recorded between 0.3μ and 0.4μ are thought to be produced by diffused light. In the subsequent use of this curve, the ordinate of the dotted horizontal line was considered as representing zero intensity.

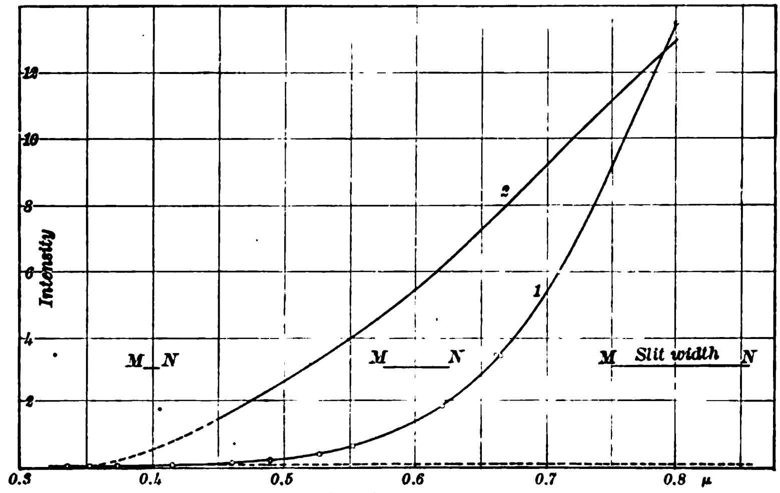

Fig. 6.

Curve 2 is the result when proper corrections are made for slit widths. This method of correction will be explained under IV. The relation between the ordinates of any wave-length in the two curves has no significance, both being plotted to a convenient scale.

III. OBSERVATIONS IN THE INFRA-RED.

1. *Cylindrical Flame.*

Owing to the difference in the dispersion of the prism, a slight change in the relative positions of the spectrometer and radiometer was of greater consequence in the infra-red than in the visible portion of the spectrum.

Correct results could be obtained only by making a series of observations in a small part of the spectrum and then checking back upon the sodium line. Many results had to be discarded because the relative position of the instruments had changed by an appreciable amount.

TABLE II.

	Wave Length.	Zero.	Reading.	Zero.	Def.
2′	0.665μ	41.75	39.50	41.61	2.18
4′	.775	41.28	33.92	41.20	7.32
5′	.855	40.80	29.40	40.69	11.35
6′	.952	40.50	24.81	40.38	15.63
7′	1.062	40.20	20.05	40.12	20.11
8′	1.196	39.85	19.00	39.80	20.82
9′	1.351	39.70	19.79	39.70	19.91
10′	1.504	39.70	21.31	39.80	18.44
[1]					
10′	1.504	39.97	21.59	40.00	18.40
11′	1.676	40.10	24.00	40.00	16.05
12′	1.847	39.90	26.02	39.90	13.88
13′	2.000	39.90	28.20	39.90	11.70
14′	2.140	39.90	30.20	39.90	9.70
15′	2.285	39.82	31.82	39.82	8.00
16′	2.419	39.70	33.69	39.80	6.06
17′	2.550	39.80	34.50	39.81	5.31
18′	2.664	39.81	34.53	39.79	5.27
19′	2.785	39.78	35.38	39.80	4.41
20′	2.904	39.80	36.10	39.80	3.70
[2]					
10′	1.504	39.78	21.50	39.71	18.25
[3]					
10′	1.504	39.50	21.05	39.30	18.35
20′	2.904	39.00	35.22	38.90	3.63
21′	3.012	38.80	35.80	38.70	2.95

[1] Here spectrometer was re-set on Na, and found 5″ in error.

[2] Returned to 10′.

[3] Spectrometer re-set on Na, and found 4″ in error.

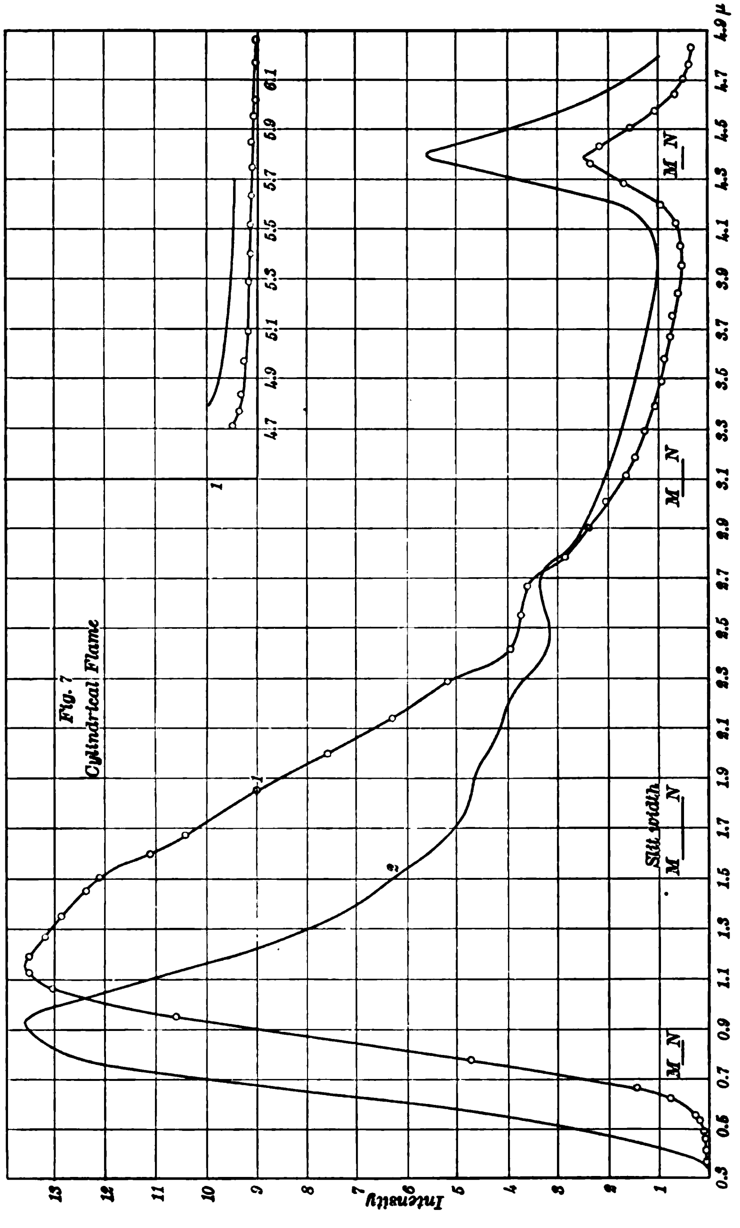
Fig. 7
Cylindrical Flame
Intensity
13
12
11
10
9
8
7
6
5
4
3
2
1
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
2.7
2.9
3.1
3.3
3.5
3.7
3.9
4.1
4.3
4.5
4.7
4.9 μ
4.7
4.9
5.1
5.3
5.5
5.7
5.9
6.1
1
2
Slit width
M N
M N
M N
M N

Fig. 7.

The preceding table, giving a series of observations covering a large part of the spectrum, shows the method used. The angles given in the first column signify the rotation of the prism table from the starting point, *i. e.*, the sodium line.

Curve 1, Fig. 7, obtained from this and other data, shows the distribution of energy as observed throughout the spectrum. It was useless to go any further than 6.0 μ, for readings beyond this region are perhaps false on account of diffused light. If all the observations taken were plotted, all the points would not lie upon this line, but the variation would not be 1 per cent. As an example of this variation, the maximum point, which is somewhat indefinite anyway, was found to shift only about 0.03 μ, from 1.14 μ to 1.17 μ. In an experiment of this sort, where the error due to the setting is so large in comparison with the errors in radiometer deflections, it is more satisfactory to draw a curve representing one series of observations carefully taken, than to draw an average curve. Curve 2 is the corrected curve which will be discussed under IV.

The most striking characteristic of curve 1 is the elevations which are due to the emission bands of the gases in the flame. If the acetylene is pure, the gases to be expected are CO_2 and H_2O.

Paschen[1] has made a study of the emission bands of both of these and has found the wave-lengths of the maxima under different conditions and temperature. He gives the following data for the bands found in the radiation from a bunsen burner.

H_2O SPECTRUM.

Maximum points, 1.46μ, 1.90μ and 2.83μ.

CO_2 SPECTRUM.

Maximum points, 2.71μ (2.68μ when CO_2 is not dry) and 4.40μ.

Although these values are computed from a dispersion curve different than the one used in our computations, yet the difference at the points considered would not be 0.01μ.

The energy curve shows elevations whose maxima agree very closely with these values, with the exception of the band 2.83μ

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[1] Paschen, Annal. der Phys., 53, 1894, p. 334.

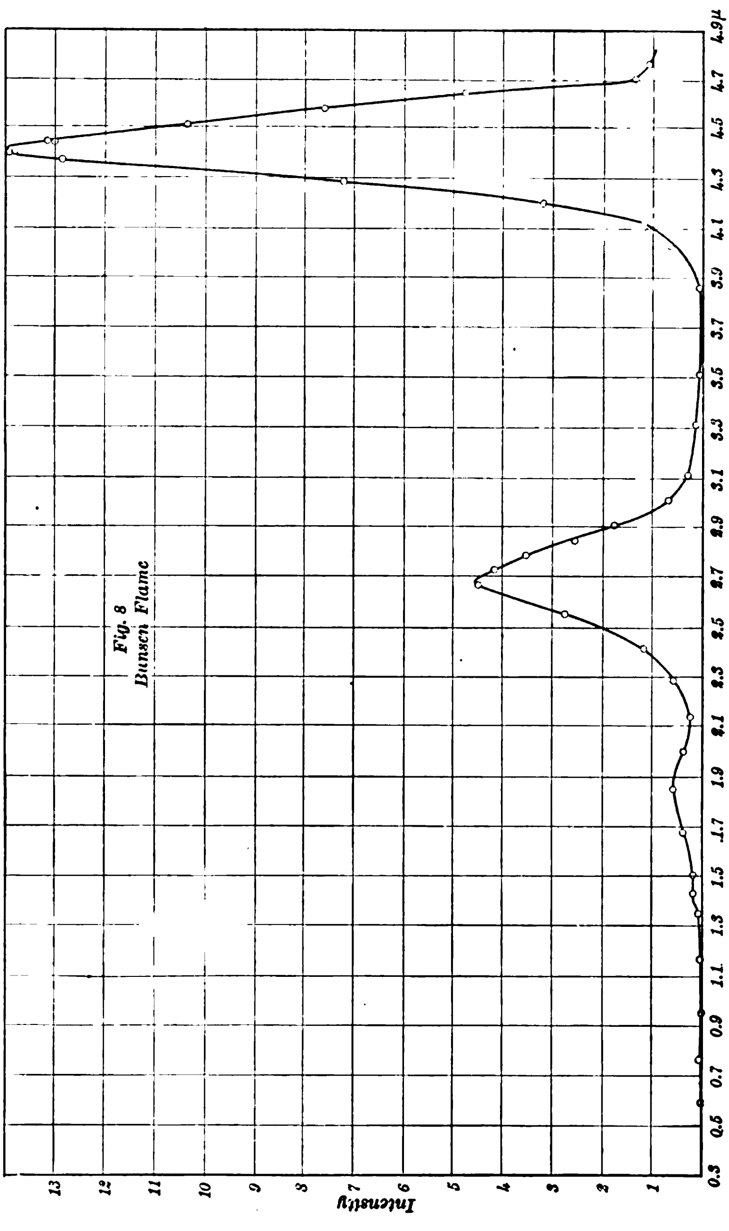
Fig. 8
Bunsen Flame
Intensity
13
12
11
10
9
8
7
6
5
4
3
2
1
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
2.7
2.9
3.1
3.3
3.5
3.7
3.9
4.1
4.3
4.5
4.7
4.9μ

Fig. 8.

which is perhaps too small. By means of these points, we have an easy and accurate method of testing the spectrometer adjustment. The slight elevation at 2.3μ was obtained several times, and may be due to the emission of some impurity in the acetylene.

2. Bunsen Flame.

The curve for the acetylene bunsen flame is shown in Fig. 8. In this curve all five emission bands appear, and the values of the wave-length at the maximum points agree with the values quoted above. The bands being more distinct than in Fig. 7 we have still a better test for the adjustment of the apparatus. If we compare the bunsen and cylindrical flame curves, we see that they do not agree in all respects. The elevations due to the emission of the H_2O gas have not the same relative values in the two cases, but this discrepancy disappears when both curves are corrected for slit width. The band at 2.3μ does not appear in the bunsen flame, and this fact makes its presence in the other somewhat in doubt. That the apparatus permits only a very small amount of stray radiation, is well shown by this bunsen flame curve. In the neighborhood of 3.7μ the deflection is practically zero, while at 4.4μ it is very large.

3. Flat Flame.

Observations were next taken upon the flame of a two-jet Naphey burner. The relative positions of the flame and the spectrometer slit are shown in Fig. 9. The work upon the flat flame is not yet finished, the results obtained up to the present time, shown in Fig. 10, curve 1, being quite unsatisfactory. The lack of consistency in the observations is, in all probability, due to an unsteadiness in the flame itself. This might be due to the fact that the layer of burning gas is thin enough to be affected easily by air draughts.

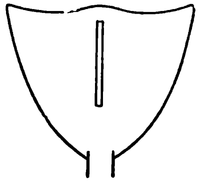

Fig. 9.

The fact that the maximum with the Naphey burner is further toward the violet, is in agreement with the observations of Hartman,[1] who showed photometrically that the Naphey burner flame was

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[1] E. L. Nichols, Journ. Frank. Inst., Nov., 1900.

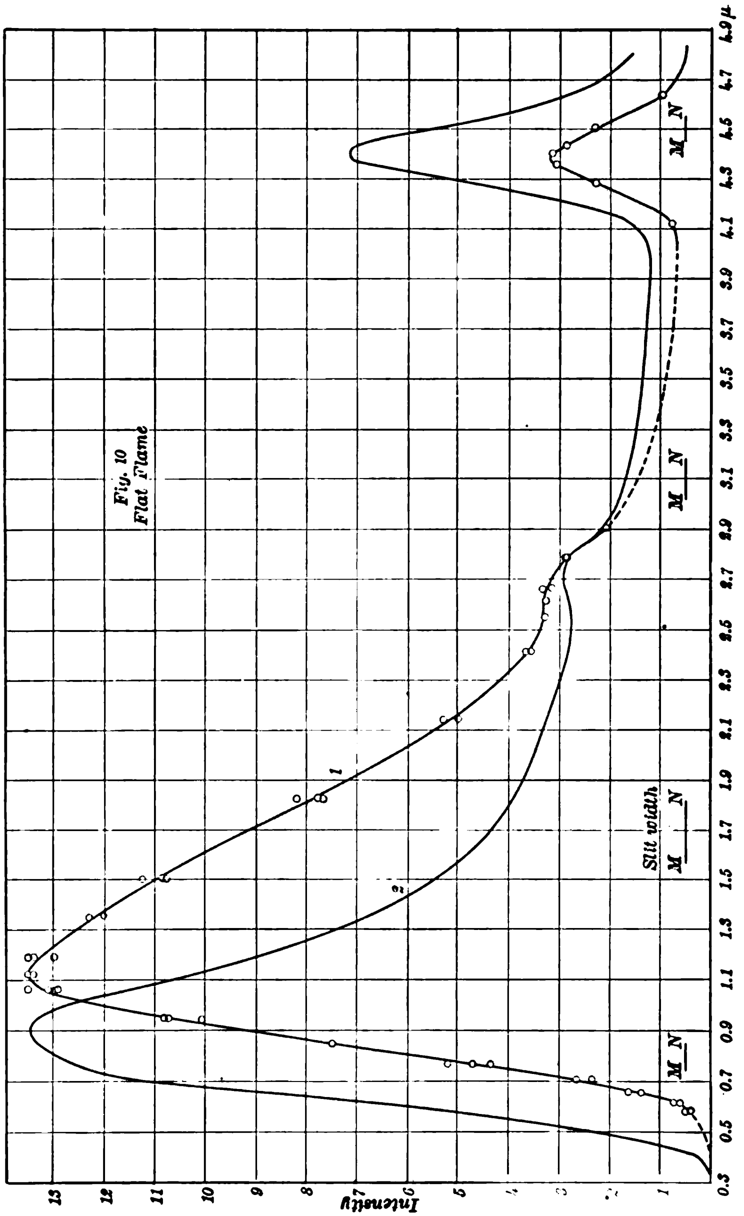
Fig. 10
Flat Flame
Slit width
M N
M N
M N
M N
1
2
Intensity
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
2.1
2.3
2.5
2.7
2.9
3.1
3.3
3.5
3.7
3.9
4.1
4.3
4.5
4.7
4.9μ
1
2
3
4
5
6
7
8
9
10
11
12
13

Fig. 10.

much richer in the violet. Curve 2 is the corrected curve and will be discussed later.

IV. DISTRIBUTION IN THE PURE NORMAL SPECTRUM.

1. Slit Width Correction.

The curves of observations do not represent the actual distribution of energy in the pure spectrum. No matter how narrow the slit, a correction must always be made, for in such measurements we are concerned not so much with the actual as with the relative widths in the different parts of the spectrum. The method of this correction will be discussed in considerable detail, as there does not seem to be uniformity among the various writers who have had occasion to make such a correction.

When I first considered the matter, the following seemed to be an approximately correct method. Consider the spectrum falling upon the radiometer slit, or the spectrometer spectrum. In different parts of this spectrum the slit has different widths when expressed in wave-lengths, and if b and b' are the intensities in the spectrometer spectrum and a and a' the corresponding slit widths in wave-lengths, then the deflection obtained when the radiometer is at b is proportional to ab, and when at b', to $a'b'$. To find the distribution of energy in the spectrometer spectrum, we would then need only to divide the ordinates of the observed curve by the slit widths expressed in wave-lengths. But this spectrum is really composed of an infinite number of overlapping images of the spectrometer slit. Consider each wave-length to give such an image. Then at any point the number of overlapping images would be proportional to the width of the spectrometer slit expressed in wave-lengths. Hence, the intensity at any point in the spectrometer spectrum is proportional to the intensity in the pure spectrum and also to a, where a is the width of the spectrometer slit expressed in wave-lengths. Then to find the pure spectrum when the spectrometer spectrum is known, one has only to divide the intensities of the latter by a, a', etc. Combining these two steps, the slits being of equal width, the distribution of energy in the pure spectrum is obtained from the observed distribution by dividing the intensities of the latter by the square of the slit widths expressed in wave-lengths. This method is accurate only in so far as the pure spectrum curve may be considered a straight line for a length corresponding to a slit width.

Langley's[1] method of correcting for the bolometer strip width is, in effect, dividing the observed energy by the strip width in wave-lengths, and does not seem to take consideration of the spectrometer slit width.

Paschen[2] has given an accurate method of correction, taking into account both the strip width and spectrometer slit width. An approximation of his equation shows that he divides only by the slit or strip width, while on the other hand, Runge[3] in a later article, divides by the square of the slit width.

In recent articles on the radiation of black bodies, I find references to the method of Paschen and to the method of Runge, and one article refers to both as though they were the same. This is rather confusing. In my opinion Paschen is in error, and I shall repeat here part of his derivation of the equation to be used, at the same time making a correction.

In a pure spectrum, let $f(x)$ be the intensity per unit wave-length at the point x, where x is measured in wave-lengths. Let the radiometer slit or bolometer strip, and likewise the spectrometer slit, have a width a, where a is expressed in wave-lengths. If the radiometer slit is at x, then an image of the spectrometer slit due to the wave-length x will fall entirely within the radiometer slit, and the intensity of this image is $af(x)$. The intensity of the wave-length $x+v$ falling upon the radiometer slit at x is $(a-v)f(x+v)$. If $F(x)$ represents the total energy falling upon the radiometer slit, we then have,

$$F(x)=\int_0^a (a-v)[f(x+v)+f(x-v)]\,dv.$$

This differs from Paschen by the factor a. He says that the energy in the image of x is $f(x)$ and in the image of $x+v$ is $\frac{a-v}{a}f(x+v)$, and so his equation is

$$F(x)=\int_0^a \frac{a-v}{a}[f(x+v)+f(x-v)]\,dv.$$

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[1] Langley, Phil. Mag., 17, 1884, p 184.

[2] Paschen, Annal. der Phys., 60, 1897, p. 712.

[3] Runge, Zeitschr. für Math. und Phys., 42, 1897, p. 205.

Making the change in his work, we have as a final equation,

$$a^2 f(x) = F(x) - \tfrac{1}{6}F_1(x) + \tfrac{2}{45}F_2(x) \cdots$$

where

$$F_1(x) = \frac{F(x+a) + F(x-a)}{2} - F(x)$$

and

$$F_2(x) = \frac{F_1(x+a) + F_1(x-a)}{2} - F_1(x).$$

I have used this equation in correcting the curves obtained from observations. $F_1(x)$ and $F_2(x)$ can be found graphically as explained by Paschen. $F(x)$ is observed, $F_1(x)$ determined from the curve of $F(x)$, and $F_2(x)$ determined from the curve of $F_1(x)$.

In my curves, the correction due to $F_2(x)$ was too small to be of any consequence, and so $f(x)$ was found simply by dividing $F(x) - \tfrac{1}{6}F_1(x)$ by the square of the slit width in wave-lengths. This slit width was determined from a curve showing the relation between wave-lengths and angular rotations of the prism table, and for the sake of greater accuracy, the values obtained were plotted with wave-lengths and a smooth curve drawn through these points.

2. *The Corrected Curves.*

This correction for slit width and impurity of the spectrometer spectrum would not be of great importance in parts of the spectrum where the dispersive power of the prism changes slightly, but in my curves, particularly in the visible portion of the spectrum, the corrected curve differs greatly from that observed. The absorption of the CO_2 and H_2O in the air between the source and the radiometer, make the observed curves very much in error at the points where we obtained emission bands. So the accurate method of correction was applied only from 0.4μ to 1.9μ and the remainder of the curve was obtained by dividing by the square of the slit width in wave-lengths.

In Fig. 7, it will be noticed that the maximum point has shifted from about 1.16μ on the observed curve, to about 0.93μ on the corrected one. Likewise in Fig. 10, the change has been from 1.12μ to 0.90μ. The observed maximum depends upon the values of the energy in the pure spectrum and upon the slit widths in wave-lengths and should lie between the maxima of these two variables. This is verified, for the slit width has its maximum value at about 1.6μ.

A discrepancy in the relative intensities of the H_2O emission bands, 1.46μ and 1.90μ, in the curves of the bunsen and cylindrical flames, has already been mentioned. The bunsen shows that the band at 1.90μ is the more intense, and this is in agreement with the work done by Paschen. In the observed curve of the cylindrical flame, Fig. 7, curve 1, the opposite seems to be the case. However in the corrected curve, the band at 1.46μ scarcely shows, while the one at 1.90μ is quite distinct. The change brought about in the correction is due to the fact that the maximum value of the slit width is at about 1.6μ, and this has a tendency to exaggerate the elevation due to the band at 1.46μ.

3. *Temperature of the Acetylene Flame.*

The change in position of the maximum intensity in the spectrum of a black body with changes of temperature, is in agreement with the equation

$$\lambda_m T = A,$$

where λ_m is the wave-length of the maximum intensity, T the absolute temperature, and A a constant.

Lummer and Pringsheim[1] have found for A the value of 2940 and this agrees approximately with the results obtained by Paschen[2] and Mendenhall and Saunders.[3] Lummer and Pringsheim have also found the value of A to be 2630 in the case of bright platinum. They are of the opinion that the temperature of flames can be calculated from the formula, and that the true value for the temperature. lies between the values obtained by assuming the two values for A.

Applying the formula, we obtain the following values:

CYLINDRICAL FLAME.

$$\lambda_m = 0.93\,\mu, \quad A = 2940, \quad T = 3160°, \text{ or about } 2890° \text{ C.}$$
$$A = 2630, \quad T = 2830°, \text{ `` `` } 2560° \text{ C.}$$

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[1] Lummer and Pringsheim, Verh. Deutsch. Phys. Gesell., 1899, p. 214.

[2] Paschen, Sitz. Kgl. Preuss. Akad. der Wiss. Ber., 1899, p. 959.

[3] Mendenhall and Saunders, Astrophys. Jl., Jan., 1901, p. 25.

FLAT FLAME,

$\lambda_m = 0.90\mu$, $A = 2940$, $T = 3270°$, or about 3000° C.
$A = 2630$, $T = 2920°$, " " 2650° C.

The temperature of the acetylene flame as determined by Nichols,[1] is about 1900° C. I am at a loss to understand the cause of the discrepancy in the application of this formula. The temperatures of different sources of light, calculated by Lummer and Pringsheim (loc. cit.) in a similar manner, are certainly more nearly within range of the correct values. A difference might arise in the method of correcting the observed curve. They say they reduce their curves to the normal spectrum, but fail to give the method. In a later article,[2] they state that when their curves were corrected according to the method of Runge, the resulting change was not more than 5 per cent.

Mendenhall and Saunders in the article already referred to say they have corrected their curves by dividing by $\frac{d\lambda}{d\delta}$ and that corrections were also made for the impurity of the spectrum according to Runge. It seems to the writer that these two methods are at variance, the one being in effect, dividing the observed values by the slit width in wave-lengths, while Runge divides by the square of the slit width in wave-lengths.

If calculations are made from our observed curves, the resulting temperature is yet about 100° C. too high.

It is quite possible that the value for A decreases with higher temperatures. Mendenhall and Saunders find a slight decrease, while Paschen finds an increase.

4. *Radiant Efficiency.*

In the study of devices for artificial lighting, the determination of the efficiency is of great importance, and to obtain this determination, the ratio of the radiation in the visible portion of the spectrum to the whole radiation, must be known. This ratio, or radiant efficiency, has already been determined for acetylene by Stewart and Hoxie.[1] They used a modification of the Melloni method, and, from a large number of readings, obtained the value of 0.105 for the flat flame of a Naphey burner.

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[1] E. L. Nichols, PHYS. REV., X., 1900, p. 234.

[2] Lummer and Pringsheim, Verh. Deutsch. Phys. Gesell., 1900, p. 163.

Langley was the first to utilize spectral energy curves for accurate determinations of the radiant efficiency. His method consisted in finding the ratio of the area in the visible portion of the spectrum to the area of the entire curve. My corrected curves were treated in this same manner, the visible spectrum being considered as ending at 0.76μ. The cylindrical flame curve, Fig. 7, curve 2, gave 0.100, and the flat flame curve, Fig. 10, curve 2, 0.131. I think there is no doubt but that the efficiency of the flat flame is the greater, but I do not place much reliance on the value 0.131, because the curve from which it was obtained is not satisfactory. The difference in the radiant efficiency of the two flames might be anticipated from the difference in temperature and color. If the absorption of the atmosphere were eliminated, these two values would be decreased by perhaps more than 0.01.

V. SOURCES OF ERROR.

1. The acetylene was made by the wet process and there was no attempt at purification. Mr. H. A. Rands made three analyses of the gas. The greater part of the acetylene was absorbed by fuming sulphuric acid, a solution of caustic potash removed the sulphur trioxide, oxygen was then removed with alkaline pyrogallol, and the remaining traces of acetylene were absorbed by cuprous chloride. He obtained 98.7, 98.4 and 99.0 as percentages of purity.

2. The gas coming from the reservoir passed through a pressure regulator devised by Prof. Moler.[2] Although several others were using gas at the same time, this device kept the pressure constant to within less than 1 per cent. Just how the variation in pressure would affect the energy entering the radiometer, depended not only upon the change in the flame itself, but also upon the position of the flame in regard to the spectrometer slit. It was found that if the pressure were changed 1 per cent., the change in the radiometer deflection would be only about 0.2 per cent.

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[1] E. L. Nichols, PHYS. REV., XI., 1900, p. 215.

[2] E. L. Nichols, Journ. Frank. Inst., Nov., 1900.

3. Before reaching the radiometer vanes, the radiation from the acetylene flames passed through a fluorite prism, a fluorite plate and a very thin sheet of mica. The selective absorption of fluorite[1] does not begin until 6μ, and this is entirely out of our region. The mica being very thin, no appreciable absorption was to be expected. However, a test throughout the region in question was made, and this expectation verified.

4. The mirrors of the spectrometer, silvered according to Brashear's[2] method, were not highly polished. Such a surface causes diffused light and also makes the difference in percentage of reflection of the short and long waves greater than when the mirrors are highly polished. The diffused light was small, as can be seen from the visible spectrum curve, Fig. 6, and the bunsen curve, Fig. 8. Paschen[3] has recently shown that with a freshly silvered and highly polished mirror, the reflection at 0.7μ is 94 per cent. and at 6μ is 98 per cent., while in the case of a mirror four years old and having a distinct brownish tint, the reflection at 0.7μ is 85 per cent. and at 6μ is 95 per cent. Judging from these figures, it is probable that the reflection from our mirrors at these wave-lengths would differ by about 6 per cent. No attempt was made to correct the curves for this error.

5. The angle at which the radiation falls upon the face of the prism is different for different wave-lengths, and it is desirable to know how much error is made when this is not taken into consideration. Calculation by the Fresnel energy equation shows that when the radiometer is at 0.6μ, 3.2 per cent. is reflected, and when at 6.0μ, 2.7 per cent., a difference of only 0.5 per cent.

6. The percentage absorption of platinum-black for different wave-lengths is certainly different, and, moreover, is not known.

7. The absorption due to the presence of CO_2 and H_2O in the atmosphere of the room causes the curve to be in very great error at the emission bands of these gases.

8. In correcting the curves, only two terms of the series were used, but these approximated the true result within 0.2 or 0.3 at the point of greatest variation.

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[1] Paschen, Annal. der Phys., 53, 1894, p. 812.

[2] Wadsworth, Astrophys. Jl., Vol. I., 1895, p. 252.

[3] Paschen, Annal. der Phys., 1901, p. 314.

If the errors 4 and 6 are not taken into consideration, I think the portion of the curve, Fig. 7, curve 1, between 0.7μ and 2.7μ is not in error by more than one per cent.

VI. SUMMARY.

The distribution of energy in the spectrum of the acetylene flame is found in three cases by means of the spectrometer and the radiometer.

1. The cylindrical flame of a single-tip burner, distribution observed, Fig. 7, curve 1, actual distribution, curve 2, and the visible portion of the spectrum, Fig. 6, curves 1 and 2.

2. The flat flame of a two-jet Naphey burner, Fig. 10, curves 1 and 2.

3. Bunsen burner flame, Fig. 8.

These curves show the emission bands of H_2O and CO_2, and, the position of these bands being known, a check is had upon the adjustment of the apparatus.

The radiant efficiency of the cylindrical flame is found to be 0.100.

I wish to extend my thanks to Professors E. L. Nichols and Ernest Merritt, at whose suggestion and under whose direction the experimental work was performed.

THE PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
June, 1901.

TEMPORARY SET, AS ASSOCIATED WITH MAGNETOSTRICTION.

By Carl Barus.

Introduction.

IN my earlier papers[1] on the subject, I inferred that the brilliant theories of magnetostriction due to Kirchoff, Korteweg, Lorberg, Helmholtz, Maxwell and others, can have but a limited application, inasmuch as the phenomena are inherently associated with what I venture to interpret as viscosity. In other words, if deformations evoked by physical changes of molecular configuration within the metal be termed viscous, *whether gradual or sudden*, then viscosity enters essentially into magnetostriction. With each twist, however small, and within the elastic limits the wire receives a certain amount of set, instantaneously; the whole of this set may be shaken out by the first magnetization and it is thus appropriately called temporary, to distinguish it from permanent set which comes much later, as a second phase lying beyond the limits of elasticity. On removing the magnetic field, therefore, the unmagnetic wire is molecularly different from the same unmagnetic wire before magnetization; and the new molecular arrangement is permanent so long as the twist is unchanged. Any change of twist, whether increment or decrement evokes new temporary set, or a new molecular grouping, which may be converted into the permanent grouping for the given twist, by a reapplication of the magnetic shake up in question. Hence throughout the whole period within which varying torsional stresses are applied, the metal is undergoing continual change of molecular constitution, which in relation to magnetostriction are as radical as though a new metal[2] were being continually supplied for investigation. These are the occurrences which a purely elastic theory does not adequately construe.

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[1] American Journal of Science, XXXIV., p. 175, 1887; X., p. 407, 1900; XI., p. 97, 1901.

[2] One may argue that the metal is not again normal until *after* the first magnetization. This is what I have done, virtually, and it works well: but one must acknowledge the ground debatable. At all events no fundamentally correct or physically acceptable theory will overlook the temporary set, which is just as much a part of magnetostriction as the elastic phenomena which follow.

The gradual viscous deformations due to the heat motion within the wire are without bearing on the present experiments. They are of a different nature and easily eliminated.

The phenomena may then be analyzed as follows: Let a soft iron wire be subject to a fixed amount of torque, corresponding to a fixed amount of twist registered as a definite deflection (fiducial zero) on some appropriate scale. The first magnetization will produce (if twist is large enough) an increment of rigidity, registered as an increased deflection (say) on the scale. The withdrawal of the field does not reproduce the original deflection; the new deflection is in all cases smaller than the original deflection or below the fiducial zero; *i. e.*, the unmagnetic wire was virtually more rigid before than after the magnetization. If torque remains constant, the new conditions are persistent; so that on subsequent magnetizations the second deflection, on subsequent demagnetizations the third deflection, is regained almost perfectly. The inelastic feature has now been eliminated for the particular torque impressed. As the second deflection is eventually an increment, the third a decrement on the first, the elastic phenomenon is the numerical sum of both and is thus greater than either. There has been a slip of the fiducial zero favorable to greater deflection.

2. *Plan and Purpose.*—In the papers mentioned, more particularly in the last, I paid considerable attention to temporary set or *slip* as I then called it. It was found to be not very different (relatively) in the smallest (45°) and in the largest (360°) twists applied. Indeed between these limits of impressed stress, it seemed to pass through a maximum. Obviously, however, the slip must initially increase from zero with the twist. It is thus the chief purpose of the present paper to investigate the character and occurrence of temporary set for twists smaller than 45°, indefinitely. I shall moreover use a slightly different method of discussion, separating the increment of rigidity due to the first magnetization from the decrement of rigidity (slip) due to the first demagnetization for the purpose of discerning the nature of each.

In the interest of a definite scheme of investigation, the twists imparted should in some way regularly increase toward, or decrease from the highest values admissible. This is best accomplished by twisting the wire back and forth by amounts increasing in fixed steps and lying symmetrically on both sides of zero. In such a case the *antecedent* twist, *i. e.*, the total strain applied in any one direction since the last magnetization, will be the numerical sum of the positive twist and the negative twist from which the positive twist has been imparted; and vice versa. In other words, temporary set will here depend on an angle *different* from the impressed or actuating twist, while the increment of rigidity depends on the latter.

Again, if the wire is twisted in regularly increasing steps on one side of zero only, it should be brought back to zero for investigation between each twist. In this case the antecedent twist which determines the temporary set is the twist last impressed, while the actuating twist is zero.

What I make clear in this paper, is the dependence of temporary set or slip on the antecedent strain as above defined. Its direction is immaterial so far as the numerical amount of slip is concerned. The conditions under which slip is picked up by the magnetically increased rigidity in the presence of an impressed strain will also appear.

Throughout this paper in referring to magnetically increased rigidity I have in mind the simple torsion model described in an earlier paper.[1] When the wire is twisted the increment of tension along the lines of longitudinal stress due to magnetization, acts with a horizontal component in a direction equivalent to an increase of rigidity.

3. *Apparatus.*—This was the same differential apparatus heretofore described, consisting of identical upper and lower, collinear, vertical soft iron wires, joined in the middle by a light brass mirror support, and fastened at the ends to torsion heads. The upper wire was surrounded by a helix with internal water circulation. The two wires are countertwisted by the amounts θ each as given in the tables. The deflections occurring at the mirror when one wire is acted on magnetically, are observed on a centimeter scale 6 me-

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[1] American Journ., X., p. 401, 1900.

ters distant. Thus 21 cm. of deflection correspond to one degree. The wires were each usually 37 cm. long and .044 cm. in diameter. The magnetic field of the long helix may be rated at about 100 c.g.s units per ampere.

The system of two wires was kept under a definite pull of about 300 grams, an insignificant amount. If the upper wire is twisted clockwise to an eye regarding it from above, an increased deflection denotes virtually increased rigidity of the upper (magnetized) wire.

This differential method was devised as has been stated elsewhere, to obtain direct relations between the deflection increments and the corresponding changes of rigidity. For if θ be the twist imparted to each wire, n their common original rigidity, if $d\theta$ and dn be the changes due to magnetostriction, then $n(\theta + d\theta) = (n + dn)(\theta - d\theta)$, seeing that the twist of the upper wire decreases by the same amount as the twist of the lower wire increases, while torque, length and section remain unchanged. Hence

$$dn/n = 2d\theta/\theta = \mu,$$

which I have called the coefficient of magnetically increased rigidity[1]. Its value is a definite minimum when the (elastic) obliquity of the external fiber is a maximum.

RESULTS FOR SUCCESSIVELY INCREASING ALTERNATING TWISTS.

4. *Preliminary.*—These results are given in the following tables, in which the first column shows the twist applied to each wire. The second to fifth columns show the reading at the scale in centimeters, originally (off (1)), and when the current is successively made and broken as the subscripts indicate. The sixth column gives the value of the antecedent twist, being the numerical sum of the existing and the preceding (opposite) twist. Great care must be taken not to exceed the strain limits stated, as otherwise the values of the antecedent twists stated would be quite erroneous. Accidents are noted. The 7th and 8th columns show the deflections from the original zero—off (1)—due to the first magnetization (on), and the first removal of the field. When these deflections have the same sign as the twist the rigidity of the magnetized wire

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[1] American Journ., XXXIV., 1887, p. 181 et seq.

TABLE I.

Twists alternating symmetrically. Magnetic field, 380 *c.g.s. units. Soft iron; diameter* .044 *cm., length* 37 *cm., scale distance* 600 *cm., deflection* 21 *cm., equivalent to* 1°.

Twist of each Wire.	Scale Reading when Field is				Antecedent Strain.	Deflections[1] Referred to		
						Original Zero.		New Zero.
	Off (1)	On (1)	Off (2)	On (2)		On.	Off.	Total.
°	cm.	cm.	cm.	cm.	°	cm.	cm.	cm.
0	—	—	—	—	0	—	—	—
− 5	0	+1.0	+1.5	+1.0	5	− 1	−1.5	.5
+ 5	0	− .5	−1.0	−1.5	10	− .5	−1	.5
−10	0	− .5	+1.0	0.0	15	.5	−1	1.5
+10	0	− .5	−1.0	− .5	20	− .5	1	.5
−20	0	.0	+1.8	.0	30	0	−1.8	1.8
+20	0	− .6	−2.2	− .6	40	− .6	−2.2	1.6
−30	0	− .4	+2.6	− .4	50	.4	−2.6	3.0
+30	0	.0	−2.4	.2	60	0	−2.4	2.4
−40	0	− .9	+3.0	− .8	70	.9	−3.0	3.9
+40	0	+ .6	−2.8	+ .6	80	.6	−2.8	3.4
−50	0	−1.6	+2.6	−1.6	90	1.6	−2.6	4.2
+50	0	+1.4	+3.1	+1.4	100	1.4	−3.1	4.5
0	—	—	—	—	—	—	—	—
−50	0	−2.6	2.2	2.4	50	2.6	−2.2	4.8
+50	0	1.6	−2.6	1.6	100	1.6	−2.6	4.2
−60	0	−3.0	2.3	−2.8	110	3.0	−2.3	5.3
+60	0	2.2	−2.9	2.2	120	2.2	−2.9	5.1
−50	0	−1.4	2.9	−1.4	110	1.4	−2.9	4.3
+50	0	2.0	−2.6	2.0	100	2.0	−2.6	4.6
−40	0	− .4	3.2	− .4	90	.4	−3.2	3.6
+40	0	.8	−2.8	.8	80	.8	−2.8	3.6
−30	0	.4	3.1	.4	70	− .4	−3.1	2.7
+30	0	.0	−2.6	.1	60	.0	−2.6	2.6
−20	0	1.0	2.7	1.0	50	− .1	−2.7	1.7
+20	0	.0	−2.2	− .2	40	.0	−2.2	2.2
−10	0	1.0	2.0	1.0	30	−1.0	−2.0	1.0
+10	0	− .4	−1.2	− .3	20	− .4	−1.2	.8
− 5	0	.7	1.0	.8	15	− .7	−1.0	.3
+ 5	0	− .1	− .6	− .1	10	− .1	− .6	.5
0	0	.5	.5	.5	5	− .5	− .5	.0

is increased; for the case of opposite signs rigidity is decreased. The last column (numerical sum of the preceding two), shows the persistent or elastic increment of rigidity in the wire molecularly modified for the particular twist in question, as explained in paragraph 1.

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[1] Taking the strains all positive, the deflections are changed in sign accordingly.

5. *Results.*—Omitting introductory experiments, results of a definite character are given in table 1, though the first few deflections taken but to centimeters, are too crude.

In the accompanying chart, Fig. 1, the deflections of the table are mapped out as ordinates relative to the given antecedent strains as abscissas, except in curve 3, where the impressed strains are the abscissas. The impressed strains are marked in degrees of arc on the peaks of the curves.

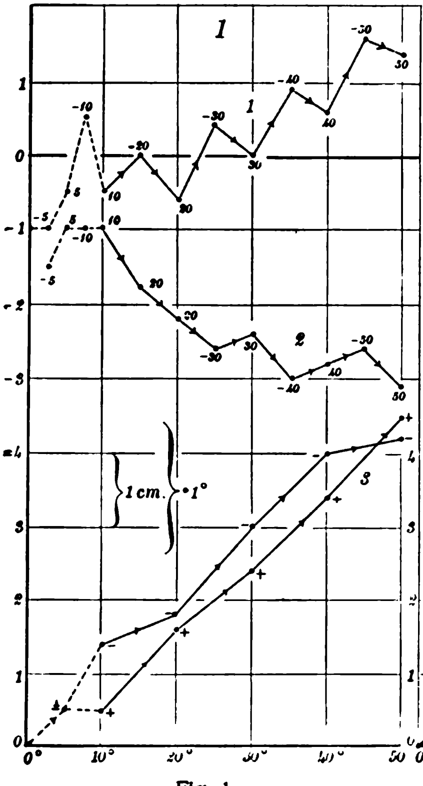

Fig. 1.

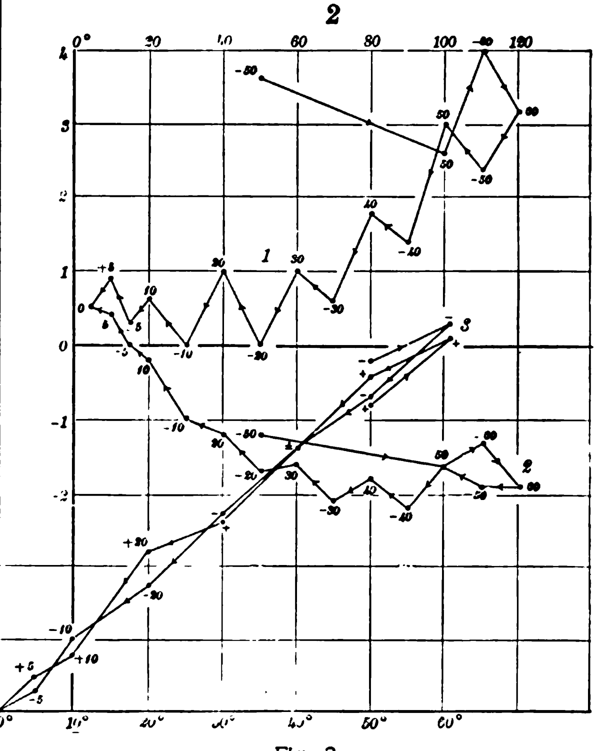

Fig. 2.

Curve 1 denotes the increase of rigidity due to the first magnetization relatively to the zero of the original unmagnetic wire. It shows the effect of temporary set or slip superposed on the effect of magnetically increased rigidity in the presence of the magnetic field and of the impressed strain stated. This curve is distinctly alternating in character, like the twists. The rigidity is always *greater* for the *negative* twists with which the experiments begin, than for the positive twists of the same character which are subsequent.

As a whole, the mean increase of rigidity beginning with negative values is accelerated as twist increases. There is indication of a minimum.

Curve 2 shows the displacement (slip) of the fiducial zero of the unmagnetic metal before and after the first magnetization. It indicates the effect of slip in the presence of the impressed strain stated and of the magnetization, being the temporary set under these circumstances. Curve 2 corresponds definitely to a decrement of rigidity, which increases numerically with the twist but soon reaches a maximum at about ± 30° or ± 40° of impressed strain. The curve is much smoother than the preceding though it eventually runs into marked alternations. The initial slope is about

$$d\theta/\theta = .0026.$$

Curve 2 in connection with curve 1 shows that whereas the effect of slip is in excess for small strains, it is soon overtaken by the magnetically increased rigidity. *The alternations of both curves correspond in the main, but are opposed in phases.*

Curve 3, the algebraic difference of the two curves, shows the persistent magnetic increase of rigidity after the first magnetization. It represents the amount of magnetically increased rigidity in the absence of slip or temporary set for the impressed strain stated. The mean increment of rigidity is initially $dn/n = 2d\theta/\theta = .008$ for the positive twists and .0096 for the negative twists.

. The second part of Table II. and the accompany chart (Fig. 2) contain corresponding results for a twist decreasing from 120° in successive steps of 10°. The curves are numbered as in the preceding chart.

Curve 1 has similar alternating characteristics to the preceding case; but the results are clearer throughout and they substantiate the occurrence of a minimum in the mean values for impressed twists of about ± 10° or ± 20°. Beyond this, however, the curve presents a new result; the rigidities of the *positive* twists are now successively in *excess* over the negative. The change of sign occurs at + 60° at the end of the curve.

TABLE II.

Twists alternating symmetrically. Magnetic field, 720 *c.g.s. Soft iron, diameter* .044, *length* 37 *cm. Scale distance* 600 *cm. Deflection,* 21° *cm. equivalent to* 1.

Twist of Each Wire.	Scale Reading for Field.				Antecedent Strain.	Deflections[1] Referred to		
	Off (1)	On (1)	Off (2)	On (2)		Original Zero.		New Zero.
						On	Off.	Total.
°	cm.	cm.	cm.	cm.	°	cm.	cm.	cm.
0	—	—	—	—	0	—	—	—
− 5	0	− .2	.2	− .4	5	.2	− .2	.4
+ 5	0	0	.8	.0	10	0	− .8	.8
−10	0	− .2	1.0	− .2	15	.2	−1.0	1.2
+10	0	− .4	−1.6	− .4	20	− .4	−1.6	1.2
−20	0	− .2	1.9	0	30	.2	−1.9	2.1
+20	0	− .2	−2.8	− .2	40	− .2	−2.8	2.6
−30	0	− .8	2.4	− .8	50	.8	−2.4	3.2
+30	0	− .2	−3.2	.2	60	.2	−3.2	3.4
−40	0	−1.2	3.0	−1.1	70	1.2	−3.0	4.2
+40	0	1.0	−3.1	1.0	80	1.0	−3.1	4.1
−50	0	−2.8	2.2	2.8	90	2.8	−2.2	5.0
+50	0	−2.0	−3.0	2.0	100	2.0	−3.0	5.0
−40	0	−1.4	2.6	−1.3	90	1.4	−2.6	4.0
+40	0	1.0	−2.9	1.0	80	1.0	−2.9	3.9
−30	0	.2	3.3	.2	70	− .2	−3.3	3.1
+30	0	.1	−3.1	.1	60	.1	−3.1	3.2
−20	0	.8	2.9	1.0	50	− .8	−2.9	2.1
+20	0	− .4	−3.0	.4	40	− .4	−3.0	2.6
−10	0	1.0	2.2	1.0	30	−1.0	−2.2	1.2
+10	0	− .2	−1.6	− .2	20	− .2	−1.6	1.4
− 5	0	1.0	1.2	1.0	15	−1.0	−1.2	.2
+ 5	0	.6	− .4	.6	10	.6	− .4	1.0
0	0	.5[2]	.2	.5	5	.5	.2	.7

Curve 2 showing the temporary set under the given conditions is again a much smoother curve initially, indicating a decrement of rigidity with a limit reached at about ±40°. Relatively to curve 1, slip is initially in excess, magnetically incremented rigidity eventually so, as before. The alternations of curves 1 and 2 correspond, but are now in the *same phase*. The initial slope is

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[1] Strains taken positive and signs of deflections changed accordingly.

[2] Zero at −7°, due to permanent set.

$$d\theta/\theta = .003.$$

Curve 3 finally oscillates in irregular fashion about a straight line such that $dn/n = 2d\theta/\theta = .0088$, remembering that the abscissas here are actuating twists and not antecedent strains.

6. *Repetition of a Complete Cycle.*—The following cycles alternate in steps of 10° as far as ±50°, but a stronger field of about 700 c.g.s. units is brought to bear. As a result the helix became warm, and gave rise to a permanent set of about 7°, the zero being found at $-7°$ on unlocking the instrument, after completing the retrograde march.

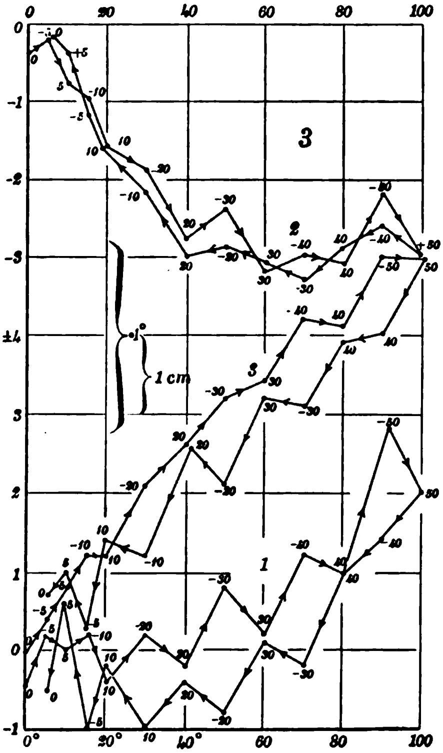

Fig. 3.

The results are given in Table II., following and constructed in in the chart Fig. 3, in the usual way. Curve 1 shows the effect of the first magnetization referred to the original molecular status of the non-magnetic wire. Arrows indicate the progress of the experiment, which begins and ends at zero. The features of the preceding observations definitely reappear. The curve alternates with the twist; its mean value increases with the twist at an accelerated rate, passing from negative values, through a minimum at about 10°; in the outgoing march, the rigidity for the negative twists is greater than the rigidity of the corresponding positive twists; the reverse holds for the return curve. The curve of temporary set, 2, is again decremental in its two branches, and the march is definitely toward a limiting value at about ± 30° or ± 40°. It is smoother at first but oscillates markedly in the region of higher twists. The slope $d\theta/\theta = .0033$.

Finally both branches of curve 3, which shows the magnetically increased rigidity relatively to the new molecular status of the unmagnetic wire, are here given in terms of the antecedent strains as abscissas, merely for the sake of a clearer location of the observations. Relatively to the impressed twists (not shown) both branches may be described as oscillating irregularly about a straight line such that the corresponding elastic increment of rigidity is

$$dn/n = 2d\theta/\theta = .010.$$

The effect of doubling the magnetic field is thus not marked, for the slopes above (.0096 and .0088) are not much smaller, while the field was about half as large. Saturation may be assumed for both cases.

7. *Remarks in Explanation.*—The inferences of my earlier papers (l. c.) will be seen to be corroborated throughout and need not therefore be repeated. What is new, viz., the growth of different phases of the phenomena within ± 50°, has also been fully presented. With regard to slip, which reaches a limit at say ± 40° in the presence of impressed strain, it appears that after passing this value temporary set is more and more fully removed by the direct effect of magnetization, finally to fall off indefinitely.

The character of the curves 1, or the deflection produced on first magnetizing the non-magnetic metal in its original molecular status for the given twist, remains for explanation. The features of this curve are its alternations, which are larger for a negative twist than for the succeeding positive twist of the same value in the outgoing series, with the reversed result in the return series; or vice versa. The peaks of the curves are of different sign in the advance and the retrogression, respectively. This observation is of considerable importance in its bearing on the present phenomena as will be seen in the sequel.

To account for the result it is at the outset sufficient to recognize (as already intimated) that the amount of temporary set or slip depends on the antecedent strain, whereas the increment of rigidity, caet. par., depends on the impressed strain or stress actually in action. Hence if a wire is twisted alternately in steps of $\pm\theta$ to a limit, and in like manner untwisted, the following series will determine the slip and the advance (incremented rigidity) due to the first magnetization of the original metal.

Slip due to Antecedent Strains.		Advance due to Impressed Strains.
θ	Outgoing	$-\theta$
2θ		$+\theta$
3θ		-2θ
4θ		$+2\theta$
$(2n-2)\theta$		$+(n-1)\theta$
$(2n-1)\theta$		$-n\theta$
$2n\theta$	Return	$+n\theta$
$(2n-1)\theta$		$-(n-1)\theta$
$(2n-2)\theta$		$+(n-1)\theta$
3θ		$-\theta$
2θ		$+\theta$
θ		zero

In the outgoing series, where the march of stress is increasing, the even antecedent twists are twice as large as the even (positive) impressed twists. In the odd twists, the former are always relatively less than this excess. Hence the slip in the even twists must be in excess of the slip in the odd twists, and thus the latter (negative) are given by peaks in the curve of rigidity, relatively to the valleys occupied by the former.

On returning the antecedent strains of the odd twists are the larger relatively to the corresponding (negative) twists in action. Hence the slip of the odd twists are relatively in excess and the even twists (positive) thus correspond to the peaks in the curve.

There can be no doubt that this explanation of the alternations of curve 1 is fundamentally correct. On closer inspection, however, an apparant discrepancy is revealed in relation to the curves 2. For whereas the increments of rigidity increase indefinitely with $n\theta$, the limit of curve 3 being as yet far off, the slips reach a minimum very early, as indicated by the curves 2 in question. One would therefore expect the oscillations of the curves 1 to cease at the same early limit.

A key to the situation is already given by the curves 2, which become markedly oscillating after the limit in question has been reached, while the character of these oscillations is reproduced and accentuated in the curves 1, either in the same or the opposite phase.[1] After passing ± 40° of impressed twist, the slip is picked up, as it were, by the concomitant advance of rigidity. Only a part of the total slip is manifest.

The probability of this state of things I have already put in evidence in the results of experiments in which slip in the absence of impressed strain was found to be a continually increasing quantity[2] even beyond 360°.

RESULTS FOR SUCCESSIVELY INCREASING, ASYMMETRICALLY ALTERNATING TWISTS.

8. *Results for Slip Alone.*—Before proceeding to further details, therefore, it is necessary to study temporary set in detail, under circumstances in which there is antecedent twist, but no impressed or acting twist. There must then be slip without the increment of rigidity. This may be done in accordance with the following scheme:

Slip due to Antecedent Strain.	Advance due to Impressed Strain.
θ	θ
θ	zero
2θ	2θ
2θ	zero

etc.,

in which for the even twists the wire is brought back to the unstrained state. In other words the wire is to be twisted successively

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[1] The present discussion is in the main experimental and qualitative. A more detailed theoretical investigation will be given elsewhere.

[2] American Jour. of Science, XI., 1911, p. 105 et seq., §§ 8, 9.

TABLE III.

Twists alternating on one side of zero. Soft iron; diameter, .044, *length* 37 *cm. Scale distance* 600 *cm. Deflection* 21 *cm. equivalent to* 1°. *Magnetic field* 400 *c.g.s.*

Twist of Each Wire.	Scale Reading for Field.[1]			Antecedent[1] Strain.	Deflection[1] Referred to		
					Original Zero.		New Zero
	Off (1)	On (1)	Off (2)		On.	Off.	Total.
°	cm.	cm.	cm.	°	cm.	cm.	cm.
+10	0	.2	− .8	10	Same as column 3	Same as column 4	1.0
0	0	.7	.7	10			0
20	0	.9	−1.1	20			2.0
0	0	1.3	1.3	20			0
30	0	1.1	−1.5	30			2.6
0	0	1.8	1.8	30			0
40	0	1.8	−2.0	40			3.8
0	0	2.6	2.6	40			0
50	0	3.0	−1.4	50			4.4
0	0	3.2	3.2	50			0
60	0	3.5	−1.7	60			5.2
0	0	3.8	3.8	60			0
70	0	4.2	−1.4	70			5.6
0	0	4.2	4.2	70			0
80	0	4.9	−1.2	80			6.1
0	0	4.6	4.6	80			0
90	0	5.2	−1.6	90			6.8
0	0	4.8	4.8	90			0
100	0	6.0	−1.2	100			7.2
0	0	9.2	——	100			0
90	0	5.5	−1.6	90			7.1
0	0	4.6	4.6	90			0
80	0	4.4	−2.0	80			6.4
0	0	4.4	4.4	80			0
70	0	4.1	−1.6	70			5.7
0	0	4.2	4.2	70			0
60	0	3.2	−1.8	60			5.0
0	0	3.6	3.6	60			0
50	0	2.8	−1.6	50			4.4
0	0	3.2	3.2	50			0
40	0	2.0	−1.9	40			3.9
0	0	2.6	2.6	40			0
30	0	1.5	−1.4	30			2.9
0	0	2.0	2.0	30			0
20	0	.8	−1.2	20			2.0
0	0	1.4	1.4	20			0
10	0	1.2	− .6	10			1.8
0	0	.8	.8	10			0

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[1] All strains taken positive for simplicity and the deflections changed in sign accordingly.

to $n\theta$, magnetized during the action of the impressed torque with a record of the scale reading before and after magnetization; then untwisted and a similar record made. Results obtained in this manner are given in Table III. in the usual way. The table consists of two parts, of which the first is the outgoing series of increasing twists, each successively brought back to zero and the temporary set shaken out by magnetization. The second part is the return series of corresponding data. The helix was provided with an internal water jacket to obviate changes of temperature, though continuous effects of this kind (ordinary viscosity) would be eliminated without this precaution. As a whole the two series are coincident, since the difference of a few millimeters in a scale distance of six meters can hardly be vouched for in experiments like the present,

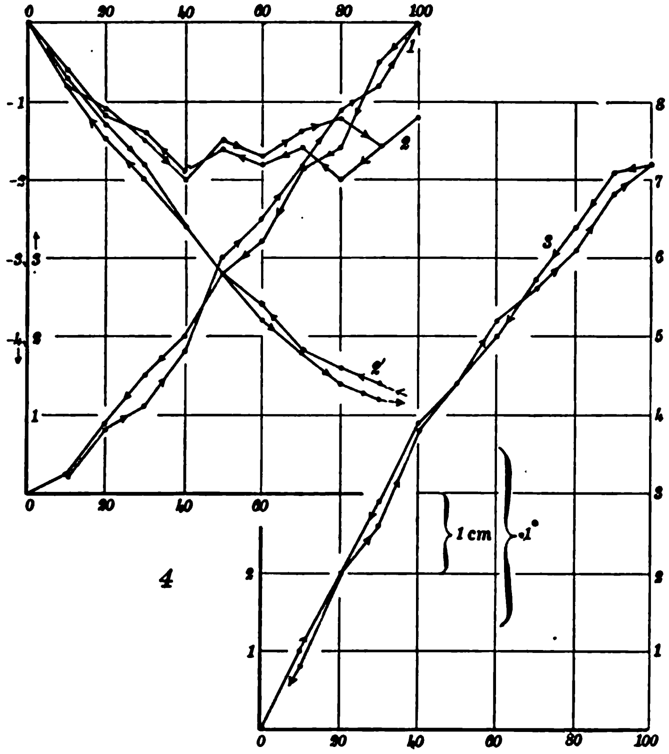

Fig. 4.

where there is inevitably much manual interference in twisting and untwisting.

The graph (Fig. 4) for this table is constructed in the usual way, either the antecedent twists or the impressed twists being the abscissas and the deflections due to changes of rigidity, the ordinates. Curve 1 shows the superposed effect of slip and advance in the presence of magnetic field and of impressed strain, the notation following the above. The curve should be smooth; it is not quite so for reasons presently to be stated. There is a gap due to curtailed ordinates below 40° and resulting from excess of slip in this region. Above 40°, the mean line is nearly straight, and if prolonged backwards would nearly intersect the origin. The slope $dn/n = 2d\theta/\theta = .003$. Direct and return curves are practically identical. The curve shows a radical departure from the corresponding case (1) in Fig. 3, as predicted.

In curve 2, showing the temporary set in the presence of the impressed strain stated, the oscillations begin markedly at about 40°, as usual, in spite of the totally new schedule of operations. The alternations of curve 1 are imaged in the same phase with accentuation throughout, the reverse having been the case above. The direct and retrograde curves are not coincident throughout. The slope is about $d\theta/\theta = .003$.

Curve 2′ is the specially new feature of the present experiments. It shows the amount of temporary set or slip for any antecedent strain in the absence of impressed strain. The smooth curve obtained in this case, practically coincident in the outgoing and the return branches, is evidence of the truth of the inferences expressed in the last paragraph. The curve also comes to a limit eventually, but the limit is considerably beyond the limits of observation. When contrasted with curve 2, the difference in ordinates shows the amount of slip *picked up* by the magnetically incremented rigidity, for any angle of twist. The initial slopes of 2 and 2′ are about identical, $d\theta/\theta = .003$, having about the same value with the sign changed as in curve 1. This curious feature is noteworthy as it occurs again below.

Curve 3 has the usual meaning, being the advance due to the magnetically increased rigidity in the absence of slip. The initial coefficient here is about $dn/n = 2d\theta/\theta = .0092$, the usual value. The curve happens to consist of two nearly linear parts, one between 0° and 40°, the other between 40° and 90°, corresponding to the oscillations of curve 2.

Looking over the whole series of phenomena, one derives the impression that the effects are not quite statistical, *i. e.*, that the number of molecules immediately engaged are not so numerous as in the kinetic theory of gases, for instance, and that for this reason the phenomena fall short of obedience to sharp laws. Sufficient variation was made in the apparatus to throw out errors due to it; but whether the mechanical interference which must inevitably accompany these experiments will not also shake out temporary set, remains to be seen.

TABLE IV.

Twists alternating on one side of zero. Magnetic field 800 *c.g.s., soft iron, diameter* .024, *length* 35 *cm., scale distance* 500 *cm.,* 18 *cm. equivalent to* 1°, *nearly.*

Twist of each Wire.	Scale Reading when Field is			Antecedent Strain.[1]	Total Deflection.[1]
	Off (1)	On (1)	Off (2)		
°	cm.	cm.	cm.	°	cm.
−360	0	9	− 7	360	16
0	0	9	9	"	0
+360	0	−11	5	"	16
0	0	−10	−10	"	0
−360	0	12	− 4	"	16
0	0	10	10	"	0
+360	0	−12	5	"	17
0	0	−10	−10	"	0
−180	0	6	− 6	180	12
0	0	8	8	"	0
+180	0	− 6	7	"	13
0	0	− 8	− 8	"	0
−180	0	6	− 5	"	11
0	0	8	8	"	0
+180	0	− 6	5	"	11
0	0	− 8	− 8	"	0
− 90	0	2	− 5	90	7
0	0	5	5	"	0
+ 90	0	− 3	4	"	7
0	0	− 5	− 5	"	0
− 90	0	2	− 5	"	7
0	0	5	5	"	0
+ 90	0	− 2	4	"	6
0	0	− 6	0	"	0

[1] Strain taken positive for simplicity and deflections changed in sign accordingly.

9. *Case of Larger Twists. Thin Wire.*—The following data for a thinner wire are taken from my earlier work where they occur in a different connection and without being numerically stated. Readings were made to centimeters, not to millimeters as above; but the wire being thinner, showed correspondingly larger deflections.

One may note that while the total deflection (increment of rigidity in the absence of slip) remains nearly constant for any twist, the ratio of the component slip and advance implied may change in marked degree. This is particularly noticeable for the twist of 360°.

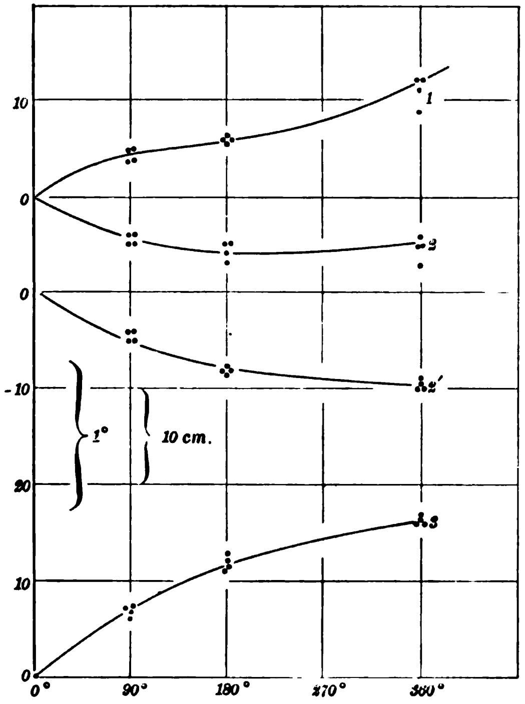

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Fig. 5.

The graphs, Fig. 5, are numbered as in the preceding chart. Remembering that, in view of smaller diameters (.024 cm.), the twists here are relatively greater, results corroborating the preceding paragraph may be deduced. Curve 2 reaches a limit much before curve 2′, the difference being the slip picked up during magnetization by the coincident increment of rigidity. The initial slopes of curves 1 and 2 ($d\theta/\theta = .003$), are nearly equal and opposite; the initial slopes of 2 and 2′ nearly identical, showing the paramount *initial* (vanishing twists) importance of slip. Curve 3 and curve 2′ show similar contour, indicating close relation of slip and advance. The former is throughout numerically somewhat less than twice the latter, in ordinate. Initially, at least, the effect of magnetization may be conceived to be expended in producing slip and advance in the ratio of 1 : 2, though necessarily in opposed directions.

In Table IV. many alternations were made consecutively for each twist selected. This method does not lend itself well for the present purposes where a continuous record is wanted. Hence the following data were investigated.

TABLE V.

Twists alternating on one side of zero. Magnetic field 400 *c.g.s., soft iron, diameter* .044, *length* 37 *cm., scale distance* 600 *cm., deflection* 21 *cm., equivalent to* 1°.

Twist of each Wire.	Scale Reading for Field.			Antecedent Strain.[1]	Total Elastic Deflection.[1]
	Off (1)	On (1)	Off (1)		
°	cm.	cm.	cm.	°	cm.
45	0	2.2	−1.8	45	4.0
0	0	3.5	3.5	"	0
90	0	5.1	−1.3	90	6.3
0	0	4.6	4.6	"	0
135	0	7.3	−1.1	135	8.4
0	0	5.6	5.6	"	0
180	0	8.6	− .8	180	9.4
0	0	5.8	5.8	"	0
225	0	9.4 10.0	− .2	225[2]	9.6 10.2
0	0	5.6	5.8	"	−.2
180	0	8.6	− .9	180	9.5
0	0	4.9	5.1	"	−.2
135	0	7.0	− .9	135	7.9
0	0	4.8	5.1	"	−.3
90	0	4.9	−1.7	90	6.6
0	0	4.3	4.5	"	−.2
45	0	1.6	−2.0	45	3.6
0	0	2.4	2.6	"	−.2

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[1] Strains taken positive and deflections changed in sign accordingly.

[2] Permanent set occurs here. After final unlocking, the zero was at —33 cm., being a set of 1.6°.

10. *Case of Larger Twists. Thicker Wire.*—The observations of Table V. were made for the same diameter and length of soft iron wire occurring throughout the bulk of the above experiments. It being the object to proceed quite up to the limits of elasticity, twists exceeding 200° were necessary, and the data are given in the following table containing both an outgoing series and a return therefrom. The inevitable occurrence of permanent set here before 225° is reached, interferes with coincidence between the two series.

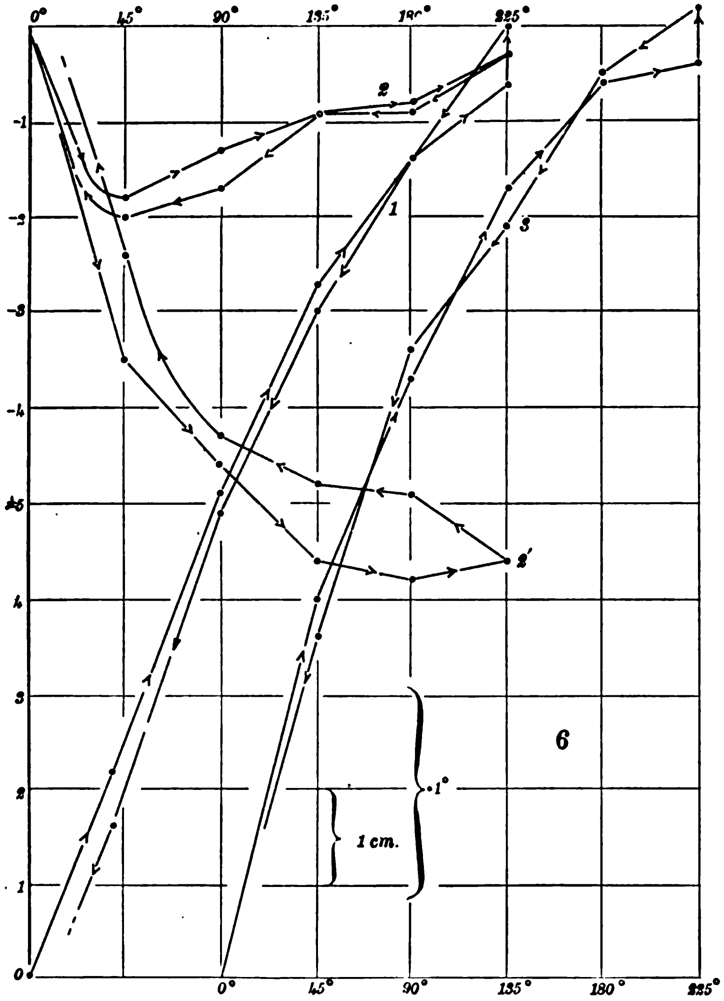

Fig. 6.

The results are given in the accompanying graph, Fig. 6, in the usual way, the abscissas being either antecedent strains or impressed strains as required. Curve 2′ shows the temporary set in the absence of impressed strain, curve 3 the advance due to magnetically increased rigidity in the absence of slip. In curve 1 there is superposed slip and advance in the presence of impressed strain and of magnetization. In curve 2, slip in the presence of impressed strain but absence of magnetization.

The smoothness of curve 1 is marked except at the end, where after the occurrence of *permanent* set, the direct and return curves are shifted laterally about 1 or 2 degrees relatively to each other. The initial slope is $d\theta/\theta = .0026$, somewhat smaller than above, but the initial curve is not so near the initial tangent. Greater smoothness corresponds to less interference, since the steps are 45°. In other respects curve 1 contains but little new information.

The case of curve 2, however, presents novelties. The above sharp minimum at about 40° is substantiated, after which the curve in its progress toward 225° nearly falls off to a vanishing deflection in the region of permanent set. The meandering path between 40° and 100° is again suggested by the new curve. The initial slope practically coincides with 2′.

Similarly definite results are obtained from curve 2′, showing the occurrence of slip alone in the absence of impressed strain. Temporary set reaches a definite limit at about 200°, after which there is *unstable* molecular equilibrium merging into permanent set. This lapse into a new and permanent molecular grouping which cannot again be transferred back by the magnetic shake-up, is the reason for divergence in the outgoing and the return series. The whole of the temporary set does not vanish, however, a natural result seeing that the yield in a practical case is always local and not distributed throughout the wire. The slope here is about $d\theta/\theta = .0037$, but it is not the initial tangent.

Curve 3, finally with an initial slope corresponding to

$$dn/n = 2d\theta/\theta = .0082,$$

which naturally falls short of the above value, rises to a definite limit with the occurrence of set. If this were obviated by much twisting, the limit would be more remote as is known from the earlier experiments.

11. *Conclusion and Summary.*—Before proceeding to definite statements it is expedient to reduce all data to common torsion standards. In my earlier work[1] the obliquity (ω radians) of the external fiber was found to be convenient for reference, while all twists were expressed (in rates τ°) per unit of length. This to some extent eliminates the arbitrary lengths and radii involved. If the twist of each wire is θ°, $\tau = \theta/l$, where l is the length of the wire of radius r. Moreover $\omega = \pi r\theta/180 \times l = \pi r\tau/180$.

For the two lengths and diameters occurring ($2r = .044$, $l = 37$ cm., and $2r = .024$, $l = 35$ cm.), the constants, are therefore, $\omega = 10 \times \theta/10^6$ nearly, and $\omega = 26 \times \theta/10^6$. Thus the limits of temporary set (curves 2′) in the cases of Tables IV. and V. occur at $\omega = .0022$ and $\omega = .0020$ radians, which are practically the same. In my old results operating with curve 3, permanent set occurred when $\omega = .003$ radians; but here the limit was pushed to the utmost verge by hardening the locally weak places by continued twisting. It is understood that ω refers to elastic deformations, no matter how much permanent set may have been previously imparted.

The minimum of curve 2, for temporary set in the presence of impressed strain and evoked by magnetization occurs for $\omega = .0004$ radians in case of Table V. In Table IV. the steps are too large, so that it can be merely located below $\omega = .0008$ radians.

The chief result of the present paper is the detection of the growth conditions of temporary set, a latent strain distinguished from permanent set in that the former may be shaken out of the soft iron wire by molecular agitation. Temporary set $d\theta$ is an instantaneous accompaniment of any twist within the elastic limits; it begins with the zero of twist at an initial rate of about $d\theta/\theta = .0037$, increasing at a retarded rate to a superior limit when the (elastic) obliquity of the external fiber exceeds .002 radians. After this the wire begins to yield locally, gradually, acquiring permanent set. The new molecular condition is stable and can not be shaken out. In soft iron temporary set is made manifest as a slip in the absence of impressed strain on instantly applying a magnetic field following an antecedent strain. The limiting amount of temporary set which can be stored may be estimated as .0015 of the limiting twist within the elastic limits.

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[1] American Jour., XXXIV., p. 182, et seq.

In the presence of impressed strain, a longitudinal magnetic field produces both magnetically increased rigidity and temporary set, as superposed phenomena. For small twists θ the rate is $d\theta/\theta = .003$ and positive. For larger angles the increase is gradually retarded but continues until permanent set is definitely established. Thus magnetically increased rigidity supervenes throughout, being initially about twice the slip. When the antecedent twist differs from the impressed twist, slip is to be associated with the former and advance with the latter and the statement of the result requires a detailed analysis. Cf. §§ 4–7.

The slip in these instances is initially quite as large as in the absence of impressed strain, the initial coefficient being $d\theta/\theta = .0035$. It soon, however, reaches a pronounced minimum (algebraically), showing that slip is more and more arrested or picked up by the concomitant magnetically increased rigidity, until with the occurrence of permanent set, slip (evoked by magnetization in the presence of impressed strain) has all but vanished. The minimum in question is determined by $\omega = .0004$ to .0008, while permanent set does not occur until $\omega = .002$ to .003.

Finally magnetically increased rigidity in the necessary presence of impressed strain and the absence of slip increases with the impressed twist at an initial rate of $d\theta/\theta = .0046$. The coefficient of rigidity is thus of the order of $\mu = dn/n = .01$. As the twist increases the coefficient μ decreases to a limit of about .004, when ω exceeds .002, elastically. If the wire is hardened by indefinite twisting beyond the elastic limits, μ seems to approach a lower limit when ω is elastically .003. In my old results I thus found

$$\mu = .002$$

in a field of about 140 c.g.s. units and a large range of diameters (.2 to .02 cm.). Mr. H. D. Day[1] showed that the limit of μ in very strong fields has not been reached, finding $\mu = .003, .005, .006$, in fields of 100, 500 and 1,500 c.g.s. units, respectively.

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[1] Day, American Journ. Sci., III., 1897, p. 449. Cf. p. 456.

The energy stored and eventually wasted in temporary set may be of passing interest. Since this work is half the product of the stress and the strain given by curves 2′, it appears that in case of Table V. ($2r = .044$) when slip has its largest value (.27°), this work does not exceed 300 ergs. Since the ratios of this work done in twisting and the energy wasted in slip are as the squares of the angles of twist and slip, about $2.4/10^6$ of the work done in twisting is lost in producing temporary set when the latter is at its maximum value just within the elastic limits. Initially the relative waste is much greater, being the square of the initial slope, $d\theta/\theta$, of curve 2′; *i. e.*, the waste is $(.0037)^2 = 14/10^6$, nearly.

The case of Table IV. hardly contains sufficiently detailed data. About 100 ergs would be stored up when the slip is at a maximum (.51°) just within the elastic limits. The corresponding final waste ratio would be but $.4/10^6$. The initial waste ratio is

$$(.0033)^2 = 11/10^6,$$

about of the same order as in the preceding case.

The energy wasted in magnetic hysteresis in double reversals per cubic cm. of saturated soft iron is about 10,000 ergs. Comparing this with the energy wasted in temporary set, Table V. shows that $300/\pi r^2 l \doteq 1,300$ would be wasted per cubic cm., and for a double reversal therefore 2,600 ergs.

From Table IV. it appears that $100/\pi r^2 l = 1,590$, or about 3,200 ergs in a double reversal.

Seeing that it is somewhat difficult to get parallel cases for comparison, it will be noticed that the energy dissipated in a double reversal and saturated soft iron, magnetically in the one case, and mechanically in the other, is of a common order of values; for temporary set must be compared with residual magnetization.

Apart from its inherent importance as an introductory phase of permanent set, temporary set has therefore the additional interest of being a clearcut instance of purely mechanical hysteresis, *without* necessary relation to magnetism, as I understand it. To further establish this it will eventually be necessary to invent means to detect temporary set in other and non-magnetic metals. It follows moreover that the Weber-Ewing model of revoluble molecular magnets is either not the final resource for the physical explanation of magnetism, or else that its polar properties have a much wider application than simply to magnetic phenomena. On the whole the analogy of temporary set with magnetization is attractive; magnetization may be shaken out of soft iron by mechanical tapping; similarly temporary may set be shaken out by magnetization, etc. In addition to the intimate connection of temporary set and magnetic hysteresis, it is also probable that the same slip lies at the root of Kelvin's "fatigue of elasticity," as yet an unclassified phenomenon in the domain of viscosity.

BROWN UNIVERSITY, PROVIDENCE, R. I.

A PRELIMINARY COMMUNICATION ON THE PRESSURE OF HEAT AND LIGHT RADIATION.

By E. F. Nichols and G. F. Hull.

MAXWELL,[1] dealing mathematically with the stresses in an electro-magnetic field, reached the conclusion that "in a medium in which waves are propagated there is a pressure normal to the waves and numerically equal to the energy in unit volume." Bartoli,[2] in 1876, announced that the existence of such a stress was essential to the validity of the second law of thermodynamics. He sought for such a pressure experimentally but failed to get conclusive results. The problem has been discussed theoretically, on the basis of thermodynamics, by Boltzmann[3] and by Galitzine.[4] Lebedew[5] applies such a pressure, opposed to the sun's gravitation, to account for the solar repulsion of comets' tails; and more recently Arrhenius[6] attempts to explain the aurora borealis on similar grounds.

Every approach to the experimental solution of the problem has hitherto been balked by the disturbing action of gases which it is impossible to remove entirely from the space surrounding the body upon which the radiation falls. The forces of attraction or repulsion, due to the action of gas molecules, are functions of the temperature difference between the body and its surroundings, caused by the absorption by the body of a portion of the rays which fall upon it, and of the pressure of the gas surrounding the illuminated body. In the particular form of apparatus used in the present study the latter function appears very complicated, and certain peculiarities of the gas action remain inexplicable upon the basis of any simple group of assumptions which the writers have so far been able to make.

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[1] J. C. Maxwell, Elec. & Mag., 1st Ed., Oxford, 1873, Vol. 2, p. 391; also 2d Ed., Oxford, 1876, Vol. 2, p. 401.

[2] S. Bartoli, Sopra i movimenti prodotti della luce e dal calore: Florence, Le Monnier, 1876; also Exner's Repert d. Phys., 21, p. 198. 1885.

[3] L. Boltzmann, Wied. Ann., 22, p. 31; also p. 291; also Wied. Ann., 31, p. 139.

[4] B. Galitzine, Wied. Ann., 47, p. 479, 1892; also Phil. Mag., 85, p. 113, 1893.

[5] P. Lebedew, Wied. Ann., 45, p. 292, 1892.

[6] S. Arrhenius, Konigl. Vetanskaps-Akademiens, Förhandlingar, 1900; also Phys. Zeitschrift, Nov. 10 and 17, 1900.

Since we can neither do away entirely with the gas nor calculate its effect under varying conditions, the only hopeful approach which remains is to devise apparatus and methods of observation which will reduce the errors due to gas action to a minimum. There are four ways in which the partial elimination of the gas action may be accomplished experimentally:

1. The surfaces which receive the radiation, the pressure of which is to be measured, should be as perfect reflectors as possible. This will reduce the gas action by making the rise of temperature due to absorption small while the radiation pressure will be increased; the theory requiring that a beam, totally reflected, exert twice the pressure of an equal beam completely absorbed.

2. By studying the action of a beam of constant intensity upon the same surface surrounded by air at different pressures, certain pressures may be found where the gas action is smaller than at others.

3. The apparatus—some sort of torsion balance—should carry two surfaces symmetrically placed with reference to the rotation axis, and the surfaces on the two arms should be as nearly equal as possible in every respect save one—the forces due to radiation and gas action should have the same sign on one side and opposite signs on the other. In this way a mean of the resultant forces on the two sides should be, in part at least, free from gas action.

4. Radiation pressure, from its nature, must reach its maximum value instantly, while observation has shown that the gas action begins at zero and increases with length of exposure, rising rapidly at first, then more slowly to its maximum effect, which, in most of the cases observed, was not reached until the exposure had lasted from 2½ to 3 minutes. For large gas pressures an even longer exposure was necessary to reach stationary conditions. The gas action may be thus still further reduced by a ballistic or semi-ballistic method of measurement.

The Torsion Balance.

The form of suspension of the torsion balance, used to measure radiation pressure in the present study, is seen in Fig. 1. The rotation axis *ab* was a fine rod of drawn glass. At *r*, a drawn glass cross-arm *c*, bent down at either end into the form of a small hook,

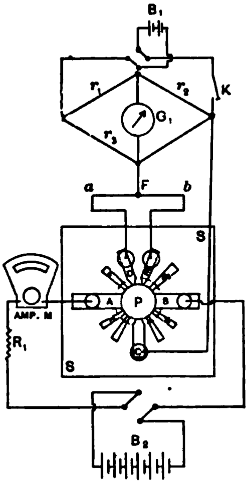

Fig. 4.

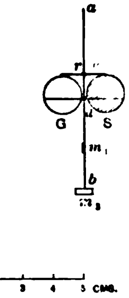

Fig. 1.

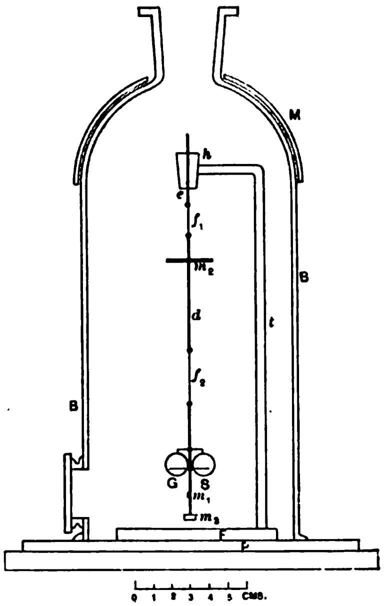

Fig. 2.

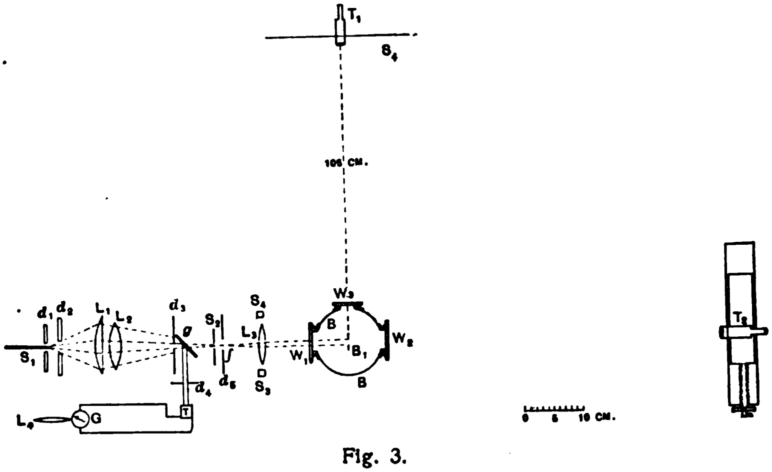

Fig. 3.

was attached. The surfaces *G* and *S*, which received the light beam, were circular microscope cover-glasses, 12.8 mm. in diameter and 0.17 mm. thick, weighing approximately 51 mgs. each. Through each glass a hole 0.5 mm. or less in diameter was drilled near the edge, by means of which the glasses could be hung on the hooks on the cross-arm *c*. Two other drawn-glass cross-arms were attached to the rotation axis at *d*, one on each side. The cover-glasses slipped easily between these, and were thus held securely in one plane. Further down on *ab*, a small silvered plane mirror m_1 was made fast at right angles to the plane of *G* and *S*. This mirror was polished bright on the silver side so that the scale at S_4 (Fig. 3), could be read in either face. A small brass weight m_3 (Fig. 1), of 452 mgs. mass and of known dimensions, was attached at the lower end of *ab*. The cover glasses which served as vanes were silvered and brilliantly polished on the silvered sides, and so hung on the small hooks on *c* that the glass face of one, and the silver face of the other, were presented to the light. A quartz fiber f_2 (Fig. 2), 3 cms. long, was made fast to the upper end of *ab*, and to the lower end of a fine glass rod *d* which carried a horizontal magnet m_2. The rod *d* was in turn suspended by a short fiber to a steel pin *e* which could be raised or lowered in the bearing *h*. The whole was carried by a bent glass tube *t*, firmly fastened to a solid brass foot *F*, resting on a plane, ground-glass plate *P*, cemented to a brass platform mounted on three levelling screws not shown in Fig. 2. A bell-jar *B*, 25 cms. high and 11 cms. in diameter covered the balance. The flange of the bell-jar was ground to fit the plate *P*. A ground-in hollow glass stopper fitted the neck of the bell-jar, which could thus be put in connection with a system of glass tubes leading to a Geissler mercury pump, a MacLeod pressure gauge, and a vertical glass tube dipping into a mercury cup and serving as a rough manometer for measuring the larger gas pressures employed during the observations. The low pressures were measured on the MacLeod gauge in the usual way. A semicircular magnet *M*, fitted to the vertical curvature of the bell-jar, was used to direct the suspended magnet m_2, and thus control the zero position of the torsion balance. By turning *M* through 180°, the opposite faces of the vanes *G* and *S* could be presented to the light.

THE ARRANGEMENT OF APPARATUS.

A horizontal section of the apparatus through the axis of the light beam, is shown in Fig. 3, which like the other figures is, in its essential features, drawn to scale. The white-hot end of the horizontal carbon *S*, of a 90° arc lamp, fed by alternating current, served as a source. The arc played against the end of the horizontal carbon from the vertical carbon which was screened from the lenses L_1 and L_2 by an asbestos diaphragm d_2. The lens L_4 projected an enlarged image of the arc and carbons on a neighboring wall, so that the position of the carbons and the condition of the arc could be seen at all times by both observers. The cone of rays passing through the small diaphragm d_2 fell upon the glass condensing lenses L_1, L_2 which formed an image of the carbon at *f*. At d_3 a diaphragm, 11.25 mm. in diameter, was interposed, which permitted only the central portion of the cone of rays to pass. Just beyond d_3, a plane plate of glass *g* was placed in the path of the beam and reflected part of the beam through the diaphragm d_4, to a thermopile *T*, connected with a galvanometer *G*. The beam which traversed the plate *g* passed on to a shutter at S_2. The shutter was worked by a magnetic escapement, operated by the seconds contact of a standard clock. The observer at T_1 might choose the second for opening or closing the shutter, but the shutter's motion always took place at the time of the seconds contact in the clock. Any exposure was thus of some whole number of seconds duration. The opening in the shutter was such as to let through, at the time of exposure, all of the direct beam which passed through d_3, but shut out the stray light. It was made to correspond in every way to the diaphragm d_4, so that for all of the reflected beam which passed d_4, the corresponding transmitted portion passed S_2 and no more. The glass lens L_3 focused a sharp image of the aperture d_3 in the plane of the vanes of the torsion balance B_1 under the bell-jar *B*. The bell-jar was provided with three plate glass windows W_1, W_2, W_3. The first two gave a circular opening 42 mm. in diameter, and the third was used in connection with the telescope T_1, by which an image of the scale S_4 was seen in the small mirror m_1 (Figs. 1, 2). The lens L_3 was arranged to move horizontally between the stops

S_3 and S_4. The stops were so adjusted that when the lens was against S_3 the sharp image of the aperture d_3 fell centrally upon one vane; and when against S_4 the image fell centrally upon the other. This adjustment, which was a very important one, was made by the aid of a telescope T_2, mounted on the carriage of a dividing engine. This was used to observe and measure the positions of the vanes and the rotation axis, as well as the positions of the images of d_3, when the lens L_3 was against the stops S_3 and S_4. For the latter measurements, the vanes could be moved out of the way, by turning the suspension through 90° by the control magnet M (Fig. 2).

METHODS OF OBSERVATION.

The observations leading to the results given later were of three different kinds: (1) The calibration of the torsion balance; (2) the measurement of the pressure of radiation in terms of the constant of the balance, and (3) the measurement of the energy of the same beam in electrical units by the aid of a bolometer described later.

1. The determination of the constant of the torsion balance was made by removing the vanes G and S and accurately measuring the period of vibration. Its moment of inertia was easily computed from the masses and distribution of the separate parts about the axis of rotation. From the moment of inertia, period, the lever arms of G and S, and the distance of the scale S_4, the constant of the balance for a centimeter deflection was found to be 4.65×10^{-6} dynes.

2. In the measurements of radiation pressure, it was easier to refer the intensity of the beam at each exposure to some arbitrary standard which could be kept constant than to try to hold the lamp as steady as would otherwise have been necessary. For this purpose, the thermopile T (Fig. 3) was introduced, and simultaneous observations were made of the relative intensity of the reflected beam by the deflection of the galvanometer G, and the pressure due to the transmitted beam by the deflection of the torsion balance. The actual deflection of the balance was then reduced to a deflection corresponding to a galvanometer deflection of 100 scale divisions. The galvanometer sensitiveness was carefully tested at

intervals during the work, and was found as nearly constant as the character of the observations required. All observations of pressure were thus reduced to the pressure due to a beam of fixed intensity.

At each gas pressure in the bell-jar, at which radiation pressure measurements were taken, two sets of observations were made. In one of these sets, static conditions were observed, and in the other, the deflections of the balance due to short exposures were measured. In the static observations, each vane of the balance was exposed in turn to the beam from the arc lamp, the exposures lasting in each case until the turning points of the swings showed that static conditions had been reached. The combined pressure, due to radiation and gas action would thus be equal to the product of the angle of deflection and the constant of the balance. The torsion system was then turned through 180° by rotating the outside magnet, and similar observations made on the reverse side of the vanes, after which the system was turned back to its first position, and the earlier set of observations repeated, and so on.

The considerations underlying the above method of procedure, are as follows: The beam from the lamp, before reaching the balance, passed through three thick glass lenses and two glass plates. All wave-lengths destructively absorbed by the glass, were thus sifted out of the beam by the time it reached the glass balance vanes. The silver coatings on the vanes absorbed therefore more than the glass. The radiation pressure was always away from the source irrespective of the way the vanes were turned, while the gas action would be exerted mainly on the silvered side of the vanes. As the face of the vane on one side of the balance was silver, and on the other side glass backed by silver, the two forces should act in the same direction on one vane, and oppositely on the other. If each silver coating reflected equally on its two faces, the average of the deflections on the two arms of the balance, should be free in part, from gas action.

This partial elimination of gas action would be more complete but for two reasons: (1) The reflection at the exposed glass face of one vane diminishes the energy incident upon the silver coating behind it and the resultant absorption in the same proportion. (2) The glass faces of the two vanes, when exposed, will not, in all probability, be equally heated. In the case of the vane which receives the light on its silvered face, the glass face can be warmed only by the conduction of heat from the silver coating through the glass, while in the vane receiving light on its glass face, some heat from absorption of the beam in passing through the glass will doubtless be added to this heat due to conduction. Although the vanes, when freshly silvered and polished, were so brilliant on the silvered sides that it was often difficult to tell which face was silver and which glass, the coatings were found in use to deteriorate rapidly, and the silver faces whitened earlier than the glass. After several hours of observing, vanes freshly silvered at the beginning had to be removed and resilvered. A higher rate of deterioration was noticeable when working in low gas pressures than in high.

By the reversal of suspension and averaging the results gained with the suspension direct and reversed, nearly all errors due to lack of symmetry in the balance, or in the position of the light images with reference to the rotation axis ; or errors due to lack of uniformity in the distribution of radiant intensity in different parts of the image, as well as errors due to certain differences between the mirrors themselves, could be eliminated.

Static observations were made at eight different gas pressures between 0.06 mm. of mercury and 96 mm. Within this range, as was afterwards shown by the series of semiballistic in connection with the static observations, the gas action varied in magnitude between one-tenth, and 6 or 7 times, the radiation pressure. The gas action was, however, at certain pressures so complicated that no rational way of treating the static measurements at the different gas pressures succeeded in establishing the existence of a constant force in the direction of propagation of the light beam. It was plain, therefore, that further elimination of the gas action must be sought in exposures so short that the gas action would not have time to reach more than a small fraction of its stationary value. This led to the method of semiballistic observations.

If the vibrations of any suspension be undamped, a constant deflecting force, acting for one-fourth of the period of oscillation, will cause a throw equal to $\sqrt{2}$ times the permanent deflection for the same force. A correction for damping when considerable may also be introduced. The period of the balance as described was 23.1 secs. This was increased to 24 secs. by adding masses to the system. Deflections due to six seconds' exposure were then observed in the same manner, with reversals and all, as has been already described under the static observations. Because of the shorter time involved in making individual observations, a considerably greater number of ballistic measurements were made at each gas pressure. In all the observations yet made, not a single ballistic deflection contrary to the direction of radiation pressure was obtained, and in the static observations, the first throw on exposing was invariably in the radiation pressure direction, irrespective of the direction of the final deflection.

The average ballistic deflection obtained, reduced to the equivalent permanent deflection for each of eight different gas pressures, is given in the accompanying table. The column p gives the gas pressure in the bell-jar in millimeters of mercury, and column d the reduced permanent deflection in millimeters for the radiation pressure.

p	d	p	d
96.3	19.7	33.4	21.1
67.7	21.0	1.2	20.9
37.9	21.6	0.13	26.8
36.5	22.1	0.06	23.2
Ave.			22.5

For gas pressures between 45 and 30 mm. the gas action is small and increases toward both higher and lower pressures in the range studied. Taking the product of the average deflection in cms. by the constant of the torsion balance, the radiation pressure was $2.25 \times 4.65 \times 10^{-5} = 1.05 \times 10^{-4}$ dynes.

The Bolometer.

To test the above value of radiation pressure with the theory, it was necessary to measure the energy of the radiation causing the pressure. This was attempted with the aid of a bolometer of special construction.

On a sheet of platinum 0.001 mm. thick, rolled in silver (by the firm Sy & Wagner, Berlin), a circle *P* (Fig. 4), 11.3 mm. in diameter, was drawn. The sheet was cut from the edges inward to the circumference of the circle, in such a way as to leave five principal strips *A*, *B*, *C*, *D*, *E*, connected to the circle in the manner shown. Other narrower strips, as *e*, *m*, *n*, *o*, etc., were left to give the disk additional support. The disk, by means of the connecting arms, was mounted with asphalt varnish centrally over a hole 14 mm. in diameter, bored through a slab *S* cut from a child's school slate. Portions of the silver not to be removed by the acid were carefully covered by the asphalt varnish. Thus on the strips *A* and *B*, the silver was protected to the very edge of the circle, while on all the other arms, the silver was left exposed back to the edge of the boring in the slate. The whole system was then plunged into warm nitric acid, and the silver eaten away from all unprotected surfaces, leaving only the thin platinum sheet, which was blackened by elec tric deposition of platinum, by the method used in the manufacture of the Reichsanstalt bolometers. At *A*, *B*, *C*, *D*, *E*, holes were bored extending through the slate, and copper washers were soldered to the silver strips, and binding posts were attached.

The torsion balance was removed from under the bell-jar, the bolometer put in the place of one of the vanes, and the bell-jar replaced. Connections to the bolometer were made as schematically shown in Fig. 4. The disk *P* was the exact size of the light image thrown on the vanes in the pressure measurements, and the intention was to heat the disk by allowing the image to fall on it, and then with the light turned off, to heat it to the *same temperature* by sending a current through it from *A* to *B*. If r be the resistance from *A* to *B* in ohms, when exposed to the lamp, and i be the current in ampères which gives the same temperature in *P* as that given by the absorbed radiation, then $i^2r \times 10^7$ will be the activity of the beam in erg-seconds. The temperature of the disk, whether exposed to the radiation or heated by the current, was shown by the resistance of the disk in the direction *C* to *D* and *E* which was made one arm of a Wheatstone Bridge. The relation of the heating current to the bridge was adjusted as follows: With the key K open, so that no current flowed through the bridge, the heating current from six storage cells B_2 was turned on, and the sliding contact at F so set that the bridge galvanometer zero was not changed by reversing the heating current. The equipotential point to c was found very near the middle of the wire ab, which showed the current distribution of P to be symmetrical with respect to a diameter at right angles to AB. The key K was then closed making the bridge current, and the bridge was balanced. The bolometer was next exposed to the radiation, and simultaneous observations were made on the two galvanometers G (Fig. 3) and G_1 (Fig. 4) and the deflection of the latter, was reduced to that corresponding to a deflection of 100 divisions of galvanometer G, as was done in the radiation pressure observations. During the pressure, as well as the bolometer measurements, the electric lamp was regulated by the observer at the galvanometer G, and the lamp held reasonably close to this standard value. The reductions were, therefore, of the simplest kind. After shutting off the light, the heating current was turned on and regulated by means of the variable resistance R_1 (Fig. 4), so that nearly the same throw was obtained from the galvanometer G, as when the bolometer was exposed to the lamp. All deflections of the galvanometer G_1 were taken with the bridge current both direct and reversed, to eliminate any local disturbances in the bridge. With the heating current on, galvanometer deflections were read with the heating current both direct and reversed. The heating current was read in ampères from a Siemens & Halske direct-reading precision milliampèremeter. From repeated observations the current which gave the same heating effect as the light beam, was $i = 0.865$ amp. The resistance between the binding posts A and B, was measured with the lamp on, and gave $r = 0.278$ ohm. The intensity of the beam in erg-seconds was thus $= 0.278 \times 0.75 \times 10^7$.

Considering the beam as a cylinder containing energy which is transmitted at the velocity v of radiation, the energy contained in a length of 3×10^{10} cms. would, according to the bolometer results be $i^2r \times 10^7$, and the consequent pressure in dynes, against the totally absorbing end of the cylinder must, according to Maxwell, equal the energy per unit length, or

$$p = \frac{i^2r \times 10^7}{v}.$$

If the incident beam falls upon a totally reflecting surface the energy per unit volume is doubled and the deduction from Maxwell is that the pressure is equal to $\frac{2i^2r \times 10^7}{v}$.

According to Bartoli and Boltzmann, the pressure due to a beam totally reflected must on thermodynamic grounds also be $\frac{2i^2r \times 10^7}{v}$. If the beam is but partially reflected, however, the pressure would be

$$p = \frac{(1 + r_1)i^2r \times 10^7}{v},$$

where r_1 equals the percentage of the incident energy reflected Considering the character of the light from the arc lamp, and the heavy infra-red absorption of the glass masses traversed, the reflecting power of the bright silver coatings was estimated to be nearly that for the wave-lengths of the D lines, or about 92 per cent.[1] Putting $1 + r = 1.92$ we have from the bolometer results

$$\frac{1.92 \times 0.278 \times 0.75 \times 10^7}{3 \times 10^{10}} = 1.34 \times 10^{-4} \text{ dynes}$$

as the pressure which the torsion balance should have shown, instead of the observed value 1.05×10^{-4}. Some of the mirrors used in the pressure observations were already well below this maximum reflecting power, so that the value 1.34×10^{-4} should doubtless be still further reduced, but no quantitative value can be assigned to such a correction.

Taking the results as they stand, the measured radiation pressure is to the radiation pressure which the theory applied to the bolometer measurements would require as 1.05 : 1.34 or as 78 : 100.

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[1] E. F. Nichols, PHYS. REV., 4, p. 303 also F. Paschen, Ann. Phys., 4, p. 304, 1901.

Some sources of uncertainty in the pressure measurements have already been mentioned, but certain sources of error in the bolometer measurements in which the nature of the effect upon the result is known, should be more clearly indicated.

1. Errors in the bolometer which would tend to produce a greater divergence between the two results obtained are:

(*a*) The reflection of radiation by the platinum-black coating (this error is small) and (*b*) the heat conductivity of the black coating. In the case of the lamp, the transformation of radiant energy into heat takes place near the outer surface of the platinum-black covering, and the electrical resistance of P is affected only after the heat has passed by conduction through this coating to the sheet of platinum underneath. In the case of heating by the current, on the other hand, the heat is generated in the conducting sheet directly, and is dissipated at the outer surface after conduction through the black layer. This error is probably small also.[1]

2. Errors which would tend to bring the two results into closer agreement are two in number. (*a*) The measured resistance between the binding posts A and B must be greater than that of the disk P by the resistance of the silver strips leading from the binding posts on both sides to P, and by any contact resistances involved. The magnitude of this error is unknown, but it may be large. (*b*) The distribution of current lines of flow for both heating current and bridge current would be such as to cause an error in the result of the same sign as the resistance error. The maximum heating would occur where the silver strips A and B join the disk, for here the stream lines will be most congested; on the other hand, the stream lines of the bridge current in these regions will be most sparse because nearly all the lines traversing the disk on the two sides will be drawn off into the edges of these same silver strips by the higher conductivity of the latter. This error may also be considerable. In a new bolometer now under construction the errors due to false resistance and unfavorable distribution of flow lines will be greatly reduced. Another series of radiation pressure measure-

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[1] See Kurlbaum: Temperaturdifferenzen zwischen der Oberfläche und dem Innern eines strahlendes Korpers. Ann. Phys., 2, p. 554.

ments, made under more favorable conditions than those here reported, is also in progress.

The writers believe that the observations already in hand are sufficient to prove experimentally the existence of a pressure, not due to gas molecules, of the nature and order of magnitude of radiation pressure, but toward a close quantitative measurement of this pressure much remains still to be done.

THE WILDER LABORATORY, DARTMOUTH COLLEGE,
HANOVER, N. H., August, 1901.

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THE

PHYSICAL REVIEW.

DISCHARGE OF ELECTRICITY FROM GLOWING PLATINUM AND THE VELOCITY OF THE IONS.[1]

BY E. RUTHERFORD.

THE fact that glowing bodies discharge positive and negative electricity with unequal facilities, has been shown by many observers. The most detailed examination of this subject has been made by Elster and Geitel.[2] They found that when a platinum wire was heated to a bright red heat, in an atmosphere of air or oxygen, at low pressure, a cold metal plate in its neighborhood discharged negative electricity more readily than positive. If, however, a thin platinum wire or carbon filament is heated in an atmosphere of hydrogen at low pressure, the cold plate discharges positive electricity more readily than negative.

The object of the present investigation was to test the validity of certain results obtained from a theoretical consideration of the conductivity of gases when the ionization was confined mainly to the surface of one electrode. A very simple means of obtaining strong surface ionization is to heat a platinum wire or plate to a red heat by an electric current, and this method has been employed in the course of the investigation. An account of the theory as given in a recent paper by the author,[3] in order to explain the differences observed in the value of the current through ionized gases when the electric field was reversed. When these investigations were in progress C. D. Child[1] published a paper in which a similar theory was worked out and applied to determine the velocity of ions drawn from a flame and from a carbon arc.

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[1] A paper read at the Denver meeting of the American Association for the Advancement of Science, August, 1901.

[2] Wied. Annal., 37 and 38.

[3] Phil. Mag., August, 1901.

A brief account of the theory will first be given and then a comparison between the theoretical and experimental results.

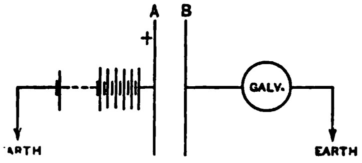

Fig. 1.

Let A (Fig. 1) be a platinum plate, the surface of which is to be a source of ionization; B, a metal plate, placed parallel to it and distant d apart. The plate A is connected to one pole of a battery of small accumulators, the other pole of which is to earth. The plate B is connected to earth through a sensitive galvanometer. Let v be the P.D. between A and B.

Suppose the plate A charged positively. When the plate A is heated to a red heat, a current is observed to pass through the galvanometer. In order to produce this current, positive ions, produced either from the heated plate or the gas in its immediate neighborhood, pass across from A to B.

Let $i =$ current per sq. cm. through the gas.

$n =$ number of ions per cc. at any point distant x from A.

$e =$ charge on the ion.

When the current is steady, the same current passes over any any cross section between the plates,

$$i = -Kne\frac{dv}{dx},$$

where $K =$ velocity of positive ion per unit potential gradient and $\frac{dv}{dx}$ is the electric force.

By Poisson's equation,[2]

$$\frac{d^2v}{dx^2} = -4\pi ne.$$

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[1] See J. J. Thomson, Phil. Mag., 47, p. 465.

[2] Physical Rev., Feb. and March, 1901.

Eliminating n between these two equations, we obtain

$$2\frac{dv}{dx}\cdot\frac{d^2v}{dx^2}=\frac{8\pi i}{k}=a, \text{ say.}$$

Integrating

$$\left(\frac{dv}{dx}\right)^2=ax+B.$$

Now B is the value of $\left(\frac{dv}{dx}\right)^2$ at the surface of the heated plate and can be made as small as we please by increasing the number of ions produced at the surface of the plate. This can be easily seen from first principles, for the higher the temperature the greater the number of ions produced per second and consequently the smaller the value of the electric field $\frac{dv}{dx}$ necessary to withdraw from the neighborhood of the plate, the number of ions requisite to convey the current. The potential gradient between the plates was experimentally examined and it was found that the slope of potential was extremely small near the heated plate and was greatest near the plate B. If the ionization is intense we can thus make B zero without any appreciable error.

Then

$$\frac{dv}{dx}=-a^{\frac{1}{2}}x^{\frac{1}{2}}.$$

Integrating between the limits 0 and d

$$v=-\tfrac{2}{3}a^{\frac{1}{2}}d^{\frac{3}{2}}$$

or

$$v=-\tfrac{2}{3}\sqrt{\frac{8\pi i}{k}}\cdot d^{\frac{3}{2}}$$

and

$$i=-\frac{9v^2k}{32\pi d^3}.$$

The ionization theory, applied to the question of the current between two parallel plates, when the ionization is powerful and confined mainly to the surface of one electrode, thus leads to the following conclusions :

1. The current is independent of the intensity of the ionization, *i. e.*, of the temperature of the plate, beyond a certain limiting value; for the equation, giving the current, does not involve the intensity of the ionization at the surface of the plate.

2. The current is directly proportional to the square of the voltage between the plates.

3. The current is directly proportional to the velocity of the ions and inversely as the cube of the distance between the plates.

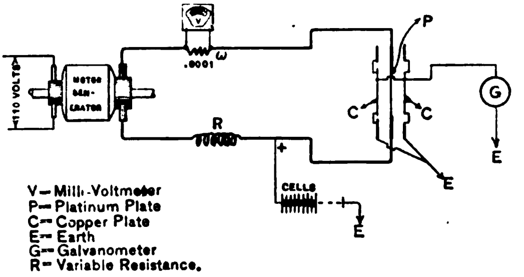

Fig. 2.

Experimental Arrangement.

The platinum foil, which served as a source of ionization, was rectangular in shape, 14 cms. long, 7.5 cms. wide, and .002 cm. thick. It was fastened at each end to solid copper plates by means of silver solder.

The upper plate was attached to a copper rod, through which the current entered, and the lower plate dipped freely into a mercury trough. The foil was thus always in a vertical plane and no correction was necessary for sag of the foil with rise of temperature. The experimental arrangement can be clearly seen from Fig. 2. The current was supplied from a motor generator, from which a current of 150 ampères, at 15 volts, could be readily obtained. The current was measured by a Weston millivoltmeter off a standard .0001 ω and could be varied by a low resistance rheostat in the circuit. The whole system was moderately well insulated and could be charged up to 600 volts above the earth, by means of a battery of small accumulators, one pole of which was connected to earth. The current was measured by means of a sensitive galvanometer between the platinum plate and two vertical copper plates *c* and *c*, placed at equal distances from opposite sides of the platinum foil. In order to obtain a uniform electrostatic field these copper plates were surrounded by guard-rings and additional side plates were fixed near platinum plate and in the same vertical plane.

The galvanometer was always slightly disturbed by the magnetic field due to the large current circulating in the leads. This was compensated for by placing a coil of wire, shunted off the .0001 ω near the galvanometer.

The sensitive galvanometer employed was of about 100,000 ω resistance and gave a deflection of 1 mm. at about 2 meters distance for a current of 3.50 10^{-10} amps.

Experimental Results.

The platinum plate began to discharge positive electricity at a temperature just below a dull red heat. The deflection for a given P.D. between the plates increased rapidly with rise of temperature. The effect of increase of temperature of the platinum plate on the amount of current for a given P.D. varies with the distance between the plates. For a distance of 3 cms. and over, the current first increased rapidly to a maximum, remained practically constant over a short range, and then steadily diminished with increase of temperature to a white heat. For a distance of 2 cms. and under, the current first increased rapidly and then more slowly with rise of temperature, but did not pass through any maximum.

Curves I., II., III. and IV., Fig. 3, show the general nature of the results, where the ordinates represent current (on different scales) and abscissæ, ampères, through the platinum plate. The plate begins to discharge positive electricity at about 70 amèpres. For distances 3 to 8 cms. a practical maximum is reached for a current of about 100 ampères.

Between 2 and 3 cms. the curves change in shape.

The shape of the curves for a given distance between the plates was altered to some extent by increase of the P.D. between the plates.

According to the theory, already considered, we should have expected the current to reach a maximum for a given P.D. and then remain constant, however much the amount of ionization at the surface of the plate was increased. In the experimental case, however, the theoretical conditions are not fulfilled. In order to increase the rate of production of ions, the temperature of the platinum plate is increased and the air between the plates is heated. Since the ions move faster in the heated gas and the current depends on the velocity of the carrier, it would naturally be expected that the current would first increase rapidly and then more slowly, as the temperature of the plate was raised. This is experimentally observed for a distance of 2 cms. and under.

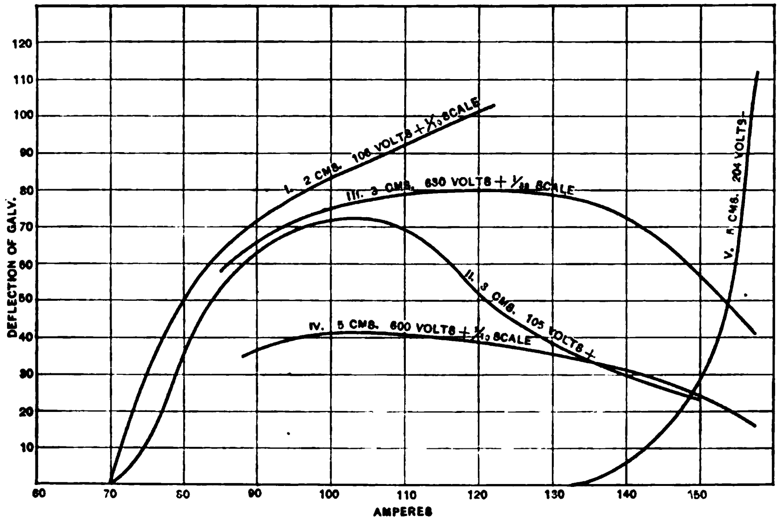

Fig. 3.

The explanation of the decrease of the current with rise of temperature for distances greater than 3 cms. will be given in detail later in the paper (page 329). It is there shown that the diminution is probably due to the presence between the plates of some slowly moving carriers.

Negative electricity was not discharged from the platinum plate till a white heat, with a maximum current of about 135 ampères. Over the wide range of temperature corresponding to the difference between the currents 70 and 135 ampères, positive electricity only can be discharged.

Variation of the Current with Voltage.

The current through the platinum plate was kept approximately constant at 100 ampères. This corresponds to the maximum current through the gas for the distance of 3 cms. and above. The current was found to vary approximately as the square of the P.D. between the plates.

Distance between platinum and copper plate, 2 cms.
1 mm. of scale $= 3.50\ 10^{-10}$ amps.

P.D. in Volts.	Deflect. of Galv. in mms.	$\frac{\sqrt{\text{deflection}}}{\text{volts.}}$
26	27	.200
52	115	.206
104	470	.208
156	1150	.217
208	2178	.224
312	4540	.216
364	6120	.215
468	10340	.217
624	18800	.219

Suitable shunts to galvanometer were employed for the larger currents. In the third column is given the value of the square root of the deflection divided by the P.D. in volts. This should be a constant if the current is proportional to the square root of the voltage. The above results show that this is very approximately the case. The interpretation of the results for lower voltages is complicated by the P.D. between terminals of platinum plate due to the passage of the current through it.

Plates 3 cms. apart.

P.D. in Volts.	Deflection.	$\frac{\sqrt{\text{deflection}}}{\text{volts.}}$
52	15	.074
104	63	.076
156	142	.076
208	267	.078
312	667	.080
468	1466	.082
624	2690	.083

In both these cases, the deflection increases slightly more rapidly than the square of the P.D. This is probably due to the fact that the ions increase in size and thus diminish in velocity, the longer the time taken to travel between the electrodes. The results obtained for distances up to 8 cm. are similar to those given above.

Variation of the Current with Distance Between the Plates.

From the theory, the current between the plates for a given P.D. should vary inversely as the cube of the distance between them. This relation is, however, widely departed from in the experimental case.

The following table shows the way in which the current, for a temperature corresponding to about 100 amps., varies with the distance between the plates for a P.D. of 624 volts.

Distance d in cms.	Current i.	$i \times d^3$
2	18800	150,000
3	2690	72,600
4	835	53,400
5	296	37,000
6	156	33,700

The above values were taken from observations at different times and are only approximate.

The current thus falls off more rapidly than the inverse cube of the distance. The value id^3 should be equal if the ionic velocities are constant for different distances between plates. It will be shown later, however, that in consequence of the air being heated near the platinum plate, the velocity of the carrier is greater near the platinum plate and diminishes as we proceed from the plate.

The average velocity (as will be shown later) of the ion for the plate 2 cms. apart, is about 7.8 cms. per second, for a potential gradient of 1 volt per cm. and for 5 cms. 4.7 cms. per second. Since K is proportional to $\frac{id^3}{v^2}$, when K is increased, $\frac{id^3}{v^2}$ increases in like ratio.

The results on the variation of the current with voltage and the variation with distance between the plates, are thus in general agreement with the theory of ionization, taking into account the differences between the theoretical and experimental conditions.

The diminution of current with distances of 3 cms. and over for high temperatures, is, however, quite contrary to the result to be expected from the simple theory and has therefore been made a matter of detailed investigation.

The phenomena can be satisfactorily explained if we suppose that, above a certain temperature, the current is carried by two or more kinds of carriers, some of which move much slower than the others. Let us for simplicity suppose the carriers between the plates are of two kinds, but of the total current i per sq. cm. between the plates, the current i_1 is conveyed by carriers of velocity K_1 and the current i_2 by carriers of velocity K_2. Then $i = i_1 + i_2$.

$$\frac{d^2 v}{d x^2} = -4\pi (n_1 + n_2) e,$$

where n_1 and n_2 are the number of ions of the two kinds at any point distant x from the platinum plate ;

$$i_1 = -K_1 n_1 e \frac{dv}{dx},$$

$$i_2 = -K_2 n_2 e \frac{dv}{dx}.$$

Therefore

$$\frac{i_1}{K_1} + \frac{i_2}{K_2} = -(n_1 + n_2) e \frac{dv}{dx},$$

or

$$2 \frac{dv}{dx} \cdot \frac{d^2 v}{dx^2} = 8\pi \left(\frac{i_1}{K_1} + \frac{i_2}{K_2}\right).$$

Integrating twice as before, and putting $\frac{dv}{dx} = 0$ at the origin, as in the simpler theory, we finally obtain,

$$\frac{i_1}{K_1} + \frac{i_2}{K_2} = -\frac{9v^2}{32\pi d^3}$$

or

$$\frac{i}{K_1}\left[1 + \frac{i_2(K_1 - K_2)}{iK_2}\right] = -\frac{9v^2}{32\pi d^3}.$$

For a given voltage and distance, the right hand side is a constant and the value of i depends on the value of the second term of the left hand side of the equation. If K_2 be small compared with K_1, the value of the term may become considerable even though i_2 may be small compared with i_1. For example, if

$$K_1 = 100K_2$$

and

$$i_1 = 100i_2$$

the limiting value of the current i is only half of its value if no such slow moving carriers were present.

The effect of the presence of slow moving carriers between the plates can be readily shown by a simple experiment. The current between heated platinum plate and copper plate 3 cms. apart, was observed on the galvanometer. A slight current of tobacco smoke, or smoke from mouldering cotton waste, was allowed to pass between the plates. The current instantly fell to less than half its value. When the current of smoke ceased, the current immediately regained its former value. The explanation is simple. Some of the positive ions in their passage between the plates became attached to the large smoke nuclei, which in consequence of their size moved very slowly in the electric field. Although the number of such carriers was probably extremely small compared with the number of ions at any given time between the plates, the effect on the value of the current was very marked.

Some information as to the presence of slow moving carriers can be gained by directing a rapid current of air between the plates. In a special experiment, a current of about 100 ampères was kept through the platinum plate, which corresponded to about the temperature of the maximum current between plates 5 cms. apart. On directing a current of air from bellows between the plates the deflection of the galvanometer rose from 65 to 84 divisions and remained constant if current of air was kept constant. The current through the platinum plate was then raised to 150 ampères and the current without the blast fell to 30, and on directing a current of air of the same velocity as before between the plates, the current rose to 60. In the one case, the current increased twice and in the other, 1.3 times. The current of air was not sufficiently rapid to blow out between the plates many of the swifter moving ions, but even a comparatively slow current of air increased the current quite appreciably, showing that some slow moving carriers were probably present at the lower temperature. The greater increase of current with the blast for the high temperature would show that more slow moving carriers were present between the plates than at a lower temperature. This is in agreement with the results on the decrease of current between the plates with temperature.

The shapes of the curves expressing the relation between current and ampères through the platinum plate (Fig. 2) are seen to depend both on the distance between the plates and the P.D. A few examples, sufficient to illustrate the general result, are given in the curves of Fig. 2. For a distance of 2 cm. and under, the current (curve I.) increased steadily with the temperature, whatever the voltage. For a distance of 3 cm. and over, the current always reached a maximum for a current of about 110 amps. and then diminished. The ratio to which the current was decreased by rise of temperature was smaller, the larger the P.D. acting. This is shown by the curves II. and III., for plates 3 cm. apart and 105 and 630 volts between the plates. For 630 volts, the current does not decrease so soon or so much as for 106 volts. For intermediate voltages, the general shapes of the curves are intermediate between curves II. and III.

The relation between temperature of the plate and the current for a given voltage receives a general explanation, if we suppose that the number of the slow moving carriers, appearing between the plates, increases with the temperature and the voltage, but that their number does not increase so rapidly as the current between the plates, which increases, as we have seen, nearly as the square of the P.D. This will suffice to explain the differences between the curves II., III. and IV. For a change of distance from 3 to 2 cm., the current increases over 10 times and in a greater proportion than the number of the slow moving carriers. The increase of velocity of the ions, due to the average rise of temperature of the gas with increase of current through the platinum plate, more than compensates for the increase of the number of slow moving carriers and the current consequently increases with rise of temperature of the platinum plate.

At the present state of our knowledge, no very definite explanation can be given of the origin of the slow moving carriers. As no sign of smoke or fumes could be seen rising between the plates, the action was not likely due to such nuclei. There are, however, two possible explanations of the presence of these large carriers. In the first place it is well known that a glowing platinum wire loses in weight. It is probable that this loss of weight is due to the disintegration of the surface of the platinum. The particles of metal shot off would probably be large compared with a molecule, and in their slow passage between the plates would, as in the case of the smoke nuclei, acquire a charge from the ions through which they move. This explanation would also serve to account for the diminution, in some cases, of the current with rise of temperature where the plate is negatively charged (see Fig. 8, curves for 53 and 102 volts).

On the other hand there is considerable evidence that in the discharge of electricity by flames and glowing platinum, the ions between the plates are not all the same size.

Whether these carriers are initially unequal in size or have different powers of acting as nuclei for the condensation of matter around them is difficult to settle experimentally. There seems to be little doubt that some ions, for example those present in flame gases, increase rapidly in size as the flame gases diminish in temperature. The results of Maclelland[1] on the velocity of ions drawn from a flame point strongly to this conclusion.

We have also the well-known fact that negative ions produced by Röntgen rays increase in size when the gas in which they are produced is saturated with water vapor.

DISCHARGE OF NEGATIVE ELECTRICITY.

Negative electricity could not be discharged for low voltages until the platinum plate had nearly reached a white heat with a current of about 135 ampères. The relation between current and P.D. could only be examined over a range of 135 to 165 ampères, which was the maximum current that could be employed in the experiments. The results are embodied in curves in Figs. 4 and 5.

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[1] Proc. Roy. Soc., 1899.

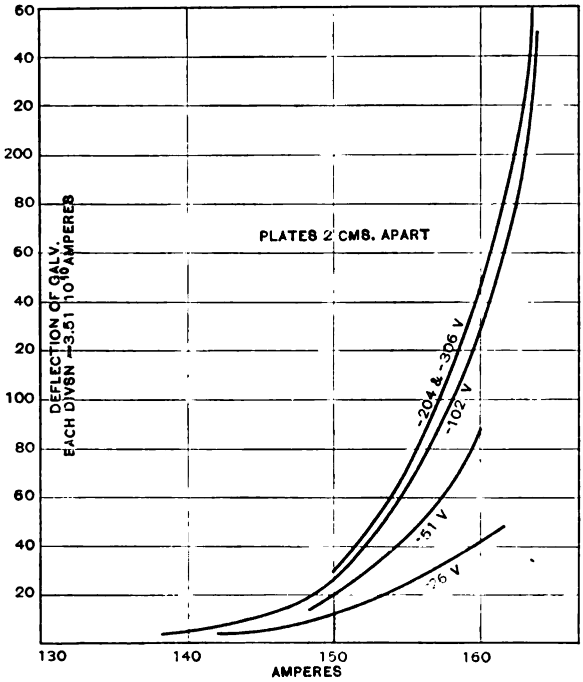
60
40
20
200
80
60
40
20
100
80
60
40
20
DEFLECTION OF GALV.
EACH DIVSN =3.51 10-10 AMPERES
PLATES 2 CMS. APART
-204 & -306 V
-102 V
-51 V
-26 V
130
140
150
160
AMPERES

Fig. 4.

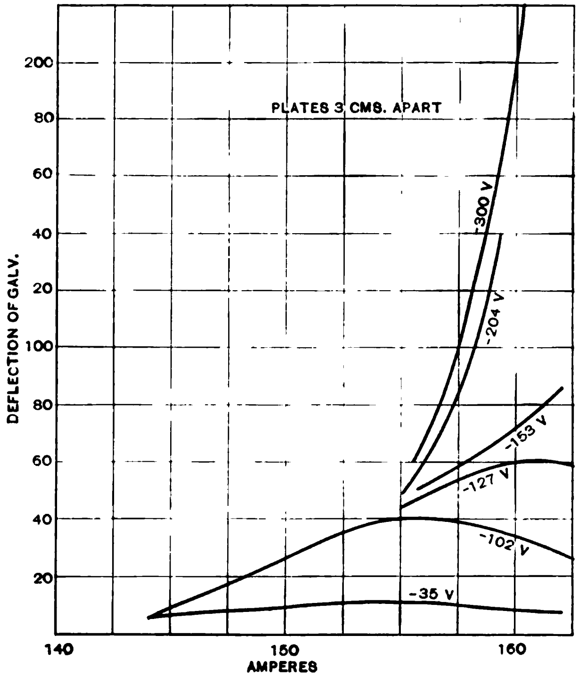
200
80
60
40
20
100
80
60
40
20
DEFLECTION OF GALV.
PLATES 3 CMS. APART
-300 V
-204 V
-153 V
-127 V
-102 V
-35 V
140
150
160
AMPERES

Fig. 5.

In Fig. 4 the results are given for a distance of 2 cms. between the plates for voltages varying from —26 to —306 volts. It will be observed that in all cases the current increases rapidly with increase of temperature. There is very distinct evidence that for a given temperature the current reaches a maximum and is not much affected by increase of voltage from —200 to —300 volts, showing that all the negative ions produced are carried across. As a very slight variation of the temperature causes a very large increase in the current, the results given for comparison are only approximate.

For plates 3 cms. apart a very distinct difference is observed. In Fig. 5 the results are given for P.D. ranging from 53 to 300 volts. From —53 and to — 102 volts, the current reaches a maximum and then decreases with rise of temperature, and, in this respect resembles curves for the positive discharge. For 127 volts P.D., the current is nearly constant at its higher portions, while from 153 to 300 volts the current is found to increase continuously and rapidly with increase of temperature. The temperature of the platinum plate has not been pushed high enough to obtain sufficient ionization at the surface for the current, due to negative ions, to vary as the square of the P.D. For this reason no attempt has been made to compare the velocity of the positive and negative ions.

Temperature Gradient Between Plates.

In the experimental arrangement the plates were vertical and exposed freely to the air on all sides. The average temperature of the gas between the plates is shown in Fig. 6, for distances of 2 and 3 cms., when a current of 124 amps. was passing through the plates, which corresponded to a bright red heat.

The temperatures were determined by a platinum rhodium thermocouple, with a special apparatus, designed and calibrated by H. M. Tory, M.A., of McGill University, for the measurement of high temperatures.

It was observed that the platinum plate was appreciably cooled by conduction at the point of contact of the junction with its surface. For this reason the indicated temperature of the plate was much too low. For a distance of 2 cms. between the plates, the indicated temperature of the platinum plate was 991° C., for a current of 134 amps., and was probably at least 100° C. too low. The temperature fell extremely rapidly near the surface of the platinum plate, became approximately constant for some distance midway between the plates, and increased slightly near the copper plate. This is to be expected since the copper plate was heated by radiation to a higher temperature than the surrounding air. The lowest temperature observed for the air between the plates was 169° C. for 3 cms. apart and 281° C. for 2 cms. With increase of distance the average temperature between plates steadily decreases.

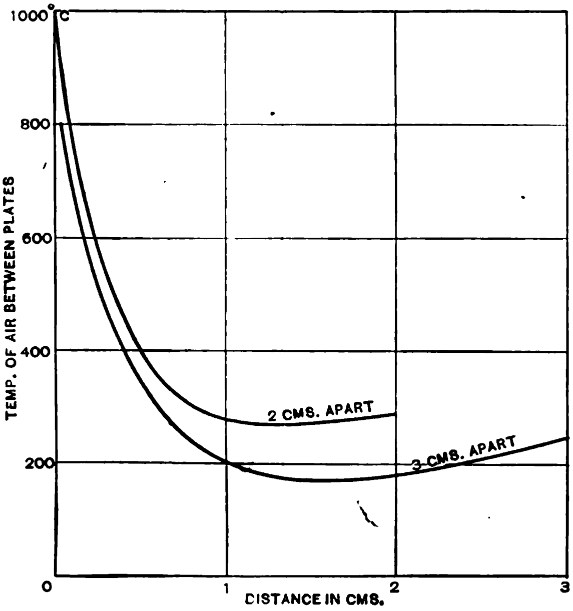

Fig. 6.

Velocity of the Ions.

From the theory discussed in the beginning of the paper, the velocity of the positive ion is given by

$$k = \frac{32\pi i d^3}{9v^2}.$$

At first sight this equation offers a simple and fairly accurate method of determination of the velocity of the ions, since i, the limiting current (which is measured by a galvanometer or electrometer), is independent of the intensity of the ionization at the surface of the platinum plate, and the quantities v and d can be accurately measured. The current has been shown to vary very approximately as the square of the P.D., so that the velocity deduced is independent of the voltage employed.

The following table is an example of the determination of velocities for distances between the plates varying from 2 to 7 cms., by means of the theory.

Distance Between Plates.	P.D. in Volts.	Deflection of Galvanometer.	Calculated Velocities for 1 Volt per cm.
2	52	103	5.5 cms. per sec.
3	103	68	3.17 " "
3	155	155	3.19 " "
4	155	53	2.59 " "
4	256	150	2.69 " "
5	255	63	2.20 " "
5	306	90	2.20 " "
7	282	24	1.90 " "

Current through platinum plate about 110 amps.
1 scale division of galvanometer $= 3.50\ 10^{-10}$ amps. $= 1.05$ E. S. units per sec.

Area of each plate surrounded by guard ring, 96 sq. cms.

The current observed, except for distance of 2 cms., was a maximum. In the above table the value of i, the current per sq. cm., is equal to the deflection divided by 192, since the current was measured to two plates, 96 sq. cms. in area, at equal distances on each side of the platinum plate. In order to calculate the velocity, i and V are expressed in electrostatic units, and the value K is then given for a potential gradient of 1 E. S. unit per second, The velocity for 1 volt per cm. is obtained by dividing this value by 300.

It will be observed that the calculated value of the velocity is greatest when the plates are closest together and gradually diminishes with the distance. For a distance of 2 cms. between the plates, the gas through which the ions travel is at a much higher average temperature than for a distance, say, of 7 cms.

A very important question now arises whether we are justified in assuming that the calculated value of the velocities of the ions given above represents even approximately the average velocity of the ions through the gas. The theory is based on the supposition that the ions are all of the same kind and size and that the velocity is unaltered in passing from one plate to the other. These conditions are, however, widely departed from in practice. Since the temperature of the gas between the plates is greatest near the platinum surface and gradually falls off to the copper plate, a given ion will decrease in velocity, for a given potential gradient, as it passes from the hot to the cold plate. The experiments of Maclelland[1] also show the velocity of the ions drawn from flame gases diminish rapidly as the temperature of the gas, through which they move, decreases. This decrease of velocity is much greater than can be accounted for by the lowering of the temperature of the gas and seems to be due to an actual increase of size of the ion. It is probable that the ions from a heated platinum surface will also show similar behavior.

It will also be shown later that the velocity of the ions at a given point of the gas is not a constant, but varies between fairly wide limits. We have also seen that some very slow moving carriers are present between the plates, the effect of which is in some cases to greatly reduce the current.

Taking all these divergences between the actual and theoretical conditions into consideration, it cannot be assumed without a special investigation that the velocities of the ions tabulated from the equation represent even approximately the true values.

Direct Determination of Velocity of the Ions.

In order to settle the amount of agreement between the calculated and actual velocities, a direct method of measuring the velocity of the ions was employed.

The method is identical in principle with that previously employed by the author[2] to determine the velocity of the negative ion produced by ultra-violet light falling on a negatively electrified surface.

A direct P.D. was changed to an alternating P.D. of known frequency by means of a suitable revolving commutator. This alternating P.D. was applied to the platinum plate. Suppose the current through the platinum plate is large enough to produce a large supply of positive ions, but not large enough to discharge negative electricity. The instant the platinum plate is charged positively, the positive ions start to travel towards the copper plate. If the sign of the platinum plate is reversed before the leading ions reach the negative electrode, the positive ions travel back again and no current passes through the galvanometer.

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[1] Proc. Roy. Soc., 1899.

[2] Proc. Camb. Phil. Soc., 1898.

The ions are thus alternately advancing and retreating during each successive change of the electric field.

Let d be the smallest distance between the plates such that for a P.D., v_0, no ions reach the copper plate. The electric force between the plates (assumed uniform) is $\frac{v_0}{d}$ and the distance d passed over in the time T of a half alternation is given by $d = \frac{v_0}{d}$ K.T. where K is the velocity of the ion per unit potential gradient; therefore

$$K = \frac{d^2}{v_0 T}.$$

In the special case we are considering, the electric field is not uniform. A special series of experiments showed that the potential gradient between the plates was very approximately the same as that deduced from theory.

We therefore have

$$\frac{dv}{dx} = -a^{\frac{1}{2}}x^{\frac{1}{2}},$$

and

$$v_0 = -\tfrac{3}{2}a^{\frac{1}{2}}d^{\frac{3}{2}},$$

therefore

$$\frac{dv}{dx} = \frac{3}{2}\frac{v_0 x^{\frac{1}{2}}}{d^{\frac{3}{2}}}.$$

The time dt taken to pass over a distance dx is given by

$$dt = \frac{dx}{K\frac{dv}{dx}} = \frac{2}{3}\frac{d^{\frac{3}{2}}}{Kv_0 x^{\frac{1}{2}}} \cdot dx.$$

Therefore, if in time T of a half alternation, the ions can just travel over a distance d, by integration we obtain

$$T = \tfrac{4}{3} \frac{d^2}{Kv_0}$$

or

$$K = \tfrac{4}{3} \frac{d^2}{v_0 T},$$

or for a given P.D. between the plates, the time taken for an ion to travel between them is $\frac{4}{3}$ that taken if the field were uniform.

In experiments on the velocity of the negative ions produced by ultra-violet light, the alternating E.M.F. of the Cambridge town mains was employed. It is, however, much more satisfactory to commute a direct E.M.F., as in that case the period of commutation is under control and the P.D. is constant during each half alternation. The commutator employed was a two part one of diameter 6.3 cms., driven by an electric motor. The insulation gap between each segment was .5 cm. so that the P.D. was applied during .95 of the time of alternation.

The speed of rotation of the commutator was determined by means of an apparatus devised by Prof. R. B. Owens, of McGill University. The armature of a small magneto-machine rotated on a continuation of the axis of the commutator. The E.M.F. generated, which was measured by a Weston voltmeter, was directly proportional to the speed. This apparatus is very advantageous for reading speeds accurately at a distance.

In the practical determination of the velocities, it was found convenient to keep the plates at a fixed distance apart and then decrease the P.D. until the point of no appreciable current through the galvanometer for a known speed of the commutator. Theoretically, if the ions all have a uniform velocity, the P.D. for which no ions reach the plate should be sharply defined. This was observed to be the case in the measurement of the negative ions produced by ultra-violet light, but in the first experiments it was found that the P.D. could not be adjusted accurately, but that a slight current passed through the galvanometer over a considerable range of voltage.

This result is not surprising when we consider that the ions are probably growing in size with time and thus move more slowly on their return journey. Some of the ions, therefore, will be unable to return to the platinum plate in the time of a half alternation. If this process continues, in the course of a large number of alternations, these ions will reach the copper plate and a current will be indicated on the galvanometer with a lower voltage than is actually necessary to carry over the original ion from the platinum to the copper plate.

This difficulty, however, can be overcome very simply. In series with the alternating E.M.F. $\pm E$ a battery of accumulators is placed of E.M.F. E_1 less than E. The P.D. acting for one-half alternation is $E + E_1$ and for the other half $E - E_1$. If the platinum plate is charged positively to a P.D. $E - E_1$, the returning ion moves in an electric field approximately $\frac{E + E_1}{E - E_1}$ times that for the outgoing ion. By adjustment of the values of E and E_1, the value $E - E_1$ can be made as small as we please compared with $E + E_1$, so that all the ions which set out during the one-half alternation are swept back during the next half. Under these conditions it was found that the value of the P.D. necessary to give no current could be adjusted without certain well-defined limits.

The following is an example of the determination of velocity of ions for plates 3 cms. apart, with a current through platinum plate of 105 amps. In this case, the deflection of the galvanometer was too small to observe with accuracy, so that a one-third microfarad condenser, in parallel with the copper plate, was charged up for 30 seconds and then discharged through the galvanometer.

P. D. +	P. D. —	Deflection.	Time.
46 volts	143 volts.	0	30″
48.6 "	146 "	1.5	30″
56 "	153 "	8	30″
65 "	162 "	18	30″
77 "	173 "	60	30″

Commutator made 750 revolutions per minute.

Time of application of P.D. was .0380 second, correcting for insulation distance between the segments.

Taking 46 volts as the minimum positive potential for no current and substituting in the formula

$$K = \frac{1}{2}\frac{d^2}{V_0 T}$$

we obtain $K = 7.9$ cms. per second as the maximum velocity of the ions between plates 3 cms. apart. It will be observed from the above table that the P.D. sweeping back the ions to the platinum plate is between 2 and 3 times as great as the P.D. which moves them from the platinum plate. In this way it is assured that all the ions which do not reach the copper plate are swept back to the platinum surface during the succeeding half alternation. Now the steady current for + 43 volts gave a throw in the galvanometer of 156 divisions when the condenser was charged for 30 seconds. This would correspond to a deflection $\frac{1}{2} \times .95 \times 156 = 74$ if all the ions starting out from the platinum plate during each half alternation reached the copper electrode. Assuming the current proportional to the square of the P.D., it can be readily calculated that for the voltages 48.6, 56, 65, 77, the ratio of the number of ions starting from the platinum plate which reach the copper plate, is .015, .07, .13, .28, respectively.

Taking as data that for 77 volts, .28 of the number of ions reach the copper plate, a rough average of the velocity of the ions can be deduced. For .28 of the time .038 second of a half alternation, ions reach the copper plate. The time taken for ions to cross between the plates is thus $.72 \times .038$ second, *i. e.*, .027 second. The average velocity is thus 5.8 cms. per second.

In a similar way it was found that the minimum voltage to carry over ions for a distance of 2 cms. for 1,000 revolutions per minute of the commutator, was 14.5 volts. The maximum velocity for this distance, 2 cms., is thus 13 cms. per second. This result can, however, be considered only as a rough approximation on account of the uncertainty of determining the actual positive potential of different portions of the platinum plate. It obviously varies from one end to the other on account of the P.D. sending the heating current through it.

The following method was employed to determine the average velocity of the ions between the plates.

A steady deflection corresponding to V volts between the plates was observed. The commutator was then slowly rotated and the deflection fell slightly below one-half its value. Theoretically it should fall to ½ × .95 of its value, and this was experimentally observed to be the case. On increasing the speed of rotation, the deflection steadily diminished. Let D_1 be the deflection for a period T of half alternation, and D the deflection for a very slow rotation of the commutator. The ratio of the number of ions reaching the copper plate is $\frac{D_1}{D}$ of the total starting out from the platinum surface, and if t be the time taken for the ions to cross over between the plates

$$.95T\left(1-\frac{D_1}{D}\right)=t.$$

The velocity of the ions is then determined by substituting in the equation of velocities and we obtain

$$K=\frac{4}{3}\frac{d^2}{V\times .95T\left(1-\frac{D_1}{D}\right)}$$

Using this method, a large number of determinations of velocities were made for distances 2, 3 and 5 cms., for different periods of alternation and different voltages. The mean of these values is included in the following table.

Velocity in cms. per second for 1 volt per cm.

Distance.	Maximum.	Average	Calculated.
2	13	7.8	5.5
3	7.9	5.8	3.2
5	—	4.7	2.2

The maximum velocity for a distance of 5 cms. was not determined, as the commutator sparked across before sufficient voltage could be employed to make sure of sweeping back the ions on their return journey. The current through the platinum plate, in all these cases, was kept constant at about 100 ampères, which corresponded to the temperature of maximum current at all distances except 2 cms.

It was experimentally observed that for a given distance the velocity of the ions increased with increase of temperature of the platinum plate, although the actual current observed through the galvanometer was diminished.

The results obtained from the direct measurement of the velocity of the ions may be summarized as follows:

1. The velocity of the positive ions drawn from glowing platinum is not a constant for given conditions, butmay vary within wide limits.

2. The velocities of the ions increase with increase o fthe temperature of the gas through which they pass.

3. The velocities calculated from the theoretical formula are too small. For 2 cms. distance between the plates the difference between the theoretical and experimental values is not large, but the difference increases with distance apart of the plates. For 5 cms. the calculated velocity is less than one-half of the average velocity.

The final conclusion we may draw from the results is that the velocity of the ions deduced from measure of the limiting current does not necessarily represent the average velocity and is much less than the maximum value of the velocity.

During the course of the investigation two papers appeared by C. D. Child, in the Physical Review (February and March, 1901). In these papers the velocity of the positive and negative ions drawn from the flames and the arc have been determined by direct measurement of the current, using the theory given in the early part of the paper.

His results show clearly that the current varies approximately as the square of the voltage, and falls off more rapidly than the inverse cube of the distance. The diminution of the current with distance in the case of flames is not, however, so rapid as in the case of glowing platinum.

It will be of interest to compare the values of the velocity of the positive ions drawn from flames and from glowing platinum. For purposes of comparison we will give the "calculated" values of the velocities of the positive ions in the two cases:

Ions Drawn from Flame.		**Ions Drawn from Glowing Platinum.**	
Distance from Edge of Flame.	**Calculated Velocity of Positive Ion.**	**Distance Between Plates.**	**Calculated Velocity Positive Ion.**
1 cm.	2.85 cms. per sec.	2	5.5 cms. per sec.
2 cms.	2.27 " " "	3	3.18 " " "
4 "	2.21 " " "	4	2.64 " " "
6 "	2.00 " " "	5	2.20 " " "
8 "	1.45 " " "	7	1.90 " " "

The calculated velocities of the positive ion in the two cases are thus not very different for distances of 4 to 7 cms. The velocity of the flame ions is, however, considerably smaller for distances of 2 to 4 cms.

We have shown, however, that the calculated values given above for the platinum ions are far too small on account of the divergences between experimental and ideal conditions. The method employed by Child is a very simple and accurate one of determining velocities, provided the ions all travel with the same velocity, *i. e.*, are all of the same size. The presence, however, of some slow moving carriers between the electrodes may diminish the value of the current and the resultant calculated velocity may be much too small. Whether such slow moving carriers are produced in the case of flames as in the case of glowing platinum cannot be definitely answered without a special investigation. The presence of even a small number of large nuclei, such as are produced by smoke or fumes, tend to make the calculated velocities too small. For these reasons the measurements of velocity of the ions close to the electric arc must be interpreted with great caution.

Macdonald Physics Building,
McGill University, July 1st.

A SPECTROPHOTOMETRIC COMPARISON OF THE RELATIVE INTENSITY OF LIGHT FROM CARBON AT DIFFERENT TEMPERATURES.

By Ernest Blaker.

FOR many years the idea advanced by Draper,[1] that all solid bodies, when heated, begin to give off light at the same temperature, that the rays first emitted were dull red and as the temperature was raised the shorter wave-lengths were added in succession, was accepted. E. Lecher[2] showed this to be not true and H. F. Weber[3] made an investigation of the subject in which it was shown that in general Draper's views would not hold. Weber's observations on carbon in the form of a glow lamp show that red does not appear first, but that to the unaided eye first appears a misty indistinct gray light, which is followed, as the temperature rises, by a series of characteristic changes before there is any appearance of red; passing through an ashen gray, then a pale yellow. The emission then changes to a bright yellow gray and the next addition is of an uncommonly bright red, which is intermittent at first, then is steady with no more flickering, being a fiery red. With rising temperature the light becomes orange, yellow, yellowish-white, then white. In studying the spectrum of the carbon filaments in the first stages of incandescence, Weber found the misty gray light to appear in that portion of the spectrum that is occupied by yellow and light green when the temperature was higher. With increasing current in the lamp, the gray band had a middle of yellowish-gray, shaded off at the ends into dull gray. At the time that the red flicker appears to the unaided eye the spectrum shows a delicate band of red on one edge and almost immediately a weak grayish-green border on the other side. The spectrum broadens out both ways with more current and when the center is a bright yellow gray it extends from the middle of the red into the blue. Then with a yet further increase of temperature the whole spectrum presents itself.

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[1] Draper, Phil. Mag., May, 1847, pp. 345–360.

[2] Lecher, Wied. Ann., No. 32, 1887, p. 256.

[3] H. F. Weber, Wied. Ann., No. 32, 1887, pp. 256–270.

Further heating increases the intensity of the light for all wave-lengths, but the wave-lengths of higher frequencies, as has been shown by Nichols and Franklin,[1] show the greatest increase. That this rate of change is not continuously greater toward the shorter wave-lengths, has been noted by Dr. Nichols,[2] and it was the purpose of the investigation herein described to obtain new data upon this subject.

TEMPERATURE OF GLOWING CARBON.

The lowest temperature at which the light emitted by carbon was studied in this investigation was such that in the longest red rays used the color was very dull, and no reading could be made for wave-lengths shorter than $.470\,\mu$.

The amount of light given off at certain temperatures was for some time simply a guess. M. Lucas[3] has made measurements of the temperature of glowing carbon, based on relations between resistance of the carbon and the current passing through it, and on that between resistance and temperature. In the special case treated, he found the temperature to be a simple function of the current. Lucas also measured the intensity of the light emitted.

H. F. Weber[4] has, on theoretical grounds, arrived at a relation between the radiation from incandescent carbon and its temperature, and has made many experimental determinations of the constants of the relation. His results, however, are thought to be too low.

M. Le Chatelier[5] has measured the temperature of incandescent lamps by a photometric comparison and has given his results in terms of the ratio $\frac{R_\theta}{R_0}$, where R_θ is the resistance of the lamp at a temperature θ and R_0 is its resistance cold.

M. P. Janet[6] has indicated how the temperature of incandescent carbon may be obtained, based on the mean specific heat of carbon as determined by Violle,[1] and later[2] has given some determinations on four different lamps, expressing results in terms of $\frac{R_\theta}{R_0}$ for purposes of comparison with the results of Le Chatelier. In three of the lamps the values obtained are very close to values obtained by plotting Le Chatelier's results and interpolating. The average of these three gives a temperature of 1620° C. for a value of $\frac{R_\theta}{R_0}$ equal to .53 and the interpolation gives a value of 1600° C. for the same value of $\frac{R_\theta}{R_0}$. Using Le Chatelier's results as a basis, the temperature of the carbon filament, when there was a difference of potential of sixty volts between its terminals, was 1738° C., the power loss in the lamp being 1.95 watts per candle. Lucas's results, reduced, give a temperature of about 1857° C. for the same power development and the mean of several results of Weber give a temperature of 1635° (abs.). Abney and Festing[3] conclude that the temperature appears to be nearly a simple function of the re-

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[1] Nichols & Franklin, Amer. Jour. Sci., 3d series, Vol. 38, 1889, pp. 111–115.

[2] Nichols, PHYSICAL REVIEW, Vol. XIII., p. 65, 1901.

[3] M. Felix Lucas, C. R., No. 100, 1885, pp. 454–456.

[4] H. F. Weber, PHYS. REV., Vol. II., 1894, pp. 112–121, 197–210.

[5] M. Le Chatelier, Jour. de Phys., 3d Serie, Vol. I., p. 203.

[6] M. P. Janet, C. R., No. 123, 1896, pp. 690, 691.

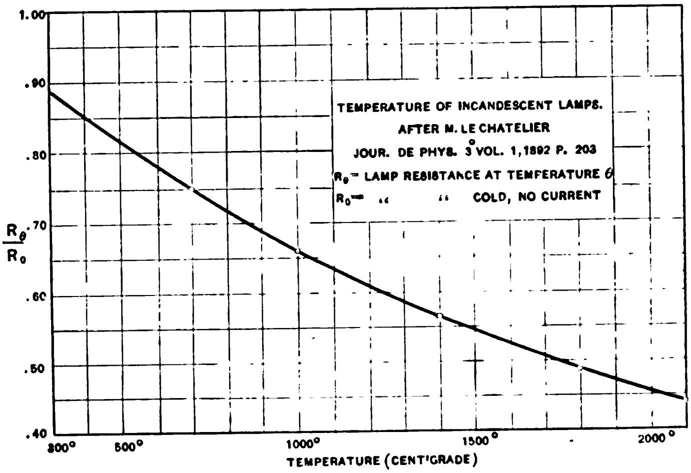

Fig. 1.

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[1] M. J. Violle, C. R., No. 120, 1895, pp. 868–869.

[2] M. P. Janet, C. R., No. 126, 1898, pp. 734–736.

[3] Abney and Festing, *Phil. Mag.* (5), No. 16, Sept., 1883, p. 225.

sistance, with which Le Chatelier's results applied to this experiment seem to agree. The results of Le Chatelier have been used as a basis for temperatures in this work, representing something near the truth.

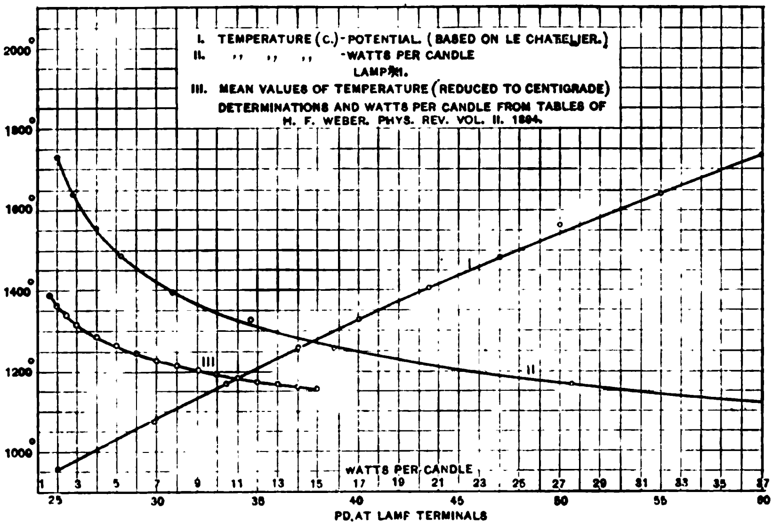

Fig. 2.

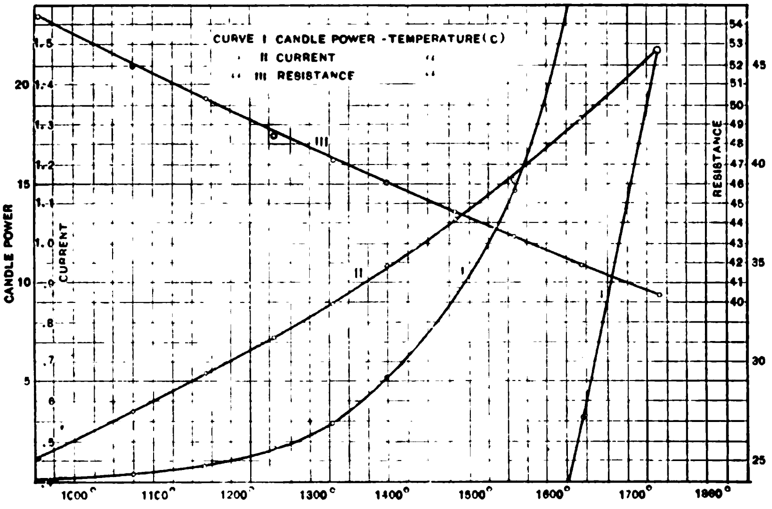

Fig. 3.

The curve in Fig. 1 shows the temperature of incandescent lamps plotted with $\frac{R_\theta}{R_0}$ from Le Chatelier's determination; Fig. 2 contains two curves showing (I.) the relation between potential differences at lamp terminals and temperature and watts per candle of the test lamp used in the measurements to be described in this paper, based on Le Chatelier's values; (II.) the temperature, watts per candle curve for the same lamp. A curve showing the relation between temperature and watts per candle as deduced from the mean of values taken from the data of several tests by Weber is given in the same figure (curve III.).

Fig. 3 shows the relation between the candle power, current and resistance of the test lamp and its temperature; the scale of temperatures being that derived from the determinations of Le Chatelier.

The following table gives the data upon which the curves relating to the test lamp were based:

TABLE I.

(*Test lamp No. 1.*)

V	I	R_θ	$\frac{R_\theta}{R_0}$	θ°C.	C. P.	W	$\frac{W}{C.P.}$
24.85	.456	54.5	.673	954		11.32	
29.90	.575	52.0	.642	1075	.33	17.18	52.
33.4	.668	50.15	·619	1168	.81	22.31	27.55
37.0	.763	48.49	.599	1255	1.8	28.24	15.7
40.0	.846	47.22	.583	1328	2.92	33.84	11.59
43.5	.943	46.02	.568	1398	5.25	40.93	7.8
47.0	1.055	44.55	.550	1484	9.62	49.59	5.16
50.0	1.16	43.29	.535	1559	14.63	58.00	3.96
55.0	1.31	41.89	.519	1643	27.00	72.05	2.67
60.0	1.48	40.54	.501	1738	45.60	88.80	1.95

$R_0 = 80.96$ *ohms at* 18° *C.*

APPARATUS AND ITS ARRANGEMENT.

The voltmeter used in this work was a large standard Weston instrument with a 150-volt, and a 3-volt scale. It was carefully calibrated for the 150-volt scale by a potentiometer method, balancing a standard Clark cell around a known resistance and then calculating the fall of potential through another known resistance. about which the voltmeter was shunted.

To measure the current, a standard Weston milli-voltmeter, with shunt box, was used. It was calibrated for two shunts; one using the one-ampère scale, and the other for the five-ampère scale, by placing it in series, first with a standard Hartmann and Braun ohm and then with a standard tenth ohm (H. & B.) around which was shunted the standard voltmeter, mentioned above.

In pursuing this investigation a Lummer-Brodhun spectro-photometer[1] was used. It consists of two collimator tubes at right angles to each other, the path of light from one source being directly through a glass cube, the path of light from the other source entering the cube and being reflected at a silvered surface within the cube in such a manner that it has the same length of path in the cube, and passes out of the cube parallel with and in the same vertical plane with the first ray, whence they are dispersed by a flint glass prism and may be examined in the telescope in juxtaposition. The instrument was supplied with two arms fastened rigidly to the base of the instrument, one being placed directly beneath and parallel with the axis of each collimator. On these arms two sources of light were placed at equal distances of 22.6 cm. from the collimator slits. One source was the standard of comparison, a flat acetylene flame from a Naphey burner, in front of which was placed a screen in which was a circular opening of 2.5 mm. radius, over which was placed a piece of glass from an incandescent lamp globe of the same type of lamp which was used as a source of incandescent carbon, on the other arm. The lamp used was one with a short thick gray carbon filament of the "hair pin" type, made to be used at 50 volts pressure. A screen was placed in front of this lamp cutting off all light from the collimator slit excepting that from a straight portion about two centimeters long, care being taken that the portion of the filament used be parallel with the axis of the slit. The instrument was so set up that this lamp was at one end of a thousand part photometer bar, at the other end of which was placed a secondary standard 16 c.p. fifty-five-volt lamp, which was used throughout below normal pressure the current being furnished by storage cells. The voltage was carefully adjusted each time the standard was used, to 50 volts. This secondary standard was standardized by comparison with an amyl-acetate lamp. The acetylene flame was used at a constant pressure throughout the work, of 7.35 cm. of water; the acetylene being produced by using commercial calcium carbide. The gas, as tested by Mr. Rands, was about 98.7 pure. For the lamp being tested, the current was furnished by a storage battery.

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[1] Zeit. für Inst., No. 12, 1892, pp. 132–139.

OUTLINE OF METHOD.

The order of experimentation was as follows: The test lamp was brought up to the desired brightness and its candle power measured by comparison with the standard lamp, using a Bunsen photometer; then with a definite opening of collimator slit the collimator slit of the standard acetylene source was varied until, for the rays being compared, the intensity was alike for both sources. In nearly all cases at least ten readings were taken for each wave-length. Eleven wave-lengths were compared, extending almost the whole length of the visible spectrum. In some instances whole sets were run over again as a check. In order that the test lamp might last throughout the experiment, it was used with small voltages and currents first, then gradually increased. The lamp was not used for a very long time at voltages near the normal and consequently did not deteriorate appreciably in candle power until used at sixty volts. Then it fell off about seven per cent. in candle power between the beginning and end of the run at that voltage. In this run the red end of the spectrum was tested first. All readings were corrected for slit width from curves based on those obtained by C. V. Capps[1] for a sixty-degree flint glass prism; both prisms being furnished on instruments made by Schmidt and Haensch.

The intensity of the acetylene source was taken as unity for the luminous intensity of the various wave-lengths and from the data obtained were drawn, for each wave-length, "isochromatic curves," using in one case, intensities and temperatures (*C*) for coordinates and in the other case log. of intensities and reciprocals of temperatures (abs.). (See Figs. 4 to 14 inclusive.) The intensity-temperature curves would have the same form whether plotted with the temperatures, centigrade or absolute, since the difference would be

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[1] Capps, Astrophysical Journal, Vol. II., January, 1900, pp. 25–35.

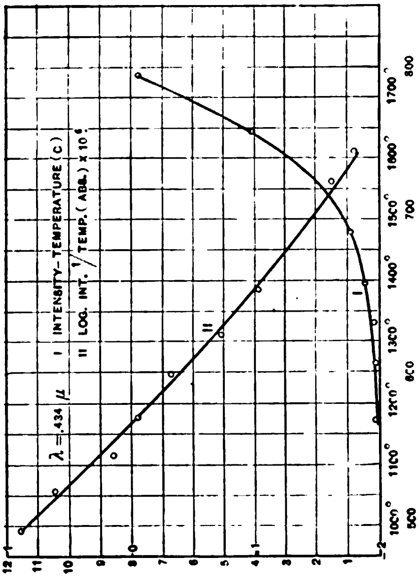

Fig. 4.

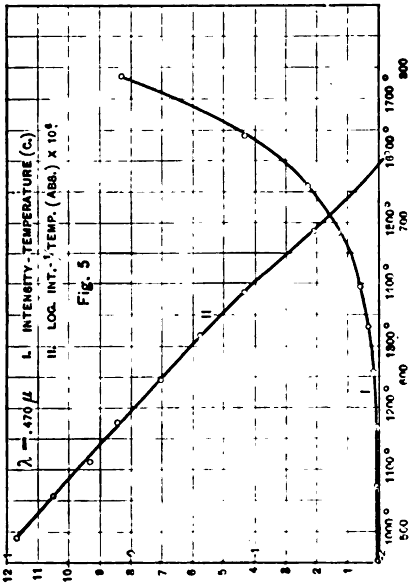

Fig. 5.

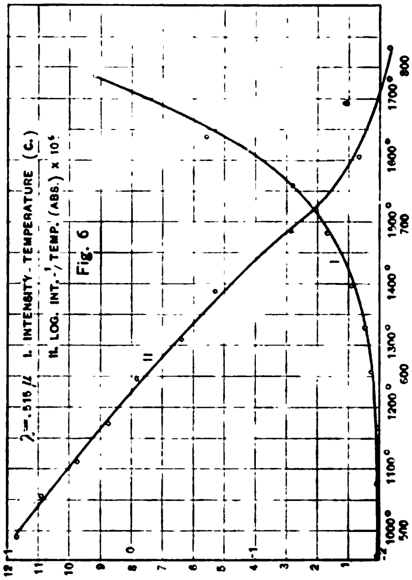

Fig. 6.

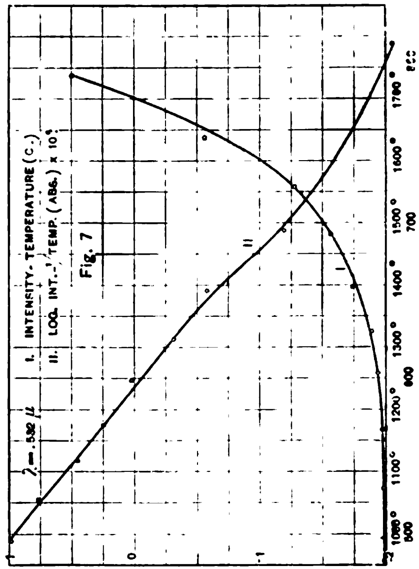

Fig. 7.

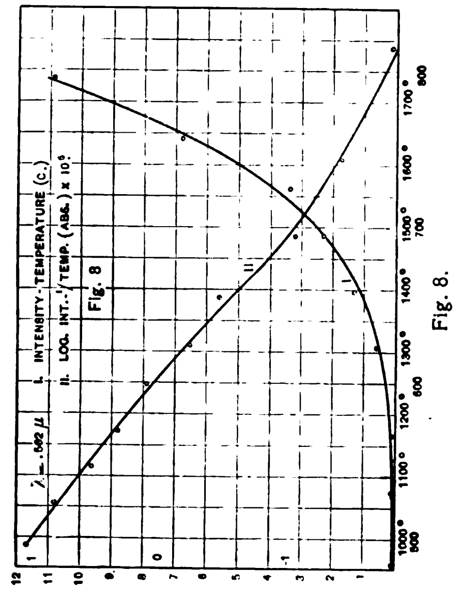

Fig. 8.

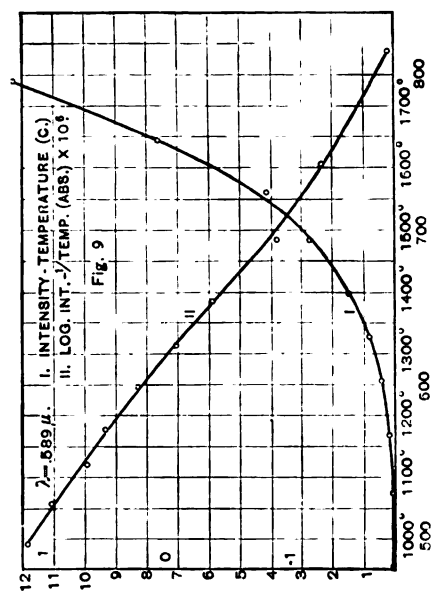

Fig. 9.

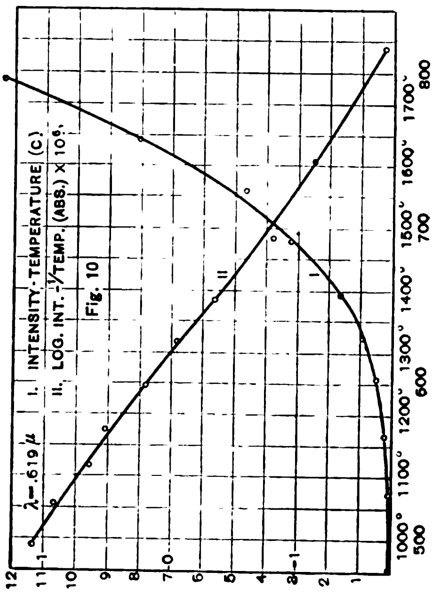

Fig. 10.

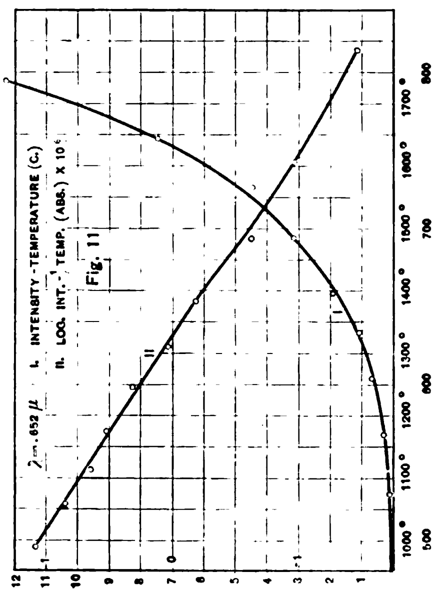

Fig. 11.

only a shifting of the origin of temperatures. From W. Wien's[1] equations for black bodies,

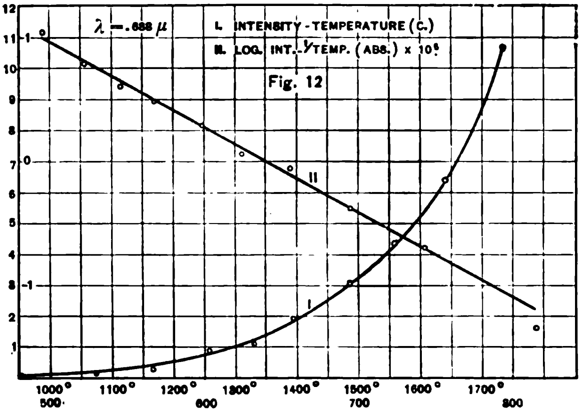

Fig. 12.

$$E = C_1 \lambda^{-5} e^{-\frac{c_2}{T\lambda}}$$

it will be seen if it be transformed into a logarithmic equation,

$$\log E = \log C_1 - 5 \log \lambda - \frac{C_2}{T\lambda} \log e,$$

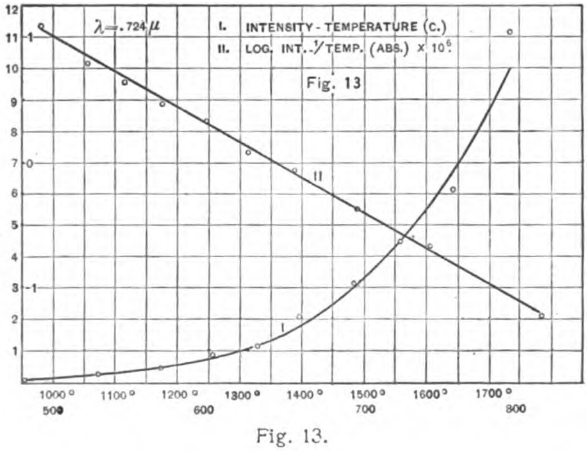

Fig. 13.

that if logarithms of intensity and reciprocals of absolute temperature be plotted as coördinates for a given value of λ, that a straight line would be the result; and that if intensity and temperature were plotted an exponential curve would result. Paschen's[1] modifications of Wien's equation,

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[1] W. Wien, Wied. Ann., 58, 1896, p. 662, etc.

$$E = C_1\lambda^{-\alpha} e^{-\frac{C_2}{T\lambda}}$$

in which the exponent of λ is changed to the more general form, shows the same general form of curves. Lummer and Pringsheim[2] in testing Wien's equation, arrive at the conclusion that C_1 and C_2 have a progressive variation such as to make the curves plotted with log I, and reciprocals of absolute temperatures, convex to the $1/T$ axis. It will be seen from the "isochromatic curves" that for the longer wave-lengths at the temperatures of these experiments the radiation seems to follow very closely the laws of radiation for "black" bodies; and that for the higher temperatures used, the intensity of all the rays followed the same laws, nearly, but that for light of a refrangibility greater than $\lambda = .619\mu$, the deviation from this becomes greater and greater, reaching a maximum at about $\lambda = .515\mu$ and then decreases again.

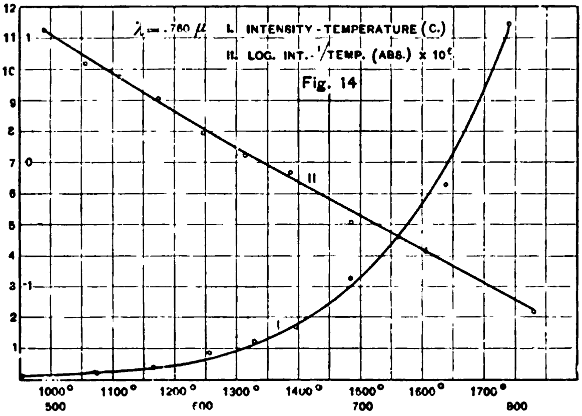

Fig. 14.

From these curves were obtained the data from which curves were drawn using intensities and wave-lengths as coördinates, which show the relative increase in intensity of the various wave-lengths as the potential, and consequently, the temperature, was increased.

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[1] F. Paschen, Astro. Phys. Jour., No. 10, June, 1899, p. 40.

[2] Lummer & Pringsheim, Verh. Deutsch. Phys. Ges., I., 1, 1900.

These curves show first that the intensity is a maximum, as compared with acetylene, in the longest wave-lengths. With the higher voltages the increase in intensity is more rapid toward the shorter wave-lengths and the ratio of increase is such that the maximum shifts toward the violet. With yet higher voltages the longer wave-lengths near the limits of the visible spectrum increase more in their intensity than near the maximum intensity, and this part of the curve also becomes concave to the axis of wave-lengths. In order to show the variation of intensity with temperature the curves for wave-length and temperature have been plotted on the same sheet. (See Figs. 15 and 16.)

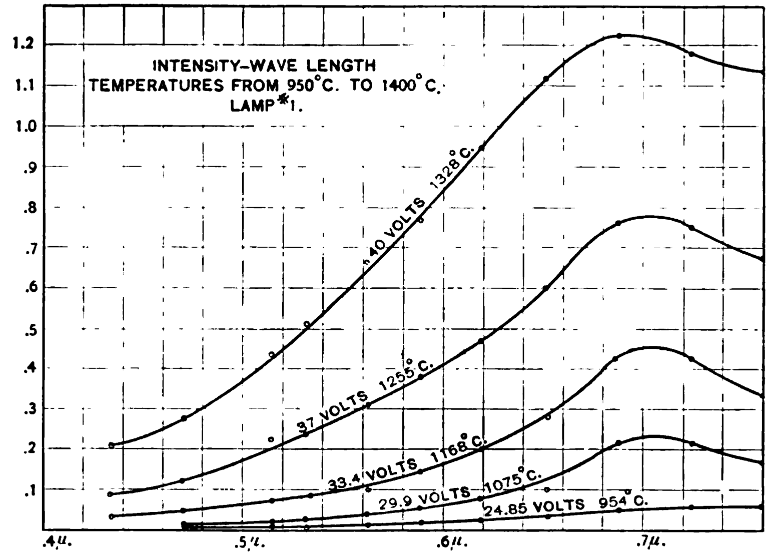

Fig. 15.

In Table II. are given the data complete for each test. *Pd* refers to the difference of potential between the terminals of the test lamp. *I* refers to the current, *W* refers to the watts, and *R* gives the resistance corresponding for the lamp. *P* denotes the pressure of the gas burned in the comparison lamp in centimeters of water ; O_a and O_A give the zero reading on the screw heads regulating the width of the slit openings, *C* referring to the test carbon source, and *A* referring to the acetylene comparison source. For opening slits the numbers on the screw heads increase so that on reaching the

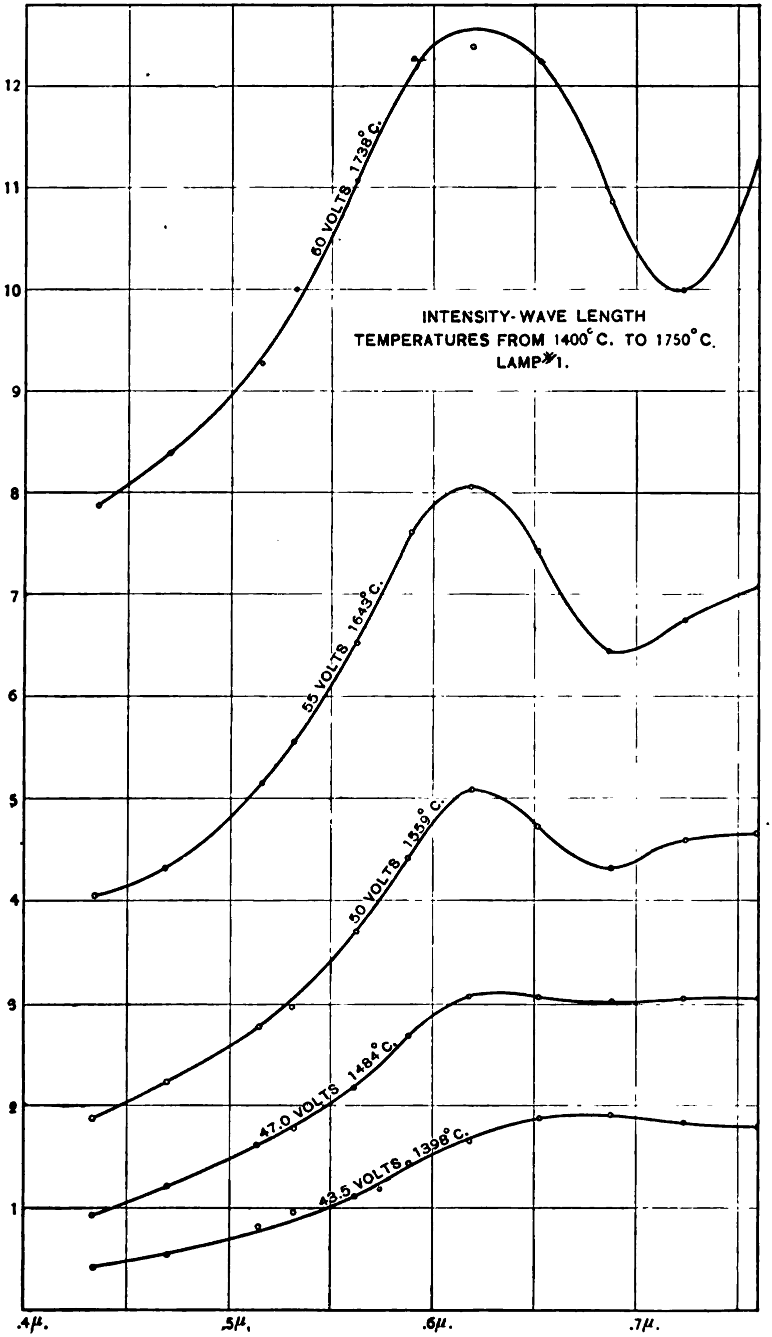
INTENSITY-WAVE LENGTH
TEMPERATURES FROM 1400° C. TO 1750° C.
LAMP #1.
60 VOLTS 1738° C.
55 VOLTS 1643° C.
50 VOLTS 1559° C.
47.0 VOLTS 1484° C.
43.5 VOLTS 1398° C.
12
11
10
9
8
7
6
5
4
3
2
1
.4μ.
.5μ.
.6μ.
.7μ.

Fig. 16.

TABLE II.

$Pd = 24.85$; $I = .456$; $W = 11.36$; $R = 54.5$; $P = 7.35$ cm.; $O_C = 95.8$; $O_A = 79$.

C Slit Open 1 mm.										
λ = .760	.724	.688	.652	.619	.589	.562	.532	.515	.470	.434
88.00	84.6	83.9	82.2	81.3	80.3	80.3	80.1	79.3	79.9	
89.60	85.6	84.1	82.8	81.6	80.6	80.3	79.9	80.0	79.8	
87.60	87.4	84.2	82.2	81.0	81.0	80.5	80.0	80.0	79.3	
90.00	85.2	84.7	82.5	81.2	80.7	80.6	80.0	79.9	79.7	
88.00	86.1	84.4	82.9	81.2	81.0	80.2	80.3	79.8	79.7	
86.60	87.0	84.0	82.3	81.2	81,3	80.6	80.1	80.1	79.6	
85.4	86.6									
88.1	86.9									
89.0	87.4									
87.2	87.2									
87.95	86.4	84.22	82.48	81.25	80.82	80.42	80.07	79.85	79.67	
A[1] open										
8.95	7.40	5.22	3.48	2.25	1.82	1.42	1.07	.85	.67	
Int.										
.06	.059	.047	.034	.02	.015	.014	.009	.008	.007	
Log. *I*.										
-2.7782	-2.7709	-2.6721	-2.532	-2.301	-2.1761	-2.1461	-3.9542	-3.9031	-3.8451	

$Pd = 29.9$; $I = .575$; $W = 17.18$; $R = 52.0$; $P = 7.35$ cm.; $O_C = 95.8$; $O_A = 79$.

C Slit Open 1 mm.									
λ = .760	.724	.688	.652	.619	.589	.562	.532	.515	.470
107.0	104.8	101.8	89.8	89.0	83.8	83.0	82.2	81.0	80.0
108.3	105.6	106.0	97.6	87.8	85.5	83.4	81.5	80.7	80.0
103.8	106.1	99.8	89.8	85.2	84.0	83.0	81.7	80.5	80.0
106.3	105.0	101.6	91.7	88.0	85.2	83.1	81.0	80.6	80.1
106.1	105.2	96.8	90.8	85.1	85.0	82.4	81.8	80.5	79.9
108.0	107.8	104.0	91.8	86.9	84.0	82.8	81.6	80.9	79.8
106.7	105.0								
104.0	107.7								
104.0	105.5								
102.6	106.6								
105.68	105.93	101.67	90.92	87.0	84.58	82.95	81.63	80.7	79.97
A open									
26.68	26.93	22.67	11.92	8.0	5.58	3.95	2.63	1.7	.97
Int.									
.194	.219	.201	.11	.076	.053	.038	.023	.015	.009
Log. *I*.									
−1.2878	−1.3414	−1.3032	−1.0414	−2.88	−2.72	−2.579	−2.352	−2.17	−3.9542

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[1] "C slit open" refers to the collimator slit opening of test lamp. "*A* open" refers to the number of scale divisions of opening of the standard. 100 divisions = 1 turn = 1 mm.

$Pd = 33.4$; $I = .668$; $W = 22.31$; $R = 50.0$; $P = 7.35$ cm.; $O_C = 95.8$; $O_A = 79.0$.

C Slit Open .5 mm.					C Slit Open 1 mm.					
λ = .760	.724	.688	.652	.619	.589	.562	.532	.515	.470	.434
100.0	104.2	118.8	106.1	96.0	90.0	88.2	86.9	84.6	82.4	81.0
96.5	102.6	117.0	105.0	96.3	92.5	88.3	85.9	85.0	82.3	81.0
100.0	103.1	115.3	104.3	96.3	90.0	87.2	86.0	84.3	82.6	81.0
100.6	100.1	117.2	105.3	96.0	91.7	87.5	87.0	84.8	82.2	80.8
97.0	103.9	117.6	103.8	96.6	93.1	87.3	85.4	85.2	82.0	81.0
96.0	99.3	115.2	106.0	95.8	91.8	87.6	86.8	84.3	81.9	81.2
96.3	100.3	117.4	105.1	95.6	91.0	87.0				
96.9	101.0	118.1	106.0	96.1	92.0	88.0				
94.2	101.3	117.5	104.8	97.1	89.8	88.2				
97.0	100.5	118.0	104.7	97.2	92.2	87.3				
97.45	101.68	117.21	105.11	96.3	91.41	87.66	86.32	84.7	82.23	81.0
A open										
18.45	22.68	38.21	26.11	17.3	12.41	8.66	7.32	5.7	3.23	2.0
Int.=										
.328	.428	.343	.247	.164	.12	.082	.065	.051	.032	.017
Log. *I*=										
−1.516	−1.631	−1.535	−1.392	−1.22	−1.07	−2.91	−2.81	−2.71	−2.51	−2.218

$Pd = 37.0$; $I = .763$; $W = 28.24$; $R = 48.49$; $P = 7.35$ cm.; $O_C = 95.8$; $O_A = 79.0$.

C Slit Open .5 mm.								C Slit Open 1 mm.		
λ = .760	.724	.688	.652	.619	.589	.562	.532	.515	.470	.434
126.8	126.0	123.7	114.8	102.4	100.2	95.8	92.9	101.6	90.6	89.2
124.5	120.0	124.1	112.2	102.5	98.1	97.0	91.3	102.0	91.8	91.9
119.1	126.0	125.6	112.0	105.0	100.0	96.0	91.8	100.9	90.0	87.8
122.0	123.7	126.0	110.0	104.7	99.0	96.4	92.0	101.9	94.0	90.0
122.3	118.0	124.6	115.0	102.0	98.1	94.8	91.9	101.3	91.8	91.0
119.0	121.5	120.4	111.2	103.0	98.9	95.6	91.4	102.1	92.6	89.5
116.0	127.0	123.1	113.1	102.1	100.3	96.0	92.4	101.5	91.2	89.8
119.0	122.6	120.8	116.7	103.2	97.0	96.0	93.8	102.2	91.5	89.0
115.8	124.0	124.7	113.2	101.2	99.0	95.7	93.1	100.0	93.6	90.6
117.2	123.0	119.8	113.3	101.8	97.2	95.2	91.9	101.7	92.0	89.0
120.17	123.18	123.28	113.15	102.79	98.78	95.85	92.25	101.52	91.91	89.78
A open										
41.17	44.18	44.28	34.15	23.79	19.78	16.85	13.25	22.52	12.91	10.78
Int. =										
.802	.874	.866	.661	.461	.386	.328	.252	.203	.119	.095
Log. *I* =										
−1.904	−1.942	−1.938	−1.820	−1.664	−1.587	−1.51	−1.40	−1.308	−1.75	−2.975

$Pd = 39.9$; $I = .845$; $W = 33.72$; $R = 47.22$; $P = 7.35$ cm.; $O_C = 95.8$; $O_A = 79.0$.

C Slit Open .25 mm.							C Slit Open .5 mm.		C Slit Open 1 mm.	
λ = .760	.724	.688	.652	.619	.589	.562	.532	.515	.470	.434
109.3	107.5	105.8	107.5	100.0	117.2	107.3	105.6	100.6	108.0	100.0
109 3	108.0	109.0	106.5	104.0	117.6	106.4	103.2	100.7	107.4	98.2
106.9	109.0	106.0	106.0	103.2	117.4	105.3	107.6	98.7	110.0	102.7
109.0	106.5	107.8	107.6	103.2	113.8	107.4	102.1	100.0	112.5	96.5
106.0	107.0	106.8	105.0	103.1	113.9	108.0	101.8	101.6	113.0	96.5
109.2	108.0	111.4	104.0	104.4	118.2	109.8	103.8	101.5	111.0	96.8
108.5	110.0	104.8	105.6	101.6	117.0	108.8	103.8	100.2	108.3	99.3
108.6	108.8	106.1	104.1	100.8	119.5	106.9	103.5	98.5	109.8	101.8
109.0	110.0	109.8	105.7	102.8	114.2	106.4	104.8	100.0	108.1	98.8
105.0	109.9	106.0	103.2	104.2	119.1	107.9	104.6	103.0	109.1	98.2
107.98	108.52	107.35	105.53	102.73	116.49	107.42	103.78	100.48	109.72	98.88
A open										
28.97	29.52	28.35	26.53	23.78	37.49	28.42	24.78	21.48	30.72	19.88
Int. =										
1.161	1.193	1.138	1.07	.945	.756	.56	.482	.413	.286	.182
Log. I =										
.065	.077	.056	.029	−1.975	−1.879	−1.74	−1.683	−1.616	−1.456	−1.259

$Pd = 43.4$; $I = .943$; $W = 40.93$; $R = 46.2$; $P = 7.35$ cm.; $O_C = 95.8$; $O_A = 79$.

C Slit Open .25 mm.									.5 mm.	1 mm.
λ = .760	.724	688	.652	.619	.589	.562	.532	.515	.470	.434
119.7	127.0	129.8	123.8	120.0	114.0	111.8	105.3	101.7	106.6	123.7
118.1	129.6	126.9	129.0	121.3	118.5	110.6	103.0	100.1	107.6	127.7
124.0	125.5	126.1	125.6	121.0	113.0	109.5	104.4	102.0	108.5	121.2
123.4	125.6	131.1	125.8	119.0	113.0	109.3	102.8	100.5	108.0	117.5
123.8	129.0	125.7	124.7	122.8	121.0	111.0	106.5	101.0	108.6	130.0
117.2	130.5	124.5	121.1	117.2	116.0	112.8	103.2	100.8	105.7	132.0
121.1	125.6	128.0	128.1	117.5	115.9	113.0	105.5	102.5	106.7	127.0
118.0	128.0	127.4	125.9	118.0	115.0	110.8	105.3	102.0	108.3	128.0
117.8	124.5	124.5	125.2	119.6	118.5	109.8	107.8	99.1	108.5	130.5
121.8	125.0	126.0	125.6	121.0	114.5	111.0	104.6	98.8	106.4	122.5
120.49	127.03	127.0	125.48	119.74	115.94	110.96	104.84	101.65	107.31	126.0
A open										
41.49	48.03	48.0	46.48	40.74	36.94	31.96	25.84	22.65	28.31	47.0
Int. =										
1.77	2.01	1.96	1.99	1.65	1.49	1.28	1.03	.91	.55	.459
Log. I=										
.248	.3032	.295	.299	.219	.173	.108	.01	−1.96	−1.747	−1.662

$Pd = 47.0$; $I = 1.055$; $W = 49.59$; $R = 44.55$; $P = 7.35$ cm.; $O_C = 95.8$; $O_A = 78.2$.

C Slit Open .2 mm.							C Slit Open .25 mm.			
$\lambda = .760$	.724	.688	.652	.619	.589	.562	.532	.515	.470	.434
134.0	135.6	135.0	144.6	141.0	131.0	119.6	121.0	116.7	107.1	99.7
133.2	137.2	136.0	142.8	144.5	127.9	121.7	121.0	118.0	110.1	101.1
136.4	135.3	137.1	140.0	137.0	131.8	122.0	121.4	118.2	112.2	102.7
136.2	136.0	137.2	140.0	140.1	135.2	118.8	124.0	119.4	111.1	102.2
134.4	136.6	137.8	142.7	138.5	130.0	122.1	122.2	119.5	107.2	101.0
134.0	136.2	136.0	138.0	141.0	131.5	118.6	125.2	116.0	111.0	104.5
133.9	134.6	138.5	144.0	144.2	133.0	119.7	123.2	119.0	111.0	99.5
133.1	130.8	138.4	142.2	137.8	129.0	119.2	123.0	117.8	110.2	101.0
135.5	134.6	135.5	141.9	138.5	133.8	119.3	123.0	119.0	112.0	98.5
136.3	137.2	136.3	140.2	142.0	130.3	122.2	122.2	119.2	109.0	100.5
134.7	135.41	136.78	141.64	140.46	131.35	120.32	122.62	118.28	110.09	101.07
A open										
56.4	57.21	58.58	63.44	62.26	53.15	42.12	44.42	40.08	31.89	22.87
Int. =										
3.20	3.07	3.04	3.295	3.265	2.77	2.16	1.79	1.64	1.29	.91
Log. I =										
.505	.487	.483	.518	.514	.443	.335	.253	.214	.111	—1.957

$Pd = 50.0$; $I = 1.155$; $W = 57.75$; $R = 43.29$; $P = 7.35$ cm.; $O_C = 95.8$; $O_A = 78.2$.

C slit open .2 mm.			C slit open .10 mm.							.25 mm.
$\lambda = .760$	.724	.688	.652	.619	.589	.562	.532	.515	.470	.434
157.8	157.8	156.0	123.6	122.8	118.0	109.0	107.0	107.0	102.3	111.2
150.5	157.2	159.7	119.5	124.0	120.5	109.9	107.5	105.2	98.0	115.8
152.0	156.8	156.8	121.8	124.5	116.1	109.1	106.9	106.5	103.0	115.0
161.8	159.8	161.0	120.2	120.9	120.6	111.2	108.0	104.6	100.0	113.8
157.4	157.0	161.8	120.0	120.5	117.0	112.7	106.9	105.9	99.5	109.0
153.4	159.0	158.6	123.3	125.0	119.1	111.5	107.9	105.3	102.6	118.6
155.0	156.3	158.5	122.1	120.2	122.0	110.3	107.2	105.0	101.0	112.0
154.2	158.0	156.8	122.6	121.2	115.0	112.8	109.0	105.0	100.0	116.0
154.8	159.0	159.4	122.1	123.0	115.2	110.0	109.3	105.9	99.9	111.0
153.0	155.7	157.7	123.0	123.6	116.7	111.8	108.0	104.4	99.0	108.0
154.99	157.66	158.63	121.82	122.57	118.02	110.83	107.77	105.48	100.53	113.04
A open										
76.79	79.46	80.43	43.62	44.37	39.82	32.63	29.57	27.28	22.33	34.84
Int. =										
4.67	4.50	4.33	4.46	4.64	4.18	3.34	2.98	2.78	2.22	1.42
Log. I =										
.669	.653	.637	.649	.665	.621	.524	.474	.444	.346	.153

$Pd = 55.0$; $I = 1.31$; $W = 72.05$; $R = 41.98$; $P = 7.35$ cm.; $O_C = 95.8$; $O_A = 78.2$

C Slit Open .1 mm.		.075 mm.	C Slit Open .1 mm.			.075 mm	C Slit Open .1 mm.			
$\lambda = .760$	.724	.688	.652	.619	.589	.552	.532	.515	.470	.434
132.3	136.0	122.8	152.4	155.0	154.8	129.2	134.5	133.3	123.1	118.5
133.8	136.0	127.5	148.2	152.5	150.1	128.3	135.0	129.1	118.9	114.8
131.0	137.2	122.4	149.5	156.8	146.8	126.7	131.8	132.0	120.0	116.5
134.8	137.1	122.8	151.5	151.2	151.0	128.0	131.0	134.2	116.0	116.2
131.4	132.6	126.8	146.2	154.8	146.0	123.0	135.8	127.0	120.6	114.8
132.6	134.9	124.5	153.8	155.2	148.8	123.8	132.2	130.5	116.9	113.0
129.4	132.0	127.8	152.2	152.1	149.7	123.6	135.0	138.0	120.3	116.5
129.5	133.8	126.0	145.1	151.8	150.0	124.2	131.4	133.0	124.6	115.1
133.3	134.2	125.2	147.5	154.8	149.1	129.0	132.6	129.8	124.0	118.8
133.0	137.2	124.1	149.2	152.9	146.3	126.3	136.4	133.1	123.2	116.7
132.11	135.1	124.99	149.56	153.71	149.26	126.21	133.57	131.0	120.76	116.09
A open										
53.91	56.9	46.89	71.36	75.51	71.06	48.11	55.37	52.8	42.56	37.89
Int. =										
6.24	6.12	6.42	7.48	8.06	7.68	6.72	5.73	5.53	4.36	4.05
Log. $I =$										
.795	.787	.808	.874	.906	.885	.827	.758	.743	.639	.607

$Pd = 60.0$; $I = 1.48$; $W = 88.80$; $R = 40.54$; $P = 7.35$; $O_C = 95.8$; $O_A = 7.80$.

	C Slit Open .08 mm.									
$\lambda = .760$	.724	.688	.652	.619	.589	.562	.532	.515	.470	.434
146.0	158.1	155.0	170.6	173.6	163.5	158.8	156.6	144.0	137.5	133.2
145.1	155.6	156.2	167.8	172.5	161.4	160.8	152.0	145.2	141.0	135.2
148.6	152.2	156.4	164.3	175.0	167.0	160.4	155.9	146.9	142.2	134.4
148.8	158.2	154.0	172.8	175.2	161.2	160.4	149.4	144.1	140.2	134.8
151.9	156.6	163.2	171.5	166.2	163.2	158.6	154.8	145.1	140.0	133.6
152.0	156.0	161.2	165.3	170.6	164.1	159.4	149.4	147.2	140.6	132.4
149.6	154.4	162.2	172.9	161.0	162.8	157.5	157.1	140.6	142.0	135.7
152.5	153.1	155.6	170.5	161.5	168.0	158.6	150.3	144.2	140.2	130.0
147.8	156.7	155.1	169.4	180.9	167.2	160.0	152.4	148.0	137.0	134.4
146.3	152.8	161.8	166.7	168.0	160.0	161.1	152.6	146.1	139.1	132.0
148.86	155.37	158.07	169.18	170.45	163.84	159.56	152.45	145.14	139.98	133.57
A open										
70.86	77.37	80.07	91.18	92.45	85.84	81.56	74.45	67.14	61.98	55.57
Int. =										
11.42	11.18	10.83	12.25	12.38	12.26	10.90	10.00	9.25	8.40	7.86
Log. $I =$										
1.050	1.048	1.035	1.088	1.094	1.089	1.037	1.000	.966	.924	.895

the zero of the scale 1 was prefixed to the reading for the sake of clearness, there being 100 divisions on the screw head and one complete turn equaling an increase in opening of the slit of one millimeter. "*A* open" gives the number of divisions of the comparison slit open to produce equality of illumination and the row marked "Int" gives the ratio of the intensity of the carbon light radiation to the intensity of the acetylene for the given wave-length.

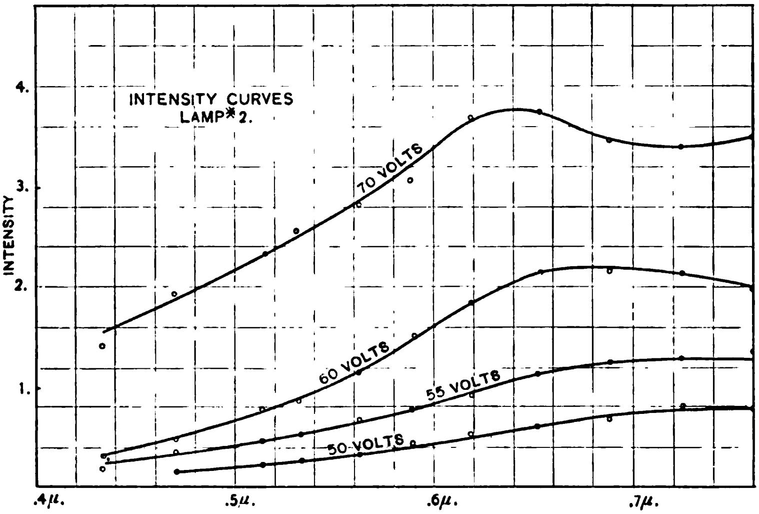

Fig. 17.

Gray and black carbon show marked differences in their efficiency, as has been shown by M. Evans[1] and confirmed by J. T. Bottomley[2] and J. C. Shedd,[3] and later by Dr. Nichols[4] in his study of the spectrum of the glow lamp. It was desired to find whether this change in the rate of change of luminosity was a characteristic of gray carbon alone, or whether it also was apparent in black carbon. A lamp was accordingly taken the filament of which had been carefully treated by coating with lampblack.[5] This lamp showed a much lower efficiency than lamp No. 1 with the gray filament. It will be noticed from the data given in Table III. that as the voltage increased the resistance also increased and that, therefore, it is impossible to apply Le Chatelier's results to determine the rise in temperature.

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[1] Mr. M. Evans, Proc. Royal Soc., Vol. XL., Feb. 18, 1886, pp. 207–214.

[2] J. T. Bottomley, Phil. Trans., 1887, p. 429.

[3] J. C. Shedd, Thesis, Cornell University Library, 1892.

[4] Dr. E. L. Nichols, PHYS. REV., Vol. II., 1895, pp. 260–276.

[5] Same, p. 264.

Table III.

V	I	W	C. P.	$\frac{W.}{\text{C. P.}}$	R.
50	1.51	75.5	4.58	16.5	33.11
55	1.65	90.75	8.68	10.45	33.33
60	1.78	106.8	14.30	7.46	33.71
70	2.04	142.8	31.24	4.57	34.31

As will be seen from Table IV. and Fig. 17 the same characteristic form of curve, with maximum shifting to the small wavelengths as the voltage rose, was likewise found in the case of this lamp.

As a final check on the results obtained using the Lummer-Brodhun spectrophotometer, a Nichols spectrophotometer was used. This instrument is made up of a horizontal collimator at one end of which are placed two totally reflecting prisms, the sources being studied being placed on a line perpendicular to the axis of the tube. The amount of incident light is regulated by means of two adjustable horizontal slits which are governed by micrometer screws.

Table IV.

$P_d = 50.00$; $I = 1.51$; $W = 75.5$; $R = 33.11$; $P = 7.35$ cm.; $O_C = 95.8$; $O_A = 78.0$.

1 mm.	C Slit Open .5 mm.									
$\lambda = .760$	.724	.688	.652	.619	.589	.562	.532	.515	.470	.434
165.0	117.1	111.1	106.4	103.4	98.8	95.0	92.0	89.3	85.8	
163.1	117.5	114.6	108.5	103.3	100.3	94.0	90.7	89.1	85.1	
160.0	118.2	111.2	108.2	102.2	99.2	93.2	91.6	88.1	84.1	
161.6	118.0	113.2	109.0	104.9	98.8	94.7	91.0	89.2	85.0	
164.1	117.8	111.3	108.9	103.2	100.0	93.0	91.1	89.0	85.0	
160.0	119.0	112.0	108.2	104.1	99.9	94.5	90.8	88.8	84.2	
162.3	117.93	112.24	108.2	103.52	99.5	94.07	91.2	88.92	84.88	
A open										
84.3	39.93	34.24	30.2	25.52	21.5	16.07	13.2	10.92	6.88	
tIn.=										
.76	.78	.655	.582	.499	.421	.312	.251	.201	.123	

$Pd = 55.0$; $I = 1.65$; $W = 90.75$; $R = 33.33$; $P = 7.35$ cm.; $O_C = 95.8$; $O_A = 78.0$.

C Slit open .5 mm.		C Slit Open .25 mm.								1 mm.
λ = .760	.724	.688	.652	.619	.589	.562	.532	.515	.470	.434
143.3	136.8	106.6	108.0	100.0	97.6	93.6	91.1	89.8	87.2	97.1
139.1	134.6	111.5	109.1	100.2	97.2	94.0	92.0	90.0	87.6	96.0
144.4	140.3	111.8	103.6	100.3	96.8	94.0	90.6	90.0	87.0	98.7
143.0	138.6	108.5	106.2	101.0	97.2	95.4	91.3	88.4	86.2	95.2
141.5	141.0	107.0	107.7	102.0	96.0	95.4	92.6	89.3	87.8	94.8
143.5	143.0	105.0	106.3	101.5	98.5	92.2	91.9	89.4	86.2	97.2
	138.2	105.7	106.2	102.1	98.2	96.0	91.0	89.1	86.0	96.8
	141.0	108.6	102.8	100.8	98.3	94.0	93.0	89.7	85.8	96.2
	144.5	108.8	103.2	99.8	97.1	95.8	92.8	89.1	86.0	95.8
	140.0	107.9	106.5	101.3	97.8	95.2	91.0	89.4	86.1	98.0
142.47	139.8	108.14	105.96	100.9	97.47	94.56	91.73	89.42	86.59	96.58
A open										
64.47	61.8	30.14	27.96	22.9	19.47	16.56	13.73	11.42	8.59	18.58
Int. =										
1.35	1.27	1.22	1.12	.91	.77	.66	.53	.45	.35	.17

$Pd = 60.0$; $I = 1.78$; $W = 106.8$; $R = 33.71$; $P = 7.35$ cm.; $O_C = 95.8$; $O_A = 78.0$.

C Slit Open .25 mm.										1 mm.
λ = .760	.724	.688	.652	.619	.589	.562	.532	.515	.470	.434
125.6	132.3	129.5	129.8	120.8	114.6	104.8	102.6	95.8	91.3	114.0
122.4	131.3	131.6	131.0	125.0	115.3	107.2	101.9	97.8	93.4	109.0
122.0	129.0	129.0	129.5	122.0	114.2	105.9	102.4	98.3	91.8	114.7
122.0	126.8	129.0	127.5	121.4	116.0	104.9	102.3	97.0	92.2	112.3
125.0	129.0	132.8	130.0	125.0	114.2	105.8	101.8	97.7	93.7	110.3
124.1	130.9	127.4	129.7	122.8	114.0	109.0	101.2	97.9	91.8	107.8
121.6	125.2	130.0								
124.7	126.5	129.0								
122.1	128.3	130.0								
124.2	126.0	131.2								
123.37	128.53	129.95	129.58	122.63	114.72	106.27	102.03	97.42	92.37	111.35
A open										
45.37	50.53	51.95	51.58	44.63	36.72	28.27	24.03	19.42	14.37	33.35
Int. =										
1.96	2.13	2.14	2.12	1.82	1.49	1.13	.96	.78	.58	.32

$Pd = 70.0$; $l = 2.04$; $W = 142.8$; $R = 34.31$; $P = 7.35$ cm.; $O_C = 95.8$; $O_A = 78$.

C Slit Open .1 mm.										.5 mm.
λ = .760	.724	.688	.652	.619	.589	.562	.532	.515	.470	.434
108.1	110.8	110.0	113.1	113.2	108.1	105.7	102.8	102.0	97.1	145.2
111.5	112.3	114.2	113.3	115.0	107.8	103.6	102.4	100.3	96.0	151.2
111.0	112.5	113.8	115.5	111.0	110.0	106.2	104.0	101.5	98.0	140.0
108.8	110.0	113.7	116.9	114.3	105.4	107.2	100.7	99.1	95.6	144.8
111.0	112.2	112.2	113.7	113.3	106.5	105.8	104.2	102.4	97.5	149.5
110.7	109.9	111.3	116.2	113.3	108.8	105.7	104.8	100.2	98.0	143.7
110.18	111.28	112.53	114.76	113.36	107.43	105.7	103.15	100.92	97.03	145.73
A open										
32.18	33.28	34.53	36.76	35.35	29.43	27.07	25.15	22.92	19.03	67.73
Int. =										
3.49	3.42	3.45	3.74	3.68	3.07	2.81	2.54	2.31	1.92	1.39

The two beams of light impinge upon a metal grating placed with the lines horizontal, so that the spectrum produced is vertical. The examining telescope swings in a vertical plane, as does also the grating, for purposes of adjustment. The spectrum produced by this instrument is a distinct normal one of about ten degrees dispersion between the limits of the visible spectrum. The character of the spectrum afforded by this instrument and the conditions of working with it differ so widely from those of the Lummer-Brodhun spectrophotometer as to offer an admirable control of the general correctness of the measurements made with the latter. Runs upon the lamp with the gray filament were made at fifty volts and at sixty-two and one-half volts. The curves obtained show the same general form with the shifting of the maximum toward the shorter wave-lengths, thus confirming the previous work and showing that the effect was a real one. No particular stress is laid on the readings for .76 μ or for .434 μ as to their real value but they do show the general trend of the curves. The data given in Table V. are for this test.

Table VI. gives the percentage, taken from the curves, for each wave-length, of the intensity at a given voltage over the value of the intensity of the same wave-length at the next lower voltage, for the lamp with the gray filament.

The results of the experiments here described are in accordance with the phenomenon observed by Dr. Nichols in his spectro-pho-

TABLE V.

$Pd = 50.0$; $I = 1.15$; $W = 57.5$; $R = 43.45$; $P = 7.35$ cm.; $O_C = 3.6$; $O_A = 88.5$.

	A Open 1 Turn.									
λ = .760	.724	.688	.652	.619	.589	.562	.532	.515	.470	.434
40.4	43.8	43.0	52.5	59.2	66.4	61.5	67.2	68.5		
46.3	44.8	42.5	53.6	55.2	60.5	67.8	66.4	73.0		
	46.4	42.2	50.8	57.5	61.6	63.8	70.5	72.5		
	44.4	41.3	51.8	55.8	62.6	66.6	69.6	71.1		
	43.4	41.2	51.9	55.7	64.2	67.1	69.5	68.0		
43.35	44.56	42.4	52.12	56.68	63.06	65.36	68.64	70.62		
C open										
39.75	40.96	38.46	48.52	53.08	59.46	61.76	65.04	67.02		
Int. =										
2.52	2.44	2.60	2.06	1.88	1.71	1.62	1.54	1.49		

$Pd = 62.5$; $I = 1.56$; $W = 97.5$; $R = 40.5$; $P = 7.35$ cm.; $O_C = 3.4$; $O_A = 88.5$.

2 Turns.	A Open 1 Turn.									
λ = .760	.724	.688	.652	.619	.589	.562	.532	.515	.470	.434
33.0	21.2	22.1	20.9	20.8	24.6	24.8	27.7	27.3	33.1	63.0
31.5	20.8	23.4	20.8	21.1	22.6	26.1	29.8	29.5	31.3	66.2
31.4	21.4	21.9	21.2	20.9	23.1	25.5	25.6	29.2	33.5	66.8
31.9	20.4	20.7	20.8	21.1	23.8	25.3	25.5	27.3	29.3	63.5
31.4	21.4	21.8	21.2	21.8	24.8	24.5	26.7	29.1	29.5	64.3
31.84	20.96	21.98	20.98	21.14	23.78	25.24	27.06	28.48	31.34	64.76
C open										
28.44	17.56	18.58	17.58	17.74	20.38	22.84	23.66	25.08	27.94	61.3
Int. =										
7.03	5.69	5.38	5.68	5.64	4.91	4.38	4.23	4.00	3.58	3.2

TABLE VI.

Volts.	Wave-Lengths.									
	.434	.470	.515	.532	.562	.589	.619	.652	.688	.724	.760
24.85	—	—	—	—	—	—	—	—	—	—	—
29.9	—	167	244	267	362	353	343	358	436	369	285
33.4	—	372	367	345	287	270	251	225	199	196	198
37.0	274	260	283	287	282	263	235	210	178	176	198
40.0	241	227	213	212	209	207	202	186	161	159	170
43.5	212	207	182	174	174	181	182	169	156	156	158
47.0	218	214	208	204	196	192	182	163	158	166	169
50.0	188	184	171	170	169	164	165	155	142	150	153
55.0	215	194	186	182	176	172	159	157	148	148	152
60.0	195	196	181	177	170	160	156	165	169	148	160

tometric observations on incandescent rods, cited above. The observations at the higher temperatures at which it was possible to work, with some fair degree approximating that of the acetylene flame, show that the distribution of energy in the spectra of acetylene and of the carbon filaments at the same temperature is not the same, as has been thought to be the case. The isotherms for the gray carbon rod, figure 22 in the paper of Dr. Nichols, begin to show the same characteristics that are brought out in the present work at the higher temperatures, namely, that after passing a maximum, compared to acetylene, in the orange a minimum is reached in the red, the comparative intensity again increasing toward the infra red. By a reference to the intensity-temperature curves it will be seen that crossing of the isochroms predicted at higher temperatures, in Dr. Nichols's paper has really taken place, the curve for wave-length .562 μ cutting those of .688 μ and .724 μ, while the curve for wave-length .589 μ cuts every one of wave-length greater than .562 μ excepting that for wave-length .619 μ which it would likely intersect at a temperature not much higher.

It is also seen by a comparison of the isotherms for gray carbon and for the black filament that the same characteristics are brought out and this phenomenon of selective radiation is one not due to the nature of the carbon surface.

I desire, in concluding, to express my gratitude to Dr. Nichols, at whose suggestion this investigation was taken up, for many valuable suggestions and his inspiring interest in the work.

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NOTE ON ELECTROMETERS.

By William Duane.

IN most symmetrical electrometers, such as the quadrant, the needle or movable conductor is made of different material from that of the fixed conductors. It is important in such cases to determine the effect, if there is one, of the electromotive force existing between two metals in contact with each other, or more strictly speaking of the difference in potential between two points in the air just outside the two metals in contact respectively. In most, and so far as the author is aware in all of the discussions and explanations of such electrometers these electromotive forces are neglected, whereas he has found both theoretically and experimentally that the Volta electromotive force under certain very common experimental conditions may introduce a large error into the measurement of a potential difference. Further he has found that, if the needle is suspended on a metal wire through which it is charged, and if the instrument is used idiostatically, the quadrant electrometer can be used to measure the Volta electromotive force with great accuracy.

To fix our ideas in regard to the Volta electromotive force suppose that V_a is the potential of a conductor A and V_b that of another conductor B. V_a and V_b are not in general the potentials in the air just outside the conductors, for owing to impressed electromotive forces in the transition layer on the surface of a conductor its potential is kept above or below that of the air in its immediate neighborhood.

Let φ_a and φ_b be the rises of potential (to be taken negative, if the potential falls) in passing from the conductors A and B respectively to the air just outside them. Then the potentials just outside the transition layers on the surfaces of A and B are $V_a + \varphi_a$ and $V_b + \varphi_b$ respectively. Suppose that the two conductors are placed in contact with each other. There is an impressed electromotive force in their surface of contact, which produces a difference of potential between them unless they are of the same material or at their neutral temperature. Call this difference of potential φ_{ab} to be taken positive if B is at a higher potential than A. Imagine a closed line drawn partly in the two conductors and partly in the air outside the two conductors. In following along this line from a point just outside of A through A then B, and then through the air back to the starting point, the potential falls by the amount φ_a in passing the transition layer of A, rises by the amount φ_{ab} in passing from A to B, and again rises by the amount φ_b in passing from B back to the air just outside it. Hence the potential must fall by the amount $\varphi_b - \varphi_a + \varphi_{ab}$ along the portion of the line drawn in the air from B to A. This fall of potential is the Volta difference of potential.

At present we will make no assumption as to the relative magnitude of φ_a, φ_b, and φ_{ab}, *i. e.*, as to whether the seat of the Volta effect is the air surfaces of the conductors or their mutual contact surface. It is sufficient to state that we know from experiment that the potential difference $\varphi_b - \varphi_a + \varphi_{ab}$ is in general a large fraction of a volt, and may under certain circumstances be larger than one volt.

To return to the quadrant electrometer, suppose that V_1 and V_2 are the potentials of the two pairs of quadrants respectively, and that V_n is the potential of the needle. If the instrument is adjusted so as to be perfectly symmetrical the formula usually quoted connecting the deflection θ with the potentials is

$$\theta = k(V_1 - V_2)[V_n - \tfrac{1}{2}(V_1 + V_2)], \qquad (1)$$

where k is a constant for small deflections. In deducing this formula any electromotive forces and differences of potential that may exist at the surfaces of the various conductors are neglected.

It is evident, however, that the potential, and therefore the electric force at points inside the space enclosed by the quadrants and outside the needle, depends upon the potentials at points in the air just outside of the various conductors and not on the potentials of the conductors themselves. Hence the mechanical forces of attraction and repulsion will depend upon the values of $V_1 + \varphi_1$,

$V_2 + \varphi_2$, and $V_n + \varphi_n$ and not, of V_1, V_2 and V_n alone, and the former should be substituted for the latter in formula (1). Hence

$$\theta = k(V_1 - V_2 + \varphi_1 - \varphi_2)[V_n + \varphi_n - \tfrac{1}{2}(V_1 + V_2 + \varphi_1 + \varphi_2)]. \quad (2)$$

If the electrometer is used heterostatically and the needle is charged to so high a potential that any changes in V_1 and V_2 that may occur are negligible in comparison with it, the quantity in square brackets is practically constant, and equation (2) becomes

$$\theta = K(V_1 - V_2 + \varphi_1 - \varphi_2). \quad (3)$$

Further, if the quadrants are all of the same material, and their surfaces in the same condition, $\varphi_1 = \varphi_2$, and

$$\theta = K(V_1 - V_2) \quad (4)$$

which is independent of the φ's.

Even if the pairs of quadrants were of different materials or their surfaces in different conditions, the Volta differences of potential would have no effect, for in order to measure a difference of potential we first connect the two pairs of quadrants together so that $V_1 - V_2 = \varphi_{21}$, and the deflection then is

$$\theta_0 = K(\varphi_1 - \varphi_2 + \varphi_{21}) \quad (5)$$

which is the practical zero. We then join wires that are of the same material to the quadrants and to the two points to be investigated, and obtain the deflection

$$\theta_1 = K(E + \varphi_{21} + \varphi_1 - \varphi_2). \quad (6)$$

Since

$$V_1 - V_2 = E + \varphi_{21}$$

where E is the desired difference of potential.

The observed deflection is

$$\theta_1 - \theta_0 = KE \quad (7)$$

and is proportional to the difference of potential E.

Hence we conclude that if the electrometer is used heterostatically, the contact potential differences have no effect and need not be corrected for.

If, however, the instrument is used idiostatically the Volta difference of potential may produce a large error. Suppose the needle is connected with the pair of quadrants (1). Usually the needle connection is made through platinum wires that dip into a concentrated solution of sulphuric acid, and if the electrodes are not exactly the same the electromotive forces at their surfaces will not exactly balance each other and there will be a resulting difference of potential produced by the electrolyte. We will assume, however, that there is no such electromotive force in the needle circuit, or better still that the connection with the needle is made entirely through metallic conductors. Under these circumstances the potential of the needle is $V_n = N_1 + \varphi_{1n}$.

Assuming the quadrants to be of the same material and, therefore, $\varphi_1 = \varphi_2$, the formula (2) becomes

$$\theta = \frac{k}{2}(V_1 - V_2)[V_1 - V_2 + 2(\varphi_n + \varphi_{1n} - \varphi_1)]. \qquad (8)$$

The expression in parenthesis $\varphi_n + \varphi_{1n} - \varphi_1$ is the Volta difference of potential between the needle and the quadrants. It appears that the deflection θ is not strictly proportional to the square of the difference of potential as is usually stated, and that if the sign of the potential difference be reversed, the deflection will change in magnitude. These conclusions can be easily tested experimentally as described below. Further it is evident that if the Volta electromotive force is neglected, and the simple formula

$$\theta = \frac{k}{2}(V_1 - V_2)^2 \qquad (9)$$

used, a large error may be made in the measurement. For example, suppose we are measuring the constant k of the instrument by means of an electromotive force of 20 volts, and that the Volta electromotive force is .5 volt (it often is larger than this); then the value of k as calculated from (9) will differ from its true value as calculated from (8) by 5 per cent.

The change in the magnitude of the deflection on reversing the sign of the potential difference $V_1 - V_2$ is not due to a lack of symmetry in the adjustment of the instrument, for the deflection is on the same side of the zero before and after the reversal. The only way to change the sign of the deflection is to join the needle to the other pair of quadrants, and when this is done we get a similar change in the magnitude of the deflection on reversing the potential difference, although in case of non-symmetry the actual magnitudes of the deflection may be much greater or less than they were before. It will be proved below that formula (8) holds true whether the instrument is exactly symmetrical or not.

The above theory was tested experimentally as follows. The bifilar suspension of an ordinary quadrant electrometer was taken out and a fine phosphor-bronze wire, through which the needle could be charged, substituted for it. A storage battery furnished a constant electromotive force of 36 volts. The apparatus was arranged according to the usual idiostatic method of measurement. It appeared that the maximum deflection always occurred on charging the quadrants connected with the needle positively. Calling θ_1 and θ_2 the maximum and minimum deflections respectively, the following numbers represent the averages of a dozen readings,

$$\theta_1 = 27.56 \text{ cm.} \quad \theta_2 = 27.06 \text{ cm.}$$

giving a difference in deflection of

$$\theta_1 - \theta_2 = .50 \text{ cm.}$$

It is well known that the Volta effect depends to a considerable extent upon the condition of the air surfaces of the metals, *i. e.*, whether they have been recently cleaned or not, etc. To test whether the difference in deflection would be altered in the same manner the needle was taken out, scraped with emery paper and replaced again. Owing to the unavoidable bending of the needle we would not expect to get the same deflection as before. Readings similar to the above gave

$$\theta_1 = 25.32 \text{ cm.} \quad \theta_2 = 22.75 \text{ cm.} \quad \theta_1 - \theta_2 = 2.57 \text{ cm.}$$

The actual deflections are slightly smaller but their difference has been increased to over five times the former difference, indicating a corresponding increase in the Volta effect. The needle was as usual made of aluminium, and this increase in the Volta effect caused by cleaning the surface of aluminium is well known.

An estimate of the magnitude of the Volta effect can be obtained as follows: Putting $V_1 - V_2 = E$ the electromotive force of the battery, formula (8) becomes

$$\theta_1 = \frac{k}{2} E[E + 2\Phi]$$

where $\Phi = \varphi_n + \varphi_{1n} - \varphi_1$ is the Volta difference of potential. Reversing the electromotive force, $V_1 - V_2 = -E$, and

$$\theta_2 = \frac{k}{2} E[E - 2\Phi].$$

By adding, subtracting and elimininating k we get

$$\Phi = \frac{E(\theta_1 - \theta_2)}{2(\theta_1 + \theta_2)}. \tag{10}$$

The quadrants of the electrometer were of brass, and the above readings substituted in formula (10) give for this particular aluminium needle, and these particular brass quadrants $\Phi = .165$ volt before cleaning and $\Phi = .962$ volt after cleaning the needle.

In order to prove that the want of symmetry in the adjustment of the electrometer is not the cause of the above-described difference in the deflections we will go back to first principles and deduce the equation for the moment of the mechanical force acting on the needle, taking account of the differences of potential at the surface of contact. As usually stated the energy of charged conductors is

$$W = \tfrac{1}{2}c_1 V_1^2 + \tfrac{1}{2}c_2 V_2^2 + \tfrac{1}{2}c_3 V_3^2 + c_{12} V_1 V_2 + c_{23} V_2 V_3 + c_{31} V_3 V_1 + \cdots \tag{11}$$

where the V's are the potentials of the conductors, and the c's the coefficients of capacity. If we use the previous notation, it is evident that the energy of the electric field will depend upon the values of the potential just outside the air surfaces of the conductors, *i. e.*, upon $V_1 + \varphi_1$, etc., and not on V_1, etc., alone. Hence the electric energy of n conductors is

$$W = \tfrac{1}{2} \sum_{p=1}^{p=n} c_p (V_p + \varphi_p)^2 + \sum c_{pq} (V_p + \varphi_p)(V_q + \varphi_q) \tag{12}$$

all possible combinations of p and q being taken once only. In the case of the quadrant electrometer

$$\begin{aligned} W = {} & \tfrac{1}{2}c_1(V_1+\varphi_1)^2 + \tfrac{1}{2}c_2(V_2+\varphi_2)^2 + \tfrac{1}{2}c_n(V_n+\varphi_n)^2 \\ & + c_{12}(V_1+\varphi_1)(V_2+\varphi_2) + c_{1n}(V_1+\varphi_1)(V_n+\varphi_n) \qquad (13) \\ & + c_{2n}(V_2+\varphi_2)(V_n+\varphi_n). \end{aligned}$$

The moment of the force Θ tending to increase the deflection θ is the derivative of W with regard to θ, the potentials being kept constant, hence

$$\begin{aligned} \Theta = {} & \tfrac{1}{2}(V_1+\varphi_1)^2\frac{\partial c_1}{\partial\theta} + \tfrac{1}{2}(V_2+\varphi_2)^2\frac{\partial c_2}{\partial\theta} + \tfrac{1}{2}(V_n+\varphi_n)^2\frac{\partial c_n}{\partial\theta} \\ & + (V_1+\varphi_1)(V_2+\varphi_2)\frac{\partial c_{12}}{\partial\theta} + (V_1+\varphi_1)(V_n+\varphi_n)\frac{\partial c_{1n}}{\partial\theta} \qquad (14) \\ & + (V_2+\varphi_2)(V_n+\varphi_n)\frac{\partial c_{2n}}{\partial\theta}. \end{aligned}$$

Assume that the quadrants are of the same material, that the needle is connected with the quadrant 1, and that the difference of potential between the quadrants is $E = V_1 - V_2$. Then

$$\varphi_1 = \varphi_2, \quad V_n = V_1 + \varphi_{1n}, \quad \text{and} \quad V_2 = V_1 - E. \qquad (15)$$

From the well-known relations between the coefficients (see Maxwell's Electricity and Magnetism, § 219)

$$\frac{\partial c_1}{\partial\theta} + \frac{\partial c_2}{\partial\theta} = \frac{\partial c_n}{\partial\theta}, \quad \frac{\partial c_1}{\partial\theta} = -\frac{\partial c_{1n}}{\partial\theta}, \quad \frac{\partial c_2}{\partial\theta} = -\frac{\partial c_{2n}}{\partial\theta}, \quad \frac{\partial c_{12}}{\partial\theta} = 0. \qquad (16)$$

Substituting (15) and (16) in (14) we get

$$\begin{aligned} \Theta_1 & = \tfrac{1}{2}\frac{\partial c_2}{\partial\theta}E[E + 2(\varphi_n + \varphi_{1n} - \varphi_1)] + \tfrac{1}{2}\left(\frac{\partial c_1}{\partial\theta} + \frac{\partial c_2}{\partial\theta}\right)(\varphi_n + \varphi_{1n} - \varphi_1)^2 \\ & \qquad (17) \\ & = \tfrac{1}{2}\frac{\partial c_2}{\partial\theta}E[E + 2\Phi] + \tfrac{1}{2}\left(\frac{\partial c_1}{\partial\theta} + \frac{\partial c_2}{\partial\theta}\right)\Phi^2; \end{aligned}$$

here Φ in the Volta potential difference as before. If the instrument is symmetrically adjusted

$$\frac{\partial c_1}{\partial\theta} = -\frac{\partial c_2}{\partial\theta}$$

and the last term of (17) vanishes.

If the sign of the electromotive force is reversed

$$\Theta_2 = \tfrac{1}{2}\frac{\partial c_2}{\partial \theta}E[E - 2\Phi] + \tfrac{1}{2}\left(\frac{\partial c_1}{\partial \theta} + \frac{\partial c_2}{\partial \theta}\right)\Phi^2$$

and the two moments differ from each other by

$$\Theta_1 - \Theta_2 = 2\frac{\partial c_2}{\partial \theta}E\Phi.$$

Hence the deflections will be different in the two cases, and this is not caused by lack of symmetry. If we had connected the other quadrants to the needle c_1 and c_2 would have been interchanged in (17), and this would have made a difference in the value of the moments only. For small deflections $\frac{\partial c_2}{\partial \theta}$ is usually constant, and in this case if the instrument is symmetrical (θ being proportional to Θ) (17) reduces to

$$\theta = \frac{k_2}{2}E[E + 2\Phi],$$

which is equation (8).

In the non-symmetrical case the last term of equation (17) simply fixes the position of the zero point as read on the scale when the quadrants were joined together and $E = 0$. So that for small deflections the zero

$$\theta_0 = \tfrac{1}{2}(k_1 + k_2)\Phi^2$$

and

$$\theta_1 = \tfrac{1}{2}k_2E[E + 2\Phi + \tfrac{1}{2}k_1 + k_2)\Phi$$

and the observed deflection from the zero is

$$\theta = \theta_1 - \theta_0 = \tfrac{1}{2}k_2E[E + 2\Phi]$$

which is exactly the same as in the symmetrical case.

A similar line of reasoning to the above will apply to every type of electrometer, and it is very important to determine the effect of the Volta potential difference unless it is definitely known that all the conductors of the instrument are of the same material and their surfaces in the same condition.

A better method of measuring the Volta effect than that described above is the following. Place a Clark cell of electromotive force e

in the needle circuit between the quadrants I and the needle. This amounts simply to increasing or decreasing the value of the difference of potential between the needle and the quadrants which before was φ_{1n}. The formula under these circumstances becomes

$$\theta_1 = \frac{k}{2} E[E + 2\Phi \pm 2e] \tag{18}$$

and the difference in deflection on reversing E is

$$\theta_1 - \theta_2 = 2kE(\Phi \pm e) = K[\Phi \pm e], \tag{19}$$

since we may assume $\frac{\partial c_1}{\partial \theta}$ to be constant over the small range of deflections $\theta_1 - \theta_2$. Experiments with the above described electrometer gave the results tabulated below.

	θ_1	θ_2	$\theta_1-\theta_2$	Φ(Volts).
No Clark cell.	24.75	22.20	2.55	.956
Positive pole to needle	26.54	20.32	6.22	.902
Negative pole to needle	22.86	24.26	−1.40	.905

The values of Φ were calculated by substituting in equation (19) and eliminating K. The temperature was 20° C., and the value of $e = 1.430$ volts.

The fact that the Volta potential difference increased enormously on simply cleaning the air surfaces of the needle while the metal contacts were left undisturbed is strong evidence for the view that most of this difference of potential is due to impressed electromotive forces in the air surfaces and not, in the metal contacts.

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A FURTHER NOTE ON THE VISIBLE RADIATION FROM CARBON.

By Edward L. Nichols and Ernest Blaker.

THE present writers have in two recent papers[1] shown the existence of a selective radiation from glowing carbon, which manifests itself in the region lying between the yellow and red. When the latter of these papers were read at the Denver meeting of the American Association for the Advancement of Science, Professor D. B. Brace suggested that this peculiarity might be connected with the anomalous dispersion of carbon noted by previous investigators. It is the object of this note to call attention to the fact that the optical properties of carbon are such as one would naturally expect to find in accompaniment with selective radiation in the given region.

While it is hardly safe to deduce from the optical behavior of a substance at ordinary temperatures its properties when heated to incandescence, it is obvious that one would expect, in so far as those properties are unchanged by the rise of temperature, that in the region where the emission curve shows a maximum there would be an absorption band and that in the neighborhood of this absorption band anomalous dispersion might be looked for. In point of fact there is evidence from measurements already published that the absorption of light by thin films of carbon is greater in the middle of the visible spectrum than in the red or violet. In 1886 one of the present writers[2] published the results of measurements upon the spectrum of two specimens of lampblack, both of which showed a greater transmitting power in the red and the violet than in the region of the D line. The maximum of absorption was found to lie in the neighborhood of the E line ($\lambda = .52\mu$). The color of lampblack was described as *a purple in which red is the dominant hue, but in which violet also greatly predominates over the wave-lengths which form the central region of the visible spectrum.* Measurements upon the age coating in incandescent lamp bulbs, made by Messrs. Moore and Ling in 1892,[3] show that the absorption of light by the deposit of carbon upon the bulb of a lamp with treated filament, after 908 hours, is greatest in the region of $.635\mu$.

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[1] Nichols: The Visible Radiation from Carbon; PHYSICAL REVIEW, Vol. XIII., page 65 (1901); also Blaker, ibid., p. 345.

[2] Nichols: Trans. of the Kansas Academy of Science, Vol. X., page 37.

[3] Nichols: American Journal of Science, Vol. XXXIV., page 277.

The curve given in Fig. 1 is plotted from their data. It is noteworthy that the maximum of absorption as indicated in this curve corresponds very closely with the maximum of radiation in the intensity curves published in our recent papers. (See Fig. 2, page 138 and Fig. 16, page 357, Vol. XIII. of the PHYSICAL REVIEW.) Knut Ångström,[1] in his paper on the transmission of heat of various wave-lengths through turbid media, explored the infra-red spectrum of lampblack. While his curves do not extend into the visible spectrum, they show the rapidly increasing opacity of this substance as the limit of visibility in the red is approached; and it is evident that we have to do here with one side of an absorption band which in the case of thick layers, at least, covers the entire spectrum. Sudden changes from opacity to transparency like that exhibited by carbon are usually indicative of anomalous dispersion and in point of fact Wood has recently[2] described measurements upon prisms of lampblack and layers of carbon deposited upon glass from the

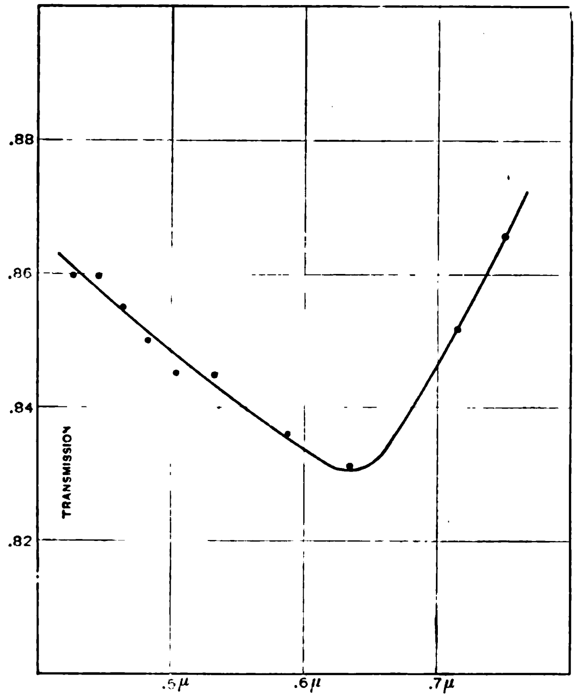

Fig. 1.

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[1] Ångström: Wiedemann's Annalen, Vol. 36, page 715 (1889).

[2] Wood: Philosophical Magazine 6, Vol. I., page 405 (1901).

filament of an incandescent lamp, which exhibited this peculiarity. It would thus appear that there is abundant experimental evidence of the existence of all those optical peculiarities in carbon which would naturally accompany the phenomenon described in our papers cited at the beginning of this note. Since the existence of a maximum of absorbing power in the middle of the visible spectrum does not appear to be universal in carbon films, neither Rosicky[1] nor Stark[2] having noted this peculiarity in their spectrophotometric studies of lampblack and Moore and Ling in their measurement of the age coating in incandescent lamps having found this maximum in a coating from a treated filament, but not in that from an untreated lamp, it has seemed worth while to make further measurements both of the absorption and dispersion of carbon.

The present writers have in hand an investigation of this character the results of which are not yet ready for presentation; but it may be stated in this connection that the spectrophotometric study of a film of carbon deposited upon glass by the method described by Wood gave indications of maximum of absorption in the region .52 μ and that measurements of the dispersion of a prism of asphalt, the optical properties of which substance, as will be shown in a forthcoming paper, are so nearly identical with those of carbon as to lead to the conclusion that the color of this substance is due to carbon particles suspended in a translucent medium consisting of hydrocarbons, gave the following results:

TABLE I.

Anomalous Dispersion in Asphalt.

Wave-Length.	Index of Refraction.
.6708 μ	1.6209
.6104 μ	1.6282
.5896 μ	1.6351
.5682 μ	1.6339

These indices are much smaller than those found for carbon by Stark and by Wood as was to have been expected in the case of what may perhaps be regarded as a solid solution of carbon in a medium of smaller refractive power. The measurements would seem to indicate that the maximum of the absorption band corresponding to this region of anomalous dispersion occurs at a somewhat shorter wave-length than in the case of the carbon film studied by Moore and Ling and of the carbons used by the present writers in their measurements upon the emissive power of that substance.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
Nov. 27, 1901.

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[1] Rosicky: Wiener Berichte, 78, II., page 407 (1878).

[2] Stark: Wiedemann's Annalen, 62, page 351.

NEW BOOKS.

High Temperature Measurements. By H. LE CHATELIER and O. BOUDOUARD. Translated by GEORGE K. BURGESS. New York, John Wiley & Sons, 1901. Pp. xi + 230.

Mr. Burgess in this volume has given us an excellent translation of the well-known work of Le Chatelier and Boudouard and he has in addition brought the subject up to date by means of a chapter of his own entitled "Recent Developments," in which the latest advances in pyrometry have been recorded. It is a great convenience to workers in this field to have in convenient form such data as those upon the expansion of nitrogen by Chappius and Harker and the results of Holborne and Day upon air thermometers. The establishment of the fact that bulbs of platinum-iridium although permeable to hydrogen are impervious to nitrogen even at a white heat is of great importance. The determinations of these authors of the melting points of silver, gold, and copper would seem to leave but little to be desired, and it is a matter of gratification to find that the art of making the platinum metals has now reached a stage in which thermo elements for high temperatures give consistent results. The bibliographical index at the close of Mr. Burgess's translation is much fuller than that in the original French edition of this work.

E. L. N.

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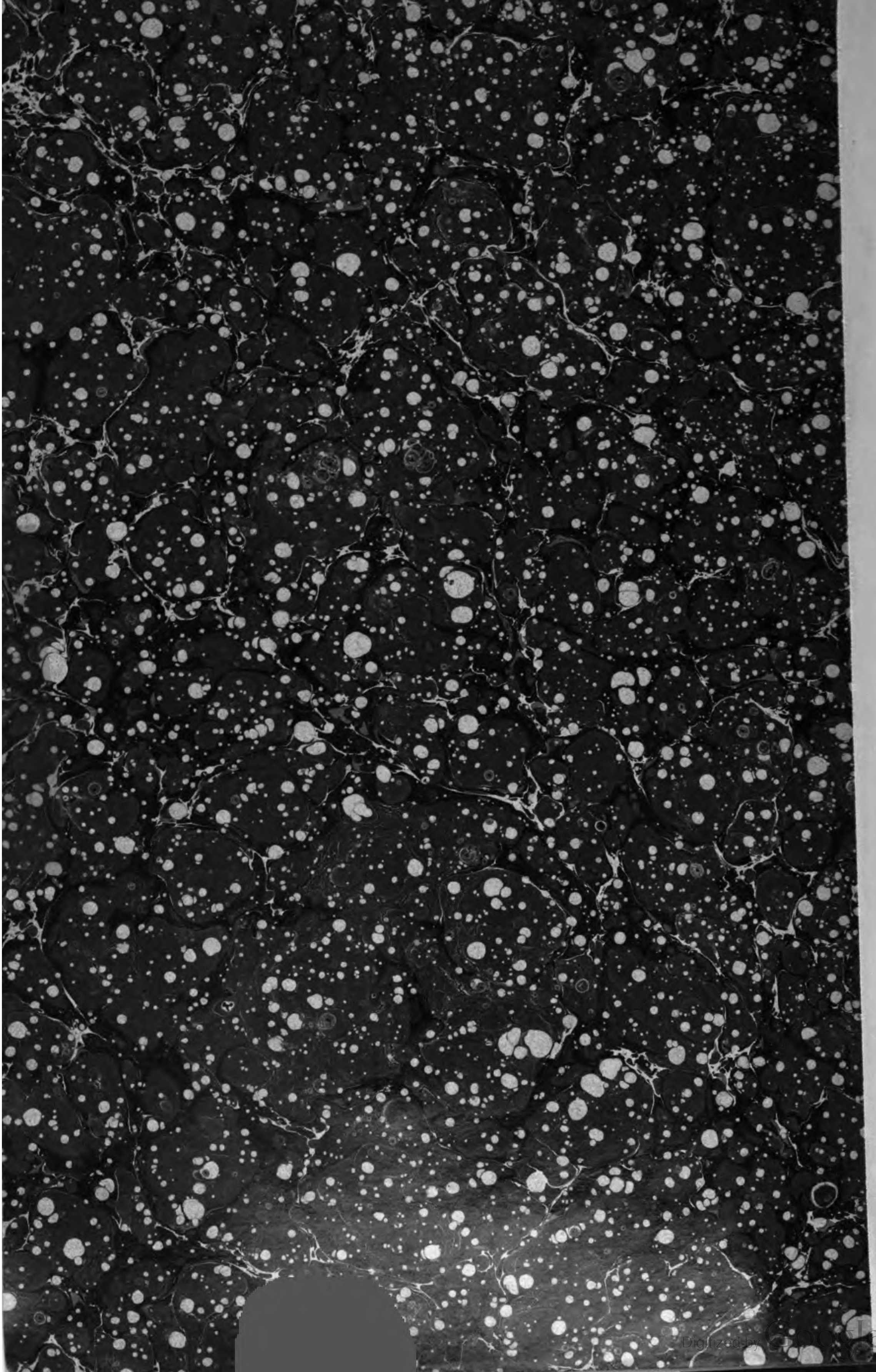

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